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EFFECT OF A PHENOMENOLOGICAL S WAVE
PION-PION INTERACTION ON THE PHOTOPRODUCTION OF
A PION PAIR IN A COULOMB FIELD

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In view of the continued interest in S wave $\pi-\pi$ scattering ¹⁾, we wish to discuss, using a simple phenomenological model, one more method of studying it experimentally, namely by observing both pions in the photoproduction of a pion pair in the Coulomb field of a nucleus. An extensive calculation of this process has been performed by Kim ²⁾, employing the Mandelstam representation to describe the S wave $\pi-\pi$ rescattering. His conclusion was that the contribution of Coulomb production to the differential cross-section $(\partial^2/\partial\omega_1\partial\Omega_1)$ was small compared to that of the production in the nuclear field, but perhaps measurable under favourable circumstances.

We wish to remark that, to lowest order in perturbation theory, i.e., Born approximation, the double differential cross-section ³⁾ for the production in a Coulomb field $(\partial^2/\partial\omega_1\partial\Omega_1\partial\Omega_2)$, has a sharp maximum and in the neighbourhood of that maximum is almost comparable to the nuclear field production; furthermore the effect of an S wave rescattering is, as we shall see, quite considerable.

By charge conjugation only $T = 0, 2$ pion states are allowed in the Coulomb field production and consequently only even angular momentum states are present. As there does not appear to be any evidence for a strong $\pi-\pi$ interaction in D or higher waves, we shall approximate them by their perturbation theory amplitudes and the total amplitude by

$$M = M_{\text{BORN APPROX.}} + f M_{\text{S BORN APPROX.}} \quad (1)$$

where the total S wave amplitude is approximated by $(f+1)$ times the S wave perturbation theory amplitude.

If we restrict ourselves to two pion intermediate states, unitarity requires that $(f+1)$ have the phase of S wave pion-pion scattering. A reasonable first approximation for $(f+1)$ is then

$$(f+1) = \frac{A_s^0 + \frac{1}{\sqrt{2}} A_s^2}{\left(A_s^0 + \frac{1}{\sqrt{2}} A_s^2\right)_{\text{BORN APP}}} \quad (2)$$

where A_s^I is the pion-pion scattering amplitude in the I th isotopic spin state, as this has the correct phase and the correct Born approximation limit. We then put A_s^2 equal to its Born approximate value, which we take to be -2λ and A_s^0 in Born approximation equal to -5λ , λ being the value of the pion-pion "coupling constant" at a particular symmetry point⁴⁾. This gives

$$(f+1) = \frac{\sqrt{2} \left(\left[\frac{q^2 + \mu^2}{q^2} \right]^{\frac{1}{2}} e^{i\delta_s^0} \sin \delta_s^0 - \lambda \right)}{-\lambda (5 + \sqrt{2})} \quad (3)$$

where q^2 is the square of the pion-pion centre of mass system relative momentum.

The lowest order perturbation theory invariant amplitude for photo-production of a pion pair in a Coulomb field is (see Fig. 1)

$$M_{\text{TOT.}} = \left[-2 \varepsilon_1 \cdot \varepsilon_2 - \frac{4 \varepsilon_1 \cdot q_1 \varepsilon_2 \cdot q_2}{(k_1 - q_1)^2 + \mu^2} - \frac{4 \varepsilon_1 \cdot q_2 \varepsilon_2 \cdot q_1}{(k_2 + q_1)^2 + \mu^2} \right] \quad (4)$$

One can, with little difficulty, expand this expression in spherical harmonics in the centre of mass system and thereby obtain the various partial wave amplitudes of the perturbation theory total amplitude. In invariant form, this gives for the S-wave

$$M_S = 2 \left(\varepsilon_1 \cdot \varepsilon_2 - \frac{\varepsilon_1 \cdot k_2 \varepsilon_2 \cdot k_1}{k_1 \cdot k_2} \right) \left[-1 + \frac{1}{k_1 \cdot k_2} \left\{ -\frac{(q_1 + q_2)^2}{2} - 2\mu^2 \sqrt{\frac{-(q_1 + q_2)^2}{(q_1 - q_2)^2}} \log \left(\frac{\sqrt{-(q_1 + q_2)^2} + \sqrt{(q_1 - q_2)^2}}{2\mu} \right) \right\} \right] \quad (5)$$

where μ is the pion mass. Even for incident photons with an energy of several GeV, the S wave amplitude is quite considerable in the region of the maximum. Physically this corresponds to the fact that if two equal particles move with lab. momentum that are even very different in magnitude, but almost parallel, their centre of mass system relative momentum is very small compared to the lab. momenta and therefore they are largely in a relative S state. From (4) and (5) one sees directly that the S wave has a denominator ^{*)} of $k_1 \cdot k_2 = -k_1 \cdot (q_1 + q_2)$ which causes peaking for both pions close to the forward direction.

Using (1), (4) and (5) we find for the scattering from a nucleus of charge Z, in the Coulomb limit, with $q_{1,2} = (\omega_{1,2}, \vec{q}_{1,2})$, $\varphi = \varphi_1 - \varphi_2$ and ρ equal to quantity contained within the square brackets of (5) ^{**)}

^{*)} $\varepsilon_1 \cdot \varepsilon_2$ is zero in the Coulomb limit as ε_1 is transverse and ε_2 longitudinal.

^{**)} q_1, q_2 and k_1 in (6) are the magnitude of the momenta and k_2^2 is the square of the momentum transferred to the nucleus.

4.

$$\frac{\partial^4 \sigma}{\partial \cos \theta_1 \partial \cos \theta_2 \partial \varphi \partial \omega_1} = \frac{\alpha^3 Z^2}{\pi} \frac{q_1 q_2}{k_2^4 k_1}$$

$$\left[\left\{ \frac{\omega_1^2 q_2^2 \sin^2 \theta_2}{k_1^2 (\omega_2 - q_2 \cos \theta_2)^2} + \frac{\omega_2^2 q_1^2 \sin^2 \theta_1}{k_1^2 (\omega_1 - q_1 \cos \theta_1)^2} + \frac{2 \omega_1 \omega_2 q_1 q_2 \sin \theta_1 \sin \theta_2 \cos \varphi}{k_1^2 (\omega_1 - q_1 \cos \theta_1) (\omega_2 - q_2 \cos \theta_2)} \right\} \right.$$

$$+ |f\rho|^2 \left\{ \frac{q_1^2 \sin^2 \theta_1 + q_2^2 \sin^2 \theta_2 + 2 q_1 q_2 \sin \theta_1 \sin \theta_2 \cos \varphi}{\left((\omega_1 - q_1 \cos \theta_1) + (\omega_2 - q_2 \cos \theta_2) \right)^2} \right\} \quad (6)$$

$$+ \frac{2\rho \operatorname{Re} f}{k_1} \left\{ \frac{1}{\omega_1 - q_1 \cos \theta_1 + \omega_2 - q_2 \cos \theta_2} \left[\frac{\omega_2 q_1^2 \sin^2 \theta_1}{\omega_1 - q_1 \cos \theta_1} + \frac{\omega_1 q_2^2 \sin^2 \theta_2}{\omega_2 - q_2 \cos \theta_2} \right. \right.$$

$$\left. \left. + q_1 q_2 \sin \theta_1 \sin \theta_2 \cos \varphi \left(\frac{\omega_1}{\omega_2 - q_2 \cos \theta_2} + \frac{\omega_2}{\omega_1 - q_1 \cos \theta_1} \right) \right] \right\}$$

The question of determining $(f+1)$ or, equivalently, $A_s^0(q^2)$ still remains. A scattering length approximation gives

$$A_s^0 = \frac{\sqrt{q^2 + \mu^2}}{q \cot \delta_s^0 - i q} = \frac{a_0 \sqrt{q^2 + \mu^2}}{1 - i a_0 q} \quad (7)$$

while a model for a resonance would be

$$A_s^o = - \left[\frac{q^2 + \mu^2}{q^2} \right]^{\frac{1}{2}} \frac{\Gamma(\nu)}{q_\pi^2 - q^2 + i\Gamma(\nu)\Theta(q^2)} \quad \Theta(q^2) = \begin{cases} 1 & q^2 > 0 \\ 0 & q^2 < 0 \end{cases} \quad (7a)$$

Taking the two to coincide at zero energy, gives $\Gamma(\nu) \sim -a_0 q q_r^2$ for small q . For $\lambda = -0.18$, $a_0 = 1.3$ (in units of $\mu = \pi = c = 1$) and assuming the resonance in (7a) to be located at a very low energy, e.g., $2\sqrt{q_r^2 + \mu^2} = 2.3 \mu$ we find greatly different values of f in the neighbourhood of (6)'s maximum, $q \approx \mu$, namely

$$\text{Scatt. length} : f(q \approx \mu) \sim -0.2 + i0.75 \quad |f|^2 \sim 0.6 \quad (8)$$

$$\text{Reson.} : f(q \approx \mu) \sim -1.25 + i0.35 \quad |f|^2 \sim 1.6 \quad (8a)$$

Measuring energy in units of pion masses ($\mu = \pi = c = 1$) we present the results of a numerical evaluation of (6) done for $k_1 = 20 \approx 2.8 \text{ GeV}$, $\varphi = \pi$ and various values of f , taking $\omega_2 = k_1 - \omega_1$.

$$\frac{\sigma^{\text{tot}}}{\cos\theta_1 \cos\theta_2 \cos\varphi \omega_1} = \frac{2.35 \times 10^{-33} Z^2 Q \text{ cm}^2}{\text{rad. sterad. energy}} \quad (9)$$

where Q is given in Table 1.

TABLE 1

Q where $\pm n$
means multiply by $10 \pm n$

ω_1	θ_1^{-1}	θ_2^{-1}	f : 0	-0.2 + i0.75	-1.25 + i0.35	1
17	20	2	5.8-1	5.5-1	7.6-1	2.3
17	20	4	8.3	1.4+1	5.3+1	1.2-3
17	17	2	1.1	1.0	6.0-1	2.9
17	17	4	4.2	1.2	5.7	2.1
17	14	3	3.7-1	2.0	1.2+1	1.4
11	13	11	8.9	4.9+1	3.1+2	3.6+1
11	13	9	5.1	1.6+2	1.2+3	3.1+2
11	13	7	7.2-1	3.8+1	3.0+2	1.3+2
11	11	11	1.3+1	2.2+2	1.6+3	3.8+2
11	11	9	3.0-1	4.8-1	1.5	6.3-3
11	9	11	2.1-2	6.6+1	5.1+2	1.8+2
11	9	9	3.7	5.4+1	4.1+2	2.2+2
11	9	7	3.8	5.9	2.4+1	3.2+1
10	12	10	9.1	2.0+2	1.5+3	3.5+2
10	12	9	1.2	1.2+2	9.2+2	2.7+2
10	12	8	1.4-1	5.4+1	4.2+2	1.6+2
10	10	11	1.3	9.3+1	7.0+2	2.0+2
10	10	9	1.1	5.1+1	4.0+2	1.8+2
10	10	8	5.6	5.0+1	3.7+2	2.2+2
10	8	11	1.7	5.3+1	4.2+2	2.0+2
10	8	9	7.4	2.9+1	1.9+2	1.4+2

As we see the cross-section is very sensitive to the value of f . Apparently not only is the S wave appreciable but the partial waves in the perturbation theory amplitude tend to cancel each other out, giving an anomalously small cross-section. The cross-section, for $\varphi = \pi$, is very small at $\theta_1 = \frac{1}{\omega_1}$, $\theta_2 = \frac{1}{\omega_2}$ and rises to maximum in the neighbourhood of that point. For $\omega_1 = \omega_2$, $\theta_1 = \theta_2$ it is in fact zero.

The competing nuclear mechanism is presumably that of Fig. 2 which has been discussed by Drell⁶⁾. It gives

$$\frac{d^4\sigma}{d\omega_1 d\theta_1 d\omega_2 d\theta_2 d\varphi d\omega_1} = \frac{\alpha}{8\pi^2} \frac{q_1^2 \sin^2\theta_1}{(\omega_1 - q_1 \cos\theta_1)^2} \frac{\omega_1 (k_1 - \omega_1)}{k_1^3} \sigma_{el,\pi p}(k_1 - \omega_1, k_2^2) \quad (10)$$

The momentum transfers in question, k_2^2 , are of the order of $(40 \frac{\text{MeV}}{c})^2$, i.e., very small. An estimate of $\sigma_{\pi p}^{el}$ can then be obtained by assuming that this momentum transfer corresponds to zero angle scattering, that the amplitude at this energy is pure imaginary and then using the optical theorem. Calling p^2 the pion nucleon centre of mass relative momentum we have

$$\sigma_{el}(k_1 - \omega_1, 0) = \frac{p^2}{8\pi} \left| \sigma_{TOT}(k_1 - \omega_1) \right|^2 \quad (11)$$

where $p^2 \approx \frac{q_{2M}^2}{(M+2\omega_2)}$. For $\omega_1 = 10$ (1.4 GeV), $\theta_1 = \frac{1}{10}$, $\theta_2 = \frac{1}{8}$ and $\varphi = \pi$, taking $\sigma_{TOT}(20-10) = 35 \text{ mb}$ ⁷⁾, expression (10) gives

$$\frac{d^4\sigma}{d\omega_1 d\theta_1 d\omega_2 d\theta_2 d\varphi d\omega_1} = \frac{7.4 \times 10^{-32} \text{ cm}^2}{\text{Mrad. en.}}$$

By comparison, the electromagnetic production double differential cross-section, at the same energies and angles, and in the same units, equals (for $Z = 1$) 1.3×10^{-32} , 1.2×10^{-31} , 8.7×10^{-31} , 5.2×10^{-31} for $f = 0, -0.2 + i0.75, -1.25 + i0.35, 1$.

As we see, production in the Coulomb field is actually larger by as much as a factor of ten than nuclear production. Final state interactions could, of course, also enhance the nuclear production, but the effect should not be nearly as considerable as it is for photoproduction ; expression (9) in any case is perhaps an overestimate of σ_{el} . There are also interference terms present, as the Coulomb production has acquired the phase of pion-pion scattering and these have not been calculated. Lastly we would like to mention that the difference between taking (8) or (8a) for f is quite considerable, as seen above.

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Fig. 1

Fig. 2