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SOME RELATIONS ON THE SCATTERING OF PSEUDOSCALAR
MESONS OFF BARYONS IMPLIED BY A RELATIVISTIC
SU(6) SYMMETRY MODEL

W. Rühl
CERN--Geneva

In some previous papers ¹⁾ we developed a relativistically invariant model of SU(6) symmetry and applied it to simple S matrix elements. Some of the results derived concerned only coupling constants, others gave relations between different form factors. These relations had the form of identities in the square of the four-momentum transfer. If predictions of this type can be shown to agree with experiment, this indicates that the SU(6) symmetry is perhaps of a basic dynamical origin and holds independently of the kinematical situation. A formulation of such a dynamics must be relativistically invariant. Our approach is purely phenomenological. It is based on the construction of a group which combines the inhomogeneous Lorentz group and the SU(3) symmetry group in a particular way. This does not mean that our group is connected with the geometry of the space as is the Lorentz group itself. We consider it as a relativistically invariant device to analyze S matrix elements for the special case of phenomena which are governed by very strong interactions.

The study of relations between scattering amplitudes is therefore of fundamental interest. Recently two relations for meson-baryon scattering in forward direction have been derived in a static SU(6) model and guessed to be valid for all centre of mass energies ²⁾. Only one of these relations, namely

$$f(K^+) = f(K^-) = f(\pi^+) - f(\pi^-) + f(K^0) - f(\bar{K}^0)$$

can be shown to hold in our model. $f(x)$ means the scattering amplitude in forward direction of a meson x at a proton, taken at a certain value of the centre of mass energy. Whether we have to take the centre of mass energy or a kinetic energy becomes important if we break the symmetry.

The S matrix element for scattering of pseudoscalar mesons on spin- $\frac{1}{2}$ baryons [the mesons belong to the 35-dimensional representation of SU(6), the baryons fill the octet of the representation $5\bar{6}$] has the form

$$\bar{u}(p_2) \left[A_1 + \frac{1}{2} (q_1^\mu + q_2^\mu) \gamma_\mu \cdot A_2 \right] u(p_1)$$

where p_1, p_2 and q_1, q_2 are the four momenta of baryons and mesons, respectively. The amplitudes A_1 and A_2 can be decomposed as

$$A_i = \sum_{j=1}^9 B_j^{(i)} P_j, \quad i=1,2$$

with

$$\begin{aligned} P_1 &= \text{Tr}(\bar{N}_2 N_1 M_1 M_2), & P_5 &= \text{Tr}(\bar{N}_2 M_1 N_1 M_2), \\ P_2 &= \text{Tr}(\bar{N}_2 N_1 M_2 M_1), & P_6 &= \text{Tr}(\bar{N}_2 M_2 N_1 M_1), \\ P_3 &= \text{Tr}(\bar{N}_2 M_1 M_2 N_1), & P_7 &= \text{Tr}(\bar{N}_2 M_1) \text{Tr}(N_1 M_2), \\ P_4 &= \text{Tr}(\bar{N}_2 M_2 M_1 N_1), & P_8 &= \text{Tr}(\bar{N}_2 M_2) \text{Tr}(N_1 M_1), \\ & & P_9 &= \text{Tr}(\bar{N}_2 N_1) \text{Tr}(M_1 M_2). \end{aligned}$$

As is well known this decomposition is not unique because the traces P_i are linearly related. In addition time-reversal invariance implies a connection between $B_5^{(i)}$ and $B_6^{(i)}$. We calculate the coefficients $B_j^{(i)}$, starting with a list of all the possible invariant interactions. There are two different types of invariants. An example of an invariant of the first kind, which we call the "regular" one, is

$$\bar{\psi}(p_2)_{ac} \epsilon^{ab} \psi(p_1)_{\dot{a}\dot{b}} \psi(q_1)^{ab} \psi(q_2)^{cd} + \text{parity transf.}$$

The second kind ^{*)}, which we name the "irregular" type, contains additional derivatives as is exhibited by the following example :

*) The author is indebted to Dr. J.S. Bell who pointed out to him the possible existence of these invariants.

$$\bar{\psi}(p_2)_{ac} \epsilon \psi(p_1)_{bd} \varphi(q_1)_{ab} \varphi(q_2)_{cd} (q_1 + q_2)_{ef} \\ + \text{parity transf.}$$

The number of regular invariants is 48, the number of irregular ones is bigger than 1000. (We take only parity invariance into account at this stage.)

These numbers show that a calculation of all the invariants and their different contents (scattering and production processes) is beset with practical difficulties. We hope that there exists a fundamental difference between the two classes of invariants, which has to be investigated from a comparison with experimental data. If we assume that only regular invariants contribute it is possible to extract all the contributions to ps meson baryon scattering. In forward direction the amplitudes A_1 and A_2 can be lumped together and the following relations are seen to hold

$$B_1 - B_2 + B_3 - B_4 = 0, \quad (1)$$

$$B_7 - B_8 = 0, \quad (2)$$

$$B_5 - B_6 = 0, \quad (3)$$

where (3) is also a consequence of time-reversal invariance. The relations (1) and (2) are identical with those used in Ref. ²⁾ and imply the relation given above for the forward scattering amplitudes, and in addition

$$f(\pi^+) - f(\pi^-) = f(K^0) - f(\bar{K}^0).$$

Here we made use of

4.

$$f(K^-) = B_1 + B_4 + B_8 + B_9,$$

$$f(K^+) = B_2 + B_3 + B_7 + B_9,$$

$$f(K^0) = B_2 + B_9,$$

$$f(\bar{K}^0) = B_1 + B_9,$$

$$f(\pi^+) = B_3 + B_9,$$

$$f(\pi^-) = B_4 + B_9.$$

Relations (2) and (3) imply a connection between two-production amplitudes in forward direction :

$$f(\pi^- p \rightarrow K^+ \Sigma^-) = f(K^- p \rightarrow \pi^+ \Sigma^-)$$

since these amplitudes are given by

$$f(\pi^- p \rightarrow K^+ \Sigma^-) = B_5 + B_7,$$

$$f(K^- p \rightarrow \pi^+ \Sigma^-) = B_6 + B_8.$$

There seems to exist no further simple relation for other cases than that of forward scattering. Of course, there exist relations between ps meson scattering and vector meson or spin- $\frac{3}{2}$ baryon production amplitudes. But we did not evaluate these contributions.

If we are interested in forward scattering alone it is practically possible to evaluate also all the irregular invariants. Their number is then considerably reduced. We have done this only so far that we were able to conclude that relation (2) remains unaltered whereas relation (1) depends on unknown parameters and holds generally only at the threshold, where baryons and mesons are at rest. Because of the mass splitting, relations which are true only at threshold cannot be compared with experiment. Relation (3) is implied by time-reversal invariance.

Thus we conclude that the equations

$$f(K^+) - f(K^-) = f(\pi^+) - f(\pi^-) + f(K^0) - f(\bar{K}^0)$$

$$f(\pi^- p \rightarrow K^+ \Sigma^-) = f(K^- p \rightarrow \pi^+ \Sigma^-)$$

are a consequence of our model, whereas

$$f(\pi^+) - f(\pi^-) = f(K^0) - f(\bar{K}^0)$$

is true under the additional assumption that irregular couplings are negligible.

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R E F E R E N C E S

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