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AN UPPER BOUND ON THE NEUTRON-DEUTERON

2 S SCATTERING LENGTH BY AN S MATRIX METHOD

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It has been known since 1951 that neutron-deuteron zero kinetic energy scattering can be fitted by two different sets of scattering lengths ¹⁾

$$\begin{aligned} \text{i)} \quad a_2 &= (0.7 \pm 0.3) \times 10^{-13} \text{ cm} & a_4 &= (6.4 \pm 0.1) \times 10^{-13} \text{ cm} \\ \text{ii)} \quad a_2 &= (8.3 \pm 0.1) \times 10^{-13} \text{ cm} & a_4 &= (2.6 \pm 0.2) \times 10^{-13} \text{ cm} \end{aligned} \quad (1)$$

where a_2 and a_4 are respectively doublet and quartet scattering lengths. Very impressive calculations ²⁾, recently performed, employing trial potentials and variational techniques, favour the first set.

We wish to present a rather simple argument which also leads to a preference of the first set. The technique employed is essentially the one expounded in a recent article by S. Weinberg ³⁾ on the "Evidence that the Deuteron is an Elementary Particle" hitherto referred to as W.

In the following we shall assume, however, that there is no elementary deuteron nor an elementary triton.

Let us begin by splitting the total Hamiltonian H into two parts

$$H = H_0 + V$$

where H_0 is the sum of the total kinetic energy and the potential which leads to the binding of the deuteron. V is the rest of the potential. Labelling a triton eigenstate as $|1\rangle$, a neutron-deuteron eigenstate of H as $|2\rangle$ and a three-particle eigenstate of H as $|3\rangle$, we first see that

$$\begin{aligned} 1 = \langle 1 | 1 \rangle &= \int \frac{|\langle 1 | H - H_0 | \alpha \rangle|^2}{(E(\alpha) + B)^2} d\alpha \\ &= \int \frac{|\langle 1 | V | \alpha \rangle|^2}{(E(\alpha) + B)^2} d\alpha \end{aligned} \quad (2)$$

2.

The states $|\alpha\rangle$ are a complete set of eigenstates of H_0 , labelled as $|\alpha_2\rangle$ or $|\alpha_3\rangle$, depending on whether they are neutron-deuteron or three-particle states. Analogously to Weinberg's treatment of neutron-proton scattering, we put

$$|\langle 1 | V | \alpha_2 \rangle|^2 = g_0^2 \quad (3)$$

where g_0^2 is a constant. Equation (2) then gives

$$1 = g_0^2 \int \frac{4\pi k^2 dk}{(E+B)^2} + \int \frac{|\langle 1 | V | \alpha_3 \rangle|^2}{(E(\alpha_3) + B)^2} d\alpha_3 \quad (4)$$

$$E = \frac{k^2}{2\mu} - b$$

where b is the deuteron binding energy, B is of course the triton binding energy and μ is the neutron-deuteron reduced mass equal to two thirds the nuclear mass. Integrating, we find

$$1 = g_0^2 \frac{4\pi^2 (2M^3)^{\frac{1}{2}}}{2 \sqrt{B-b}} + \int \frac{|\langle 1 | V | \alpha_3 \rangle|^2}{(E(\alpha_3) + B)^2} d\alpha_3 \quad (5)$$

As the integral over three particle states is positive, because both the numerator and denominator are positive, (5) implies

$$0 \leq g_0^2 \leq \frac{2\sqrt{B-b}}{4\pi^2(2\mu^3)^{1/2}} = \frac{2\sqrt{B-b}}{\pi\rho} \quad (6)$$

where $\rho = 4\pi^2(2\mu^3)^{1/2}$.

The transition operator T can be written as

$$T = V + V \frac{1}{E - H + i\varepsilon} V$$

and its matrix element, for forward direction neutron-deuteron scattering, equals

$$\begin{aligned} \langle \alpha_2 | T | \alpha_2 \rangle &= T_{22} = V_{22} + \frac{|\langle \alpha_2 | V | 1 \rangle|^2}{E(\alpha_2) + B} \\ &+ \sum_{2'} \frac{T_{22'} T_{2'2}^+}{E(\alpha_2) - E_{2'} + i\varepsilon} + \sum_{3'} \frac{T_{23'} T_{3'2}^+}{E(\alpha_2) - E_{3'} + i\varepsilon} \quad (7) \end{aligned}$$

where we have introduced a complete set of eigenstates of H

$$1 = |1\rangle\langle 1| + \sum_{2'} |2'_m\rangle\langle 2'_m| + \sum_{3'} |3'_m\rangle\langle 3'_m|$$

4.

We begin our examination of (7) by, following W., dropping the matrix element of the potential V_{22} , from the equation. Comparing the magnitude of the first two terms in (7) at zero kinetic energy, $E \rightarrow -b$, dropping V_{22} is then equivalent to assuming

$$\langle 0,d | V | 0,d \rangle \ll \frac{|\langle 0,d | V | 1 \rangle|^2}{B-b} \quad (8)$$

For the case of n-p scattering, treated by W., one is favoured by having a denominator of $b \approx 2.2$ MeV in the analogous expression as opposed to $B-b \approx 6.3$ MeV. However, counterbalancing this, we have here on the right-hand side something like the triton wave function averaged over the range of the potential and the triton wave function has a shorter range than the deuteron wave function. It is of course difficult to know just what potential to choose for the above situation, but as nuclear potentials are generally much deeper than $B-b$, we believe (8) to be a rather good approximation. In any case, since V is attractive, V_{22} is negative and therefore decreases the value of T_{22} at zero energy, the latter being proportional to the scattering length.

Returning to the Eq. (7) for $T_{22}(E) = t(E)$, we find it more convenient to study the equation for $t^{-1}(E)$. As

$$\frac{\text{Im } t^{-1}(E+i\epsilon)}{\pi} = - \frac{\text{Im } t(E)}{\pi |t(E)|^2}$$

we see that, along the physical cut $E \gg -b$

$$\frac{\text{Im } t^{-1}(E+i\epsilon)}{\pi} = \rho \sqrt{E+b} + \frac{R(E) \theta(E)}{|t(E)|^2} \quad (9)$$

where $\theta(x) = 1$ for $x \gg 0$ and zero otherwise. $R(E)$ is the discontinuity along the three-particle cut and is clearly real and positive.

In addition, we require that $t^{-1}(E)$ have a zero at $E = -B$ with residue, because of (3), equal to $1/g_0^2$, corresponding to the pole of $t(E)$. $t^{-1}(E)$ can then be written in the form

$$\begin{aligned}
 t^{-1}(E) = & \frac{E+B}{g_0^2} + (E+B)^2 \rho \int_{-b}^{\infty} \frac{\sqrt{E'+b} dE'}{(E'+B)^2 (E'-E-i\epsilon)} \\
 & + (E+B)^2 \int_0^{\infty} \frac{R(E') dE'}{(E'+B)^2 |t(E')|^2 (E'-E-i\epsilon)} \quad (10)
 \end{aligned}$$

$$= t_0^{-1}(E) + f(E)$$

where

$$\begin{aligned}
 t_0^{-1}(E) = & \frac{E+B}{g_0^2} + (E+B)^2 \rho \int_{-b}^{\infty} \frac{\sqrt{E'+b} dE'}{(E'+B)^2 (E'-E-i\epsilon)} \\
 = & \left[\frac{E+B}{g_0^2} + i\pi\rho\sqrt{E+b} + \frac{\pi\rho}{\sqrt{B-b}} \left(\frac{B-E-b}{2} \right) \right] \quad (11)
 \end{aligned}$$

and $f(E)$ is the explicit contribution of the three-particle states.

6.

We are essentially finished now. The phase shift for n-d 2S scattering can be defined by

$$e^{2i\delta(E)} = 1 - 2\pi i \rho \sqrt{E+b} \, t(E)$$

which implies that

$$\cot \delta - i = \frac{-1}{\pi \rho \sqrt{E+b} \, t(E)} = \frac{-1}{\pi \rho \sqrt{E+b}} \left[\frac{1}{t_0(E)} + f(E) \right]$$

$f(E)$ is defined in (10) and $t_0^{-1}(E)$ is given explicitly in (11). As $k = \sqrt{2\mu(E+b)}$ in the limit of $k \rightarrow 0$, or equivalently $E \rightarrow -b$

$$\begin{aligned} k \cot \delta &= -\frac{\sqrt{2\mu}}{\pi \rho} \left[\frac{B-b}{g_0^2} + \frac{\sqrt{B-b} \pi \rho}{2} + f(-b) \right] \\ &= -\frac{\sqrt{2\mu(B-b)}}{2} \left[1 + \frac{2\sqrt{B-b}}{\pi \rho g_0^2} + \frac{2f(-b)}{\pi \rho \sqrt{B-b}} \right] \quad (12) \\ &= -\frac{1}{a_2} \end{aligned}$$

From (10) we see that

$$\frac{2 f(-b)}{\sqrt{B-b} \pi \rho} = \lambda = \frac{2 (B-b)^2}{\sqrt{B-b} \pi \rho} \int_0^{\infty} \frac{R(E') dE'}{|t(E')|^2 (E'+b)(E'+B)^2} \geq 0$$

(6) then tells us that

$$\frac{2 \sqrt{B-b}}{\pi \rho g_0^2} = \alpha \geq 1$$

so finally

$$a_2 = \frac{2}{\sqrt{2\mu(B-b)}} [1 + \alpha + \lambda]^{-1} \quad (13)$$

Using $B = 8.5$ MeV, $b = 2.2$ MeV and μ , as stated before, equal to two thirds of the nucleon mass

$$a_2 = \frac{2 \times 3.2 \times 10^{-13} \text{ cm}}{[1 + \alpha + \lambda]}$$

We thus obtain bounds on a_2 :

$$0 \leq a_2 \leq 3.2 \times 10^{-13} \text{ cm}$$

The upper value can only be reached, however, when $\alpha = 1$ and $\lambda = 0$, that is to say when the contribution of the three-particle states to the scattering matrix is zero. We expect them to be actually quite large, as can be readily seen with the crudest kind of models and therefore do not find it at all surprising that the value of the 2S scattering length should be considerably below the upper bound of 3.2×10^{-13} cm.

To understand more clearly what the situation is, let us rewrite (7) at $E = -b$.

$$\begin{aligned}
 t(E) = & V_{22} + \frac{g_0^2}{E+B} - \sum_{2'} \frac{|t(E_{2'})|^2}{E_{2'} + b + i\varepsilon} \\
 & - \sum_{3'} \frac{|T_{23'}|^2}{E_{3'} + b}
 \end{aligned} \tag{14}$$

We see that all terms, except the pole term, are negative. If we neglected everything but this pole term, we would find $a_2 = 6.4 \times 10^{-13}$ cm, this corresponding to $\alpha = \lambda = 0$ and $g_0^2 = 2\sqrt{B-b}/\pi\rho$. If we include two-particle states, but neglect three-particle states, we find $a_2 = 3.2 \times 10^{-13}$ cm. Finally, taking into account three-particle states and the potential (V_{22}) term, we presumably would arrive at the value of $a_2 = 0.7 \times 10^{-13}$ cm. There is of course always an error involved in replacing $|\langle 1|V|\alpha_2\rangle|^2$ by a constant g_0^2 and then using (4) but we do not believe this affects our results by more than $O(m_\pi^{-1})$.

In conclusion, we believe we have obtained an upper bound of 3.2×10^{-13} cm on the 2S neutron-deuteron scattering length with strong evidence that the actual value is much lower, having used as known data only the binding energies of the deuteron and of the triton.

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R E F E R E N C E S

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