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THE TRILINEAR MESON-MESON COUPLING AND THE ELECTROMAGNETIC

INTERACTION OF MESONS IN A RELATIVISTIC MODEL OF

SU(6) SYMMETRY

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The trilinear meson coupling is perhaps the simplest problem to which a relativistically invariant model of SU(6) symmetry can be applied. The model to which we refer has been described in some preceding papers ¹⁾. The mesons are treated as identical particles. Mesons and photons are assumed to belong to 35-dimensional representations of SU(6) with negative parity. Because mesons are Bose particles, the vertex must be symmetric in the meson fields. Nevertheless we distinguish between the masses of the mesons in our calculation, for we hope to get an indication of the point where we must break the symmetry and insert the physical masses. We shall see that our results agree with experimental data only if the meson masses are equal to a mean mass up to the end of our calculation.

Recently a paper appeared ²⁾ in which the trilinear meson vertex was treated under the additional assumption that trilinear and quadrilinear couplings are connected as is prescribed by a gauge theory of mesons. We emphasize that we need neither a gauge theory of mesons nor any superimposed SU(12) symmetry, which some authors use in order to formulate their version of a relativistically complete ³⁾ or covariant ⁴⁾ SU(6) model.

The following set of invariant couplings is possible

$$\varphi_1^c{}_a \varphi_2^b{}_c \varphi_3^{ab} - \varphi_1^c{}_a \varphi_2^{bc} \varphi_3{}^a{}_b \quad (-) \quad (1)$$

$$\varphi_1^b{}_c \varphi_2^c{}_a \varphi_3^{ab} - \varphi_1^{bc} \varphi_2^c{}_a \varphi_3{}^a{}_b \quad (+) \quad (2)$$

$$\varphi_1^c{}_a \varphi_2^b{}_c \varphi_3^{ab} - \varphi_1^{ca} \varphi_2^b{}_c \varphi_3{}^a{}_b \quad (-) \quad (3)$$

$$\varphi_1^b{}_c \varphi_2^c{}_a \varphi_3^{ab} - \varphi_1^b{}_c \varphi_2^{ca} \varphi_3{}^a{}_b \quad (+) \quad (4)$$

$$\varphi_1^b{}_c \varphi_2^c{}_a \varphi_3^a{}_b - \varphi_1^b{}_c \varphi_2^c{}_a \varphi_3{}^a{}_b \quad (+) \quad (5)$$

2.

$$\varphi_1^{bc} \varphi_2^{ca} \varphi_3^{ab} - \varphi_1^{bc} \varphi_2^{ca} \varphi_3^{ab} \quad (+) \quad (6)$$

$$\varphi_1^{ca} \varphi_2^{bc} \varphi_3^{ab} - \varphi_1^{ca} \varphi_2^{bc} \varphi_3^{ab} \quad (-) \quad (7)$$

$$\varphi_1^c{}_a \varphi_2^b{}_c \varphi_3^a{}_b - \varphi_1^c{}_a \varphi_2^b{}_c \varphi_3^a{}_b \quad (-) \quad (8)$$

The signs behind each term indicate the type of SU(3) trace

$$(+)=M_1^i{}_j M_2^j{}_k M_3^k{}_i,$$

$$(-)=M_1^i{}_j M_3^j{}_k M_2^k{}_i.$$

In the case of the electromagnetic interaction we identify φ_3 with the photon field. Multiplying each invariant with a parameter α_i and adding them, we get the expression for the general vertex. Bose symmetrization and charge conjugation invariance restricts the number of independent parameters. The MM_γ vertex contains three parameters

$$\alpha \equiv \alpha_1 = \alpha_2 = -\alpha_3 = -\alpha_4,$$

$$\beta' \equiv \alpha_5 = \alpha_8,$$

$$\gamma' \equiv \alpha_6 = \alpha_7,$$

whereas for the MMM vertex

$$\alpha = \gamma'$$

holds in addition. α, β', γ' are real numbers.

A short and straightforward calculation (see Ref. ¹⁾) gives the following results. $P^{(8)}$ means the pseudoscalar octet, $V^{(9)}$, $V^{(8)}$, $V^{(1)}$ the vector nonet, octet and singlet, respectively. ϵ_i and m_i are the polarization vector and the mass belonging to the vector particle V_i .

$$P_1^{(8)} P_2^{(8)} V_3^{(8)} : 2\alpha [(-) - (+)] \frac{m_1 + m_2 + m_3}{m_1 m_2} ((p_1 - p_2) \varepsilon_3),$$

$$P_1^{(8)} V_2^{(8)} V_3^{(8)} : 4\alpha [(-) + (+)] i \varepsilon_{\alpha\sigma\nu\lambda} \left(\frac{p_3^\alpha}{m_3} - \frac{p_2^\alpha}{m_2} \right) \frac{p_1^\sigma}{m_1} \varepsilon_2^\nu \varepsilon_3^\lambda,$$

$$V_1^{(8)} V_2^{(8)} V_3^{(8)} : [(-) - (+)] \left\{ 4 \sum_{\substack{\text{perm} \\ 1,2,3}} (\alpha + \beta' \frac{p_2 p_3}{m_2 m_3}) (\varepsilon_1 \varepsilon_2) \left(\frac{p_1}{m_1} \varepsilon_3 \right) \right. \\ \left. - \frac{\beta'}{m_1 m_2 m_3} ((p_1 - p_2) \varepsilon_3) ((p_2 - p_3) \varepsilon_1) ((p_3 - p_1) \varepsilon_2) \right\}$$

where

$$\varepsilon_i p_i = 0,$$

$$p_1 + p_2 + p_3 = 0.$$

In the last expression the sum over permutations of the indices 1, 2, 3 takes even (odd) permutations with a positive (negative) sign.

The electromagnetic vertex has the form ($q^2 = m_3^2$, $m = m_1 = m_2$)

$$P_1^{(8)} P_2^{(8)} \gamma : G_{PP}(q^2) ((p_1 - p_2) \varepsilon_3)$$

$$P_1^{(8)} V_2^{(8)} \gamma : G_{PV}(q^2) \frac{1}{m} \varepsilon_{\alpha\sigma\nu\lambda} p_1^\alpha p_2^\sigma \varepsilon_2^\nu \varepsilon_3^\lambda$$

$$V_1^{(8)} V_2^{(8)} \gamma : G_{VV}^{(1)}(q^2) (\varepsilon_1 \varepsilon_2) ((p_1 - p_2) \varepsilon_3) \\ + (G_{VV}^{(1)}(q^2) + G_{VV}^{(2)}(q^2)) [(\varepsilon_2 \varepsilon_3) (p_2 \varepsilon_1) - (\varepsilon_1 \varepsilon_3) (p_1 \varepsilon_2)] \\ + \frac{G_{VV}^{(3)}(q^2)}{8m^2} [((p_2 - p_3) \varepsilon_1) ((p_3 - p_1) \varepsilon_2) ((p_1 - p_2) \varepsilon_3) \\ - q^2 \{ (\varepsilon_2 \varepsilon_3) ((p_2 - p_3) \varepsilon_1) - (\varepsilon_1 \varepsilon_3) ((p_1 - p_3) \varepsilon_2) \}].$$

These five form factors are related by (we drop an over-all factor $\frac{4}{m}$)

4.

$$G_{PP} = [\alpha + \frac{q^2}{2m^2} \gamma] [(-) - (+)],$$

$$G_{PV} = [\alpha + \gamma] [(-) + (+)],$$

$$G_{VV}^{(1)} = [\alpha - \frac{q^2}{2m^2} \beta] [(-) - (+)],$$

$$G_{VV}^{(2)} = [\gamma - \beta] [(-) - (+)],$$

$$G_{VV}^{(3)} = -2\beta [(-) - (+)],$$

where

$$\beta = \beta' \frac{m}{q}, \quad \gamma = \gamma' \frac{m}{q}.$$

Inserting $\frac{1}{2}(\lambda_3 + \frac{1}{\sqrt{3}} \lambda_8)$ for the photon, the SU(3) trace $[(-) - (+)]$ becomes the charge in units of the elementary charge. Therefore gauge invariance implies $\alpha(0) = 1$.

The parameter α of the MMM vertex is connected with the ρ decay coupling constant $g_{\rho\pi\pi}$ which we define as

$$L_{\rho_3 \pi_1 \pi_2} = -i g_{\rho\pi\pi} ((p_1 - p_2) \cdot \epsilon_3) (\vec{\pi}_1 \times \vec{\pi}_2) \cdot \vec{\rho}$$

We get

$$\alpha = \frac{\sqrt{2}}{12} m \cdot g_{\rho\pi\pi}$$

Similarly we define the coupling constants $g_{\omega\rho\pi}$ and $g_{\phi\rho\pi}$

$$L_{\pi_1 \rho_2 \omega_3} = \frac{g_{\omega\rho\pi}}{m_\pi} \epsilon_{\mu\nu\lambda\sigma} p_2^\mu p_3^\nu \epsilon_2^\lambda \epsilon_3^\sigma \vec{\rho} \cdot \vec{\pi}$$

If we assume the $\phi\omega$ mixing to be given by ⁵⁾

$$\omega = \left(\frac{1}{3}\right)^{\frac{1}{2}} \phi_0 + \left(\frac{2}{3}\right)^{\frac{1}{2}} \omega_0,$$

$$\phi = -\left(\frac{2}{3}\right)^{\frac{1}{2}} \phi_0 + \left(\frac{1}{3}\right)^{\frac{1}{2}} \omega_0,$$

we find

$$g_{\phi \rho \pi} = 0,$$

$$\frac{g_{\rho \pi \pi}^2}{g_{\omega \rho \pi}^2} = \frac{9}{64} \frac{m^2}{m_\pi^2}.$$

The process $\Phi \rightarrow 3\pi$ is known to be rare ; as the following table shows, the latter result is also in good agreement with experiment.

$$m \quad \left(\frac{g_{\rho \pi \pi}^2}{g_{\omega \rho \pi}^2} \right)_{th.} \left(\frac{g_{\rho \pi \pi}^2}{g_{\omega \rho \pi}^2} \right)_{exp.}$$

700 MeV	3.5	
		4 - 6
910 MeV	6	

If we want to break the symmetry by inserting physical masses, the ratio

$$\frac{4}{9} \cdot \frac{m_\rho^2 m_\omega^2 (2m_\pi + m_\rho)^2}{m_\pi^2 (m_\rho + m_\omega)^2}$$

has to be substituted for m^2 . Then the ratio of the squares of coupling constants takes the value ≈ 25 .

In a similar way we can see that the process $\Phi \rightarrow \gamma + \pi$ is forbidden, whereas the process $\omega \rightarrow \gamma + \pi$ is allowed. The known branching ratio for the latter process ⁶⁾ can be used to determine the parameter $\gamma(0)$. We get

$$\gamma = 2.2 \cdot \frac{m}{m_\omega} - 1$$

6.

which is consistent with $\gamma(0) = 1$. On the other hand, the hypothesis that vector mesons and photons are coupled to the same current predicts

$$\alpha(m^2) = \gamma(m^2).$$

Therefore the value $\gamma(0) \approx 1$ is quite reasonable.

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R E F E R E N C E S

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