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BARYONS AND MESONS IN A THEORY

WHICH COMBINES RELATIVISTIC INVARIANCE WITH $SU(6)$ SYMMETRY

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I. INTRODUCTION

In this paper we study the meson-baryon vertex. We assume that the pseudoscalar octet and the vector-meson nonet fill a 35-dimensional representation of the group $SU(6)$ whereas the baryon octet together with the decuplet of spin $\frac{3}{2}$ baryon resonances form a 56-dimensional representation. This vertex has already been examined in the first applications of the group $SU(6)$ to the theory of strongly interacting particles, Refs. ¹⁾⁻⁵⁾. Later it turned out ⁶⁾ that some of the earlier results, as the ratio D/F for the coupling of pions to baryons and the ratio of the magnetic moments of the nucleons, can be calculated only if a first-order symmetry breaking is taken into account. In these cases the breaking of the symmetry is due to the kinetic energy.

Therefore it seems to be worth while to consider a group theoretic model which is Lorentz invariant from the outset. Such a model does not distinguish between spin and orbital angular momentum and allows for an arbitrarily large recoil. As final results we can derive relations between invariant functions and not only relations between coupling constants. These results hold strongly within the limits of validity of the symmetry.

We discuss an example of such a Lorentz invariant theory which was studied with respect to some of its mathematical properties in a preceding paper ⁷⁾. We call the underlying invariance group $G_{>}$. Some of its features and of our definitions shall now be resumed.

- (1) The group $G_{>}$ is an inhomogeneous group, the homogeneous part of which is $SL(6, C)$. The invariant subgroup of $G_{>}$ consists of translations in two dual 36-dimensional spaces.
- (2) A parity transformation interchanges the role of these dual spaces.
- (3) We are concerned only with a certain subset of irreducible unitary representations of $G_{>}$. They are characterized by a condition between the two dual momenta $p, \tilde{p}$ (which are written as 6×6 matrices): $p \cdot \tilde{p}$ is proportional to the identity matrix. This condition determines the corresponding little group to be a form of $SU(6)$, if the mass does not vanish.

2.

- (4) Physical states have vanishing values for all those momentum components which are not scalars with respect to transformations of $SU(3)$.

The last assumption serves to exclude orbital contributions to the isospin.

The way of our calculation is as follows. We start with an expression for the vertex operator which is invariant with respect to the group G_7 . Then we rewrite it in such a way that its decomposition into irreducible parts under the subgroup $SL(2,C) \otimes SU(3)$ becomes apparent. Because we assume the two baryons to be free physical particles we need account only for that contribution of the momentum matrices of all the three particles which transforms as a scalar under the group $SU(3)$.

In Sections II and III we demonstrate how to describe mesons and baryons in the group G_7 . In Section IV we discuss the different types of invariant interactions between mesons and baryons. All our results are contained in the tables 1-5, for both intrinsic meson parities η . A discussion of these results can be found in Section V. Some convenient formulae are listed in an appendix.

II. THE MESON FIELD

The meson field (see Ref. ⁷⁾) is described by a $SL(6,0)$ spinor field of degree two which obeys the following four pairs of differential equations

$$(a) \quad p_{ab} \psi^{bc} = \kappa \psi_a^c, \quad \tilde{p}^{ab} \psi_b^c = \kappa \psi^{ac},$$

$$(b) \quad p_{ab} \psi_c^a = \kappa \psi_{cb}, \quad \tilde{p}^{ab} \psi_{ca} = \kappa \psi_c^b,$$

$$(c) \quad p_{cb} \psi^{ac} = \kappa \psi^a_b, \quad \tilde{p}^{bc} \psi_a^b = \kappa \psi^{ac},$$

$$(d) \quad p_{cb} \psi^b_a = \kappa \psi_{ca}, \quad \tilde{p}^{ab} \psi_{bc} = \kappa \psi^a_c.$$

We assume in addition the hermiticity of ψ^{ab} :

$$\begin{aligned} \overline{(\psi^{ab})} &= \bar{\psi}^{ab} = \psi^{*ba} \\ &= \psi^{ba} \end{aligned}$$

and that the trace of ψ_a^b and ψ^a_b vanishes:

$$\psi_a^a = \psi^a_a = 0.$$

4.

Together with the latter conditions the four pairs of field equations determine one irreducible representation of the group G_2 . The vanishing of the trace $\gamma_{\dot{a}}^{\dot{a}}$ implies a compatibility equation

$$p_{ab} \gamma^{ba} = 0,$$

which we call the "transversality condition". It means that the thirty-sixth component of the meson field, namely the $SU(3)$ trace of the longitudinal field, vanishes.

If we consider only the subgroup $SL(2, \mathbb{C}) \otimes SU(3)$ of $SL(6, \mathbb{C})$ we can write

$$\gamma^{ab} = \sigma_{\mu}^{\alpha\beta} \left[\frac{p^{\mu i}_k}{\kappa} L^k_j + T^{\mu i}_j \right]$$

The pairs of indices (α, β) and (i, j) correspond to (a, b) . L^k_j is the longitudinal, $T^{\mu i}_j$ the transversal meson field:

$$\tilde{p}^{\mu i}_k T^{\mu k}_j = 0$$

We define a physical state as such a state that

$$p^{\mu i}_k = p^{\mu} \delta^i_k.$$

At a three-particle vertex only two particles need to be physical to eliminate all non-physical momenta of the third particle even if this is unphysical (e.g., virtual). We assume that the baryons are physical states. We have then

$$\psi^{ab} = \sigma_{\mu}^{\alpha\beta} \left[\frac{p^{\mu}}{\chi} L_j^i + T^{\mu} i_j \right]$$

with

$$T^{\mu} = \sum_{A=1}^3 n_{(A)}^{\mu} T_{(A)}$$

$$p^{\mu} n_{\mu(A)} = 0, \quad n_{\mu(A)} n^{\mu}_{(A)} = -1.$$

In this way we have connected a scalar and a vector field with the field ψ^{ab} . What is the meaning of the other fields $\psi_{\dot{a}}^{\dot{b}}$, $\psi_{\dot{a}b}^a$, $\psi_{\dot{a}b}$?

We shall not answer this question completely here but give only some arguments.

We examine the expressions

$$a_{\mu\nu}^{(1)} i_j = +\frac{1}{2} \sigma_{\mu}^{\gamma\alpha} \sigma_{\nu}^{\beta\delta} \psi_{\alpha}^{\beta} i_j^{\delta}$$

and

$$a_{\mu\nu}^{(2)} i_j = +\frac{1}{2} \sigma_{\mu}^{\gamma\alpha} \sigma_{\nu}^{\beta\delta} \psi_{\beta}^{\alpha} i_j^{\delta}$$

6.

Introducing the field ψ^{ab} by means of the field equations and assuming for simplicity $p_\mu^i = p_\mu \cdot \delta^i_j$, we get:

$$\kappa \cdot a_{\mu\nu}^{(1) i}{}_j = \frac{1}{2} \sigma_\mu^{\gamma\dot{\alpha}} \sigma_\nu^{\beta\dot{\delta}} p_\lambda \sigma^\lambda_{\dot{\alpha}\delta} \psi^{\delta\beta, i}{}_j,$$

$$\kappa \cdot a_{\mu\nu}^{(2) i}{}_j = \frac{1}{2} \sigma_\mu^{\gamma\dot{\alpha}} \sigma_\nu^{\beta\dot{\delta}} p_\lambda \sigma^\lambda_{\dot{\delta}\beta} \psi^{\alpha\delta, i}{}_j.$$

we decompose ψ^{ab} into a longitudinal and a transversal part and defining tensor fields by

$$F_{\mu\nu}^i{}_j = p_\mu T_\nu^i{}_j - p_\nu T_\mu^i{}_j,$$

$$\tilde{F}_{\mu\nu}^i{}_j = \varepsilon_{\mu\nu\lambda\kappa} p^\lambda T^\kappa{}^i{}_j,$$

We arrive finally at (we use formula (5) of the appendix)

$$\kappa a_{\mu\nu}^{(1) i}{}_j = \kappa g_{\mu\nu} L^i{}_j + F_{\mu\nu}^i{}_j - i \tilde{F}_{\mu\nu}^i{}_j,$$

$$\kappa a_{\mu\nu}^{(2) i}{}_j = \kappa g_{\mu\nu} L^i{}_j - F_{\mu\nu}^i{}_j - i \tilde{F}_{\mu\nu}^i{}_j.$$

These expressions are an obvious generalization of the well-known results for the Lorentz group, where ψ^α_β and $\psi_{\dot{\alpha}}^{\dot{\beta}}$ are equivalent to self-dual and anti-self-dual tensors, respectively.

The interpretation of the remaining field $\psi_{\dot{a}b}$ is peculiar for our definition of parity. As shown in Ref. 7) it holds

$$\begin{aligned}\psi_{\dot{a}b} &= \sigma_{\mu\dot{\alpha}\beta} \left[\frac{p^\mu}{\pi} L^i_j - T^\mu{}^i_j \right] \\ &= -g_{\mu\mu} \sigma_{\mu\dot{\alpha}\beta} \left[\frac{p^\mu}{\pi} L^i_j - T^\mu{}^i_j \right].\end{aligned}$$

A parity transformation changes upper indices into lower ones and adds or takes off the points. So we have for a parity transformation

$$\psi^{ab} \longrightarrow -\eta \psi_{\dot{a}\dot{b}}$$

or

$$\begin{aligned}L^i_j &\longrightarrow +\eta \cdot L^i_j \\ T^\mu{}^i_j &\longrightarrow -\eta \cdot g_{\mu\mu} \cdot T^\mu{}^i_j.\end{aligned}$$

where η is the intrinsic parity of the mesons.

III. THE BARYON FIELD

We introduce the notation

$$\psi_{a_1 a_2 \dots a_n, b_1 b_2 \dots b_m, c_1 c_2 \dots c_p, d_1 d_2 \dots d_q} = \psi_{p, q}^{(n, m)}.$$

and neglect for the moment symmetrization and traces. The momenta p_{ab} and $\tilde{p}^{ab}$ can be written as $p_{11}^{(00)}$ and $\tilde{p}_{00}^{(11)}$. Using this notation we can ascribe a finite network of symbols $\binom{n, m}{p, q}$ to a certain field and to all its derivatives by means of p and $\tilde{p}$:

$$\begin{array}{ccc} \binom{n+1}{p} \binom{m}{q-1} & & \binom{n}{p-1} \binom{m+1}{q} \\ & \binom{n}{p} \binom{m}{q} & \\ \binom{n}{p+1} \binom{m-1}{q} & & \binom{n-1}{p} \binom{m}{q+1} \end{array}$$

where only non-negative integers are allowed. Each connecting line means a pair of field equations. In this way the meson field can be represented by

$$\begin{array}{ccccc}
 & & \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} & & \\
 & \swarrow & & \searrow & \\
 \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} & & & & \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} \\
 & \searrow & & \swarrow & \\
 & & \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} & &
 \end{array}$$

and for the baryons we choose the representation

$$\begin{array}{ccccc}
 \begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix} & & & & \\
 & \searrow & & & \\
 & & \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} & & \\
 & & & \searrow & \\
 & & & & \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \\
 & & & & \searrow \\
 & & & & & \begin{pmatrix} 0 & 0 \\ 0 & 3 \end{pmatrix}
 \end{array}$$

In order to get an irreducible representation for the baryons we symmetrize $\psi \begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix}$ and assume the validity of all the compatibility equations which are a consequence of this symmetry. Under the subgroup $SL(2, \mathbb{C}) \otimes SU(3)$ this representation can be decomposed according to

$$\begin{aligned}
 \psi^{abc} = & D^{\alpha\beta\gamma} \psi_{ijk} + \\
 & + (18)^{-\frac{1}{2}} \sum_{\substack{\alpha, \beta, \gamma \\ i, j, k \text{ cycl.}}} \epsilon^{\alpha\beta} \epsilon^{ijm} N^{\gamma, k}_m.
 \end{aligned}$$

The field D describes the decuplet of spin $-\frac{3}{2}$ baryon resonances, whereas N represents the baryon octet, $N^{\gamma, k}_k = 0$.

We are interested only in the baryon part. Together with another spin $\frac{1}{2}$ baryon octet representation $M_{\gamma}^k, N_{\gamma}^k$ makes up a Dirac spinor

$$B_m^k = \begin{pmatrix} M_{\gamma}^k \\ N_{\gamma}^k \end{pmatrix}_m.$$

We shall express our results in terms of the Dirac spinors $B, \tilde{B}$ and the vectors or scalars T, L .

IV. DIFFERENT TYPES OF INVARIANT COUPLINGS BETWEEN BARYONS AND MESONS

We use the notation Ψ for the baryons and φ for the mesons. We consider only parity conserving coupling terms. The meson field appears therefore in the coupling invariant either as the pair $\varphi^{ab}, \varphi_{ab}$ or as the pair φ_a^b, φ_b^a . Invariant forms which involve the first pair are named by us "direct couplings", if they involve the second pair "magnetic couplings". In each case there are three different possibilities.

$$\begin{aligned}
 d_1: & \bar{\Psi}_1^a{}_{bc} \Psi_2^c{}_{da} \varphi^{bd} - \eta \bar{\Psi}_1^a{}_{bc} \Psi_2^c{}_{da} \varphi^{bd}, \\
 d_2: & \bar{\Psi}_1^a{}_{bc} \Psi_2^c{}_{da} \varphi^{bd} - \eta \bar{\Psi}_1^a{}_{bc} \Psi_2^c{}_{da} \varphi^{bd}, \\
 d_3: & \bar{\Psi}_1^a{}_{bc} \Psi_2^c{}_{da} \varphi^{bd} - \eta \bar{\Psi}_1^a{}_{bc} \Psi_2^c{}_{da} \varphi^{bd}, \\
 m_1: & \bar{\Psi}_1^a{}_{bc} \Psi_2^d{}_{bc} \varphi_a^d - \eta \bar{\Psi}_1^a{}_{bc} \Psi_2^d{}_{bc} \varphi_a^d, \\
 m_2: & \bar{\Psi}_1^a{}_{bc} \Psi_2^d{}_{bc} \varphi_a^d - \eta \bar{\Psi}_1^a{}_{bc} \Psi_2^d{}_{bc} \varphi_a^d, \\
 m_3: & \bar{\Psi}_1^a{}_{bc} \Psi_2^d{}_{bc} \varphi_a^d - \eta \bar{\Psi}_1^a{}_{bc} \Psi_2^d{}_{bc} \varphi_a^d.
 \end{aligned}$$

We distinguish between the baryons Ψ_1 and Ψ_2 with the masses K_1 and K_2 . The symbol η is the intrinsic parity of the mesons.

In our calculations we proceed by substituting $\bar{\Psi}_1^{abc}$ and Ψ_2^{abc} for the different baryon spinors appearing in the coupling. This can be done by means of the field equations. The "transversality" of the meson field (remember: the elimination of the thirty-sixth component!) is conserved only by d_1 for $-\eta = +1$ and some linear combinations of d_2 and d_3 or m_1 and m_2 .

V. DISCUSSION OF THE RESULTS FOR THE PARITY $\eta = -1$

We consider only the physically more interesting case $\eta = -1$. We define a "static limit" as:

$$K_1 = K_2, \\ \kappa^2 = 0.$$

in the form factors ^{*)}. Then all the direct couplings reduce to one form, the magnetic couplings to another one. We get for the direct coupling type

<u>d:</u>	$\frac{i}{\kappa} \tilde{B}_1 \gamma_5 B_2$	$\tilde{B}_1 \gamma_\mu B_2$	$\frac{-i}{2\kappa} \tilde{B}_1 \sigma_{\mu\nu} B_2 (p_1^\nu - p_2^\nu)$
D	-12K	0	+6
F	-8K	+6	-2
singlet		+6	-4

and for the other type

<u>m:</u>	$+\frac{i}{2\kappa} \tilde{B}_1 \sigma_{\mu\nu} B_2 (p_1^\nu - p_2^\nu)$
D	+12
F	+8
singlet	+4

^{*)} We make no assumption with respect to the sign of the energy of the baryons. Our results are valid for baryons and antibaryons (see Section VI).

If we add both invariants by use of a parameter γ in the form

$$d + \gamma \cdot \frac{\pi}{K} \cdot m$$

we get

- (i) the ratio D/F for pions is $\frac{3}{2}$.
- (ii) The vector field is not coupled to the D type current of the form $\tilde{B}_1 \gamma_\mu B_2$.
- (iii) The anomalous magnetic moments of proton and neutron can be written as

$$\left. \begin{aligned} \mu_p &= -2\gamma \\ \mu_n &= -\frac{2}{3} + \frac{4}{3}\gamma \end{aligned} \right\} \begin{aligned} &= +1.80 \\ &= -1.87 \end{aligned} \quad \text{for } \gamma = -0.9$$

Eliminating γ we get the relation

$$\frac{1 + \mu_p}{\mu_n} = -\frac{3}{2}$$

In this way we have rederived some well-known formulae which are due to the SU(6) symmetry. Let us turn our attention to the more general case of arbitrary values for the recoil π^2 .

Introducing the parameters $\alpha_i, \beta_i, i = 1, 2, 3$ which depend on κ^2 , we can write the coupling as

$$\sum_{i=1}^3 (\alpha_i d_i + \beta_i \cdot \frac{2\kappa}{K_1 + K_2} m_i)$$

If the baryon masses are equal, $K_1 = K_2$, we have

$$d_2 = d_3$$

and four different ratios of parameters remain to be determined. If we postulate that the D type current of the form $\tilde{B}_1 \gamma_\mu B_2$ should vanish identically we have a condition

$$-\alpha_1 + 2\beta_1 + \beta_2 = 0, \quad (K_1 = K_2).$$

Certainly the information on the baryon resonances can also be used to gain knowledge about the parameters α_i, β_i .

We consider here the simple model

$$\alpha_1 = \beta_1 = \beta_2 = 0, \quad \gamma = \frac{\beta_3}{\alpha_2 + \alpha_3}.$$

In this case we get the following results:

(α): the ratio D/F for pions is

$$\frac{12K^2 - 3\pi^2}{8K^2 + \pi^2(1 - 6\gamma)}$$

In the limit $\pi^2 \rightarrow -\infty$ we get $D/F \rightarrow -3$, (see below).

($\alpha\alpha$): The electromagnetic properties of the nucleons are described by the form factors

$$F_{2p} = -2\gamma \cdot F_{1p},$$

$$F_{1n} \equiv 0,$$

$$F_{2n} = -\frac{2}{3}(F_{1p} + F_{2p}).$$

If

$$\lim_{\pi^2 \rightarrow -\infty} \pi^2 \frac{F_{2p}}{F_{1p}} = -2K^2 \gamma_{\infty}, \quad |\gamma_{\infty}| < \infty,$$

we get

$$\lim_{\pi^2 \rightarrow -\infty} \frac{G_{m,p}}{G_{c,p}} = \frac{1}{1 - \frac{1}{2}\gamma_{\infty}}.$$

G_m and G_c are the well-known combinations

$$G_m = F_1 + F_2,$$

$$G_c = F_1 + \frac{\pi^2}{4K^2} F_2.$$

VI. CONCLUSION

Let us state some of our most important physical predictions. Writing F_1 and F_2 for the form factors belonging to $\tilde{B}_1 \gamma_\mu B_2$ and $-\frac{i}{2K} \tilde{B}_1 \sigma_{\mu\nu} B_2 (\vec{p}_1 - \vec{p}_2)$ and then forming the linear combinations G_c and G_m as above we get the results:

$$K_1 = K_2 = K, \quad \kappa^2 = 0:$$

$$\left(\frac{D}{F}\right)_{\text{Pions}} = \frac{3}{2}, \quad G_c^D : G_c^F : G_c^{\text{singlet}} = 0 : 1 : 1$$

$$K_1 = K_2 = K, \quad \kappa^2 = 4K^2:$$

$$\left(\frac{D}{F}\right)_{\text{Pions}} = 0, \quad G_c^D : G_c^F : G_c^{\text{singlet}} = 3 : 2 : 1 \quad *)$$

For every value of κ^2, K_1, K_2

$$G_m^D : G_m^F : G_m^{\text{singlet}} \equiv 3 : 2 : 1.$$

This means that the magnetic form factors of nucleons and hyperons are all proportional, the constant being fixed by the static limit.

These results show that our theory is preferable to the spurion model for several reasons!

*) Similar results have been derived in another model by N. Cabibbo and M. Veltman (private communication).

Our theory is crossing symmetric, that means that all channels can be calculated from the same analytic expression. No ad hoc assumption concerning the introduction of a spurion in one channel but not in the other need to be made. No additional amplitudes appear as a consequence of the spurion.

We are grateful to Professors J. Prentki and D. Amati for some interesting discussions.

A P P E N D I X

We write down some convenient formulae for the σ matrices, σ^k :
 Pauli matrices, $\sigma^0 = 1$, $\tilde{\sigma}^\mu = -g^{\mu\nu} \sigma_\nu$,

$$(1) \quad \sigma_\mu^{\alpha\beta} \sigma_\nu^{\dot{\beta}\delta} + \sigma_\nu^{\alpha\beta} \sigma_\mu^{\dot{\beta}\delta} = -2g_{\mu\nu} \delta^{\alpha\delta},$$

$$\sigma_\mu^{\dot{\alpha}\beta} \sigma_\nu^{\beta\dot{\delta}} + \sigma_\nu^{\dot{\alpha}\beta} \sigma_\mu^{\beta\dot{\delta}} = -2g_{\mu\nu} \delta_{\dot{\alpha}\dot{\delta}}.$$

In matrix notation we write these formulae as

$$(2) \quad \tilde{\sigma}_\mu \sigma_\nu + \tilde{\sigma}_\nu \sigma_\mu = -2g_{\mu\nu},$$

$$\sigma_\mu \tilde{\sigma}_\nu + \sigma_\nu \tilde{\sigma}_\mu = -2g_{\mu\nu},$$

$$(3) \quad \sigma_\mu^{\dot{\alpha}\beta} \sigma_\nu^{\beta\dot{\alpha}} = \text{Sp}(\sigma_\mu \tilde{\sigma}_\nu) = -2g_{\mu\nu},$$

$$(4) \quad \sigma_\mu^{\dot{\alpha}\beta} \sigma^\mu_{\dot{\gamma}\delta} = -2(\delta_{\dot{\alpha}\delta} \delta_{\beta\dot{\gamma}} - \delta_{\dot{\alpha}\dot{\gamma}} \delta_{\beta\delta}),$$

$$(5) \quad \text{Sp}(\sigma_\mu \tilde{\sigma}_\nu \sigma_\lambda \tilde{\sigma}_\kappa) = 2[g_{\mu\nu} g_{\lambda\kappa} + g_{\mu\kappa} g_{\nu\lambda} - g_{\mu\lambda} g_{\nu\kappa} + i \varepsilon_{\mu\nu\lambda\kappa}].$$

We introduce the matrix $\tilde{\varepsilon}$ with the components $\varepsilon^{\alpha\beta}$ or $\varepsilon^{\dot{\alpha}\dot{\beta}}$,
 $\tilde{\varepsilon} = i\sigma^2$. We get:

$$(6) \quad \tilde{\varepsilon} \sigma_\mu^T \tilde{\varepsilon} = \tilde{\sigma}_\mu.$$

R E F E R E N C E S

- 1) F. Gürsey and L.A. Radicati, Phys. Rev. Letters 313, 173 (1964).
- 2) A. Pais, Phys. Rev. Letters 13, 175 (1964).
- 3) F. Gürsey, A. Pais and L.A. Radicati, Phys. Rev. Letters 13, 299 (1964).
- 4) B. Sakita, University of Wisconsin preprint, August 1964.
- 5) B. Sakita, Argonne National Laboratory preprint, September 1964.
- 6) M. Gell-Mann, Calif. Institute of Techn. preprint, November 1964.
- 7) W. Rühl, CERN preprint, December 1964.

$\eta = +1$	$+\frac{1}{\kappa} \tilde{B}_1^B B_2$	$+\tilde{B}_1 \gamma_\mu \gamma_5 B_2$	$-\frac{i}{\kappa_1 + \kappa_2} \tilde{B}_1 \sigma_{\mu\nu} \gamma_5 B_2 (p_1^\nu - p_2^\nu)$
D	$\bar{\Gamma} \left[\frac{3}{2} \kappa^2 + \frac{3}{2} (\kappa_1 - \kappa_2)^2 \right] \bar{\Gamma} (\kappa_1 - \kappa_2)$	$+3\kappa^2 + \frac{3}{2} (\kappa_1 - \kappa_2)^2 - 3(\kappa_1^2 + \kappa_2^2)$	$-\frac{3}{2} (\kappa_1^2 - \kappa_2^2)$
F	$\bar{\Gamma} \left[\frac{5}{2} \kappa^2 - \frac{1}{2} (\kappa_1 - \kappa_2)^2 + 3(\kappa_1^2 + \kappa_2^2) \right] \bar{\Gamma} (\kappa_1 - \kappa_2)$	$+2\kappa^2 - \frac{1}{2} (\kappa_1 - \kappa_2)^2 - 2(\kappa_1^2 + \kappa_2^2)$	$+\frac{1}{2} (\kappa_1^2 - \kappa_2^2)$
Singlet		$+\kappa^2 - (\kappa_1 - \kappa_2)^2 - (\kappa_1^2 + \kappa_2^2)$	$+(\kappa_1^2 - \kappa_2^2)$

$\eta = -1$	$+\frac{i}{\kappa} \tilde{B}_1 \gamma_5 B_2$	$\tilde{B}_1 \gamma_\mu B_2$	$-\frac{i}{\kappa_1 + \kappa_2} \tilde{B}_1 \sigma_{\mu\nu} B_2 (p_1^\nu - p_2^\nu)$
D	$\bar{\Gamma} \left[\frac{3}{2} \kappa^2 - \frac{3}{2} (\kappa_1 + \kappa_2)^2 \right] \bar{\Gamma} (\kappa_1 + \kappa_2)$	$-3\kappa^2 - \frac{3}{2} (\kappa_1 + \kappa_2)^2 + 3(\kappa_1^2 + \kappa_2^2)$	$+\frac{3}{2} (\kappa_1 + \kappa_2)^2$
F	$\bar{\Gamma} \left[\frac{5}{2} \kappa^2 + \frac{1}{2} (\kappa_1 + \kappa_2)^2 - 3(\kappa_1^2 + \kappa_2^2) \right] \bar{\Gamma} (\kappa_1 + \kappa_2)$	$-2\kappa^2 + \frac{1}{2} (\kappa_1 + \kappa_2)^2 + 2(\kappa_1^2 + \kappa_2^2)$	$-\frac{1}{2} (\kappa_1 + \kappa_2)^2$
Singlet		$-\kappa^2 + (\kappa_1 + \kappa_2)^2 + (\kappa_1^2 + \kappa_2^2)$	$-(\kappa_1 + \kappa_2)^2$

T A B L E 1

The coupling of mesons to baryons according to the scheme of d_1 .
A common factor in front of these expressions is $-(18\kappa_1 \kappa_2)^{-1}$.

$\gamma = +1$	$\frac{1}{\chi} \tilde{B}_1^B$	$i \tilde{B}_1 \gamma_\mu \gamma_5 B_2$	$-\frac{i}{K_1 + K_2} \tilde{B}_1 \sigma_{\mu\nu} i \gamma_5 B_2 (p_1^\nu - p_2^\nu)$
D	$3[-\chi^2 + (K_1 - K_2)^2]$	$-3(K_1 + K_2)$	$-3(K_1 + K_2)$
F	$[-\chi^2 + (K_1 - K_2)(5K_1 + K_2)]$	$-5K_1 + K_2$	$+(K_1 + K_2)$
Singlet		$-4K_1 + 2K_2$	$+2(K_1 + K_2)$

$\gamma = -1$	$\frac{i}{\chi} \tilde{B}_1 \gamma_5 B_2$	$\tilde{B}_1 \gamma_\mu B_2$	$-\frac{i}{K_1 + K_2} \tilde{B}_1 \sigma_{\mu\nu} B_2 (p_1^\nu - p_2^\nu)$
D	$-3[-\chi^2 + (K_1 + K_2)^2]$	$+3K_1 - 3K_2$	$+3(K_1 + K_2)$
F	$[-\chi^2 + (K_1 + K_2)(5K_1 - K_2)]$	$+5K_1 + K_2$	$-(K_1 + K_2)$
Singlet		$+4K_1 + 2K_2$	$-2(K_1 + K_2)$

T A B L E 2

The coupling scheme d_2 . A common factor is $-(18K_1)^{-1}$. The coupling scheme d_3 is similar: we need only interchange the indices 1 and 2 and then take the complex conjugate expression.

$\eta = +1$	$+\frac{1}{\chi} \widetilde{B}_1 B_2$	$+i \widetilde{B}_1 \gamma_\mu \gamma_5 B_2$	$-\frac{i}{K_1+K_2} \widetilde{B}_1 \sigma_{\mu\nu} i \gamma_5 B_2 (p_1^\nu - p_2^\nu)$
D	$+3\chi^2 [\chi^2 - (K_1 - K_2)^2]$	$-3\chi^2 (K_1 - K_2)$	$-(-3\chi^2 + 6K_1 K_2)(K_1 + K_2)$
F	$+\chi^2 [-\chi^2 + (K_1 - K_2)^2 + 6K_1 K_2]$	$+\chi^2 (K_1 - K_2)$	$-(+\chi^2 + 4K_1 K_2)(K_1 + K_2)$
Singlet		$+2\chi^2 (K_1 - K_2)$	$-(-2\chi^2 + 2K_1 K_2)(K_1 + K_2)$

$\eta = -1$	$+\frac{1}{\chi} \widetilde{B}_1 \gamma_5 B_2$	$+\widetilde{B}_1 \gamma_\mu B_2$	$-\frac{i}{K_1+K_2} \widetilde{B}_1 \sigma_{\mu\nu} B_2 (p_1^\nu - p_2^\nu)$
D	$+3\chi^2 [\chi^2 - (K_1 + K_2)^2]$	$-3\chi^2 (K_1 + K_2)$	$+(3\chi^2 + 6K_1 K_2)(K_1 + K_2)$
F	$+\chi^2 [-\chi^2 + (K_1 + K_2)^2 - 6K_1 K_2]$	$+\chi^2 (K_1 + K_2)$	$+(-\chi^2 + 4K_1 K_2)(K_1 + K_2)$
Singlet		$+2\chi^2 (K_1 + K_2)$	$+(-2\chi^2 + 2K_1 K_2)(K_1 + K_2)$

T A B L E 3

The coupling scheme m_1 . The common factor is $(18K_1 K_2 \chi)^{-1}$.

$\eta = +1$	$+ \frac{1}{\alpha} \widetilde{B}_1^B \gamma_{52}^B$	$+ \widetilde{B}_1 \gamma_{\mu} \gamma_{52}^B$	$- \frac{i}{K_1 + K_2} \widetilde{B}_1 \sigma_{\mu\nu} i \gamma_{52}^B (p_1^\nu - p_2^\nu)$
D	$+ \frac{3}{2} \alpha^2 \widetilde{[-\alpha^2 - (K_1 - K_2)^2]} \widetilde{]}$	$+ \frac{3}{2} \alpha^2 (K_1 - K_2)$	$+ (\frac{3}{2} \alpha^2 - 3K_1^2 - 3K_2^2) (K_1 + K_2)$
F	$+ \frac{\alpha^2}{2} \widetilde{[-5\alpha^2 - (K_1 - K_2)^2 + 6(K_1^2 + K_2^2)]} \widetilde{]}$	$- \frac{1}{2} \alpha^2 (K_1 - K_2)$	$+ (\frac{5}{2} \alpha^2 - 2K_1^2 - 2K_2^2) (K_1 + K_2)$
Singlet		$- \alpha^2 (K_1 - K_2)$	$+ (2\alpha^2 - K_1^2 - K_2^2) (K_1 + K_2)$

$\eta = -1$	$+ \frac{i}{\alpha} \widetilde{B}_1^B \gamma_{52}^B$	$+ \widetilde{B}_1 \gamma_{\mu} \gamma_{52}^B$	$- \frac{i}{K_1 + K_2} \widetilde{B}_1 \sigma_{\mu\nu} B_2 (p_1^\nu - p_2^\nu)$
D	$- \frac{3}{2} \alpha^2 \widetilde{[-\alpha^2 + (K_1 + K_2)^2]} \widetilde{]}$	$- \frac{3}{2} \alpha^2 (K_1 + K_2)$	$- (\frac{3}{2} \alpha^2 - 3K_1^2 - 3K_2^2) (K_1 + K_2)$
F	$- \frac{\alpha^2}{2} \widetilde{[-5\alpha^2 - (K_1 + K_2)^2 + 6(K_1^2 + K_2^2)]} \widetilde{]}$	$+ \frac{1}{2} \alpha^2 (K_1 + K_2)$	$- (\frac{5}{2} \alpha^2 - 2K_1^2 - 2K_2^2) (K_1 + K_2)$
Singlet		$+ \alpha^2 (K_1 + K_2)$	$- (2\alpha^2 - K_1^2 - K_2^2) (K_1 + K_2)$

T A B L E 4

The coupling scheme m_2 . The common factor is $(18K_1 K_2 \alpha)^{-1}$.

$\eta = +1$	$+\frac{1}{\chi} \tilde{B}_1 B_2$	$\frac{-i}{K_1+K_2} \tilde{B}_1 \sigma_{\mu\nu} i \gamma_5 B_2 (p_1^\nu - p_2^\nu)$
D	0	$-6(K_1+K_2)$
F	$+6\chi^2$	$-4(K_1+K_2)$
Singlet		$-2(K_1+K_2)$

$\eta = -1$	$+\frac{i}{\chi} \tilde{B}_1 \gamma_5 B_2$	$\frac{-i}{K_1+K_2} \tilde{B}_1 \sigma_{\mu\nu} B_2 (p_1^\nu - p_2^\nu)$
D	0	$+6(K_1+K_2)$
F	$-6\chi^2$	$+4(K_1+K_2)$
Singlet		$+2(K_1+K_2)$

TABLE 5

The coupling scheme m_3 . The common factor is $(18\chi)^{-1}$.