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ELASTIC PHOTON-DEUTERON SCATTERING

AND THE NUCLEON POLARIZABILITY^{*)}

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ABSTRACT

An analysis of the existing experimental data on the elastic scattering of photons by deuteron, is carried out in the framework of the impulse approximation. Nevertheless, it also attempts to take into account certain effects which are not given by the impulse approximation. Including these effects, we have been able to improve the prediction of the theory on the deuteron Compton effect. Based on this analysis we make an estimation of the neutron electric polarizability. We found it $\sim 1.2 \cdot 10^{-42} \text{ cm}^3$. In addition we discuss experiments in proton, deuteron and He^4 Compton effect with polarized γ 's, for a better specification of the polarizabilities of the proton and neutron as well.

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I. As is well known, photons are a complementary tool to electrons for the study of nucleon structure. While with electron scattering on the nucleon, one may measure the form factors G_E and G_M of the nucleon structure, with the scattering of photons one may study the photon-induced polarizability of the nucleon's charge and current cloud. This polarizability will characterize the deformation of the meson clouds under the influence of electric and magnetic fields. The polarizability will be zero if the nucleon is treated as a rigid structure^{1,2)}.

Theoretical and experimental works have been devoted to estimating the electric and magnetic polarizability of the nucleons²⁻⁸⁾. The proton polarizability has been estimated directly from the low-energy proton Compton effect by assuming that the divergence of the low-energy experimental data from the Rayleigh-Powell formula⁹⁾ is due to a secondary radiation coming from the polarization of the meson cloud, and can be accounted for by two more parameters in the low-energy amplitude, which are multiplied by the square of the radiation frequency. These two parameters, α_e and α_m , are referred to as electric and magnetic polarizability of the proton³⁾. Measurements of Goldanskii et al.⁴⁾ indicate that

$$\alpha_e = (9 \pm 2) 10^{-43} \text{ cm}^3$$

$$\alpha_m = (2 \pm 2) 10^{-43} \text{ cm}^3.$$

One may also have indirect information about the polarizability of the proton by making use of dispersion relations. In that case, one has to know the π^+ and π^0 photoproduction which contribute to the dispersion integrals of the polarizability, through the unitarity condition⁷⁾.

To estimate the neutron polarizability is more complicated. We do not have available direct experimental data on the scattering of photons by neutrons nor on π^- photoproduction^{*)}, so in order to get an idea of the magnitude of the neutron polarizability, one is forced to use indirect experimental data, for example, the data of deuteron Compton effect and the photoproduction ratio $d\sigma_{\pi^-}/d\sigma_{\pi^+}$.

*) The possibility of estimating the polarizability of the neutrons from the scattering of neutrons by nuclei will not be discussed here.

Previously it has been argued²⁻⁵⁾ that the existing data probably indicate that the electric polarizability of the neutron is larger than that of the proton. However, the experimental information¹⁰⁻¹²⁾ was insufficient and too ambiguous to make any conclusion.

Recently, new experimental data on the elastic scattering of photons by deuterons have been accumulated¹³⁻¹⁴⁾ and it is the purpose of the present investigation to consider these data and to see if they are consistent with the statement that the neutron electric polarizability is larger than the protons, and to make an estimation of the order of magnitude of the former.

It will be seen below that an electric polarizability of the neutron, greater than 40% of the proton one, appears unlikely on this basis.

The present analysis is carried out in the framework of the impulse approximation¹⁵⁾. However, it also attempts to take into account certain effects which are not given by the impulse approximation.

In addition, we shall discuss experiments in proton, deuteron and He⁴ Compton effect with polarized γ 's which seem very helpful for a better specification of the polarizabilities of the proton and neutron as well.

II. The amplitude for the elastic scattering of photons by deuterium in the framework of the impulse approximation is written

$$\langle f | F_d | i \rangle = \int d\vec{r} e^{\frac{i}{2} \vec{Q} \cdot \vec{r}} \left\{ {}^3X_{\varphi_d}(\vec{r}) [F_1 + F_2] {}^3X_{\varphi_d}(\vec{r}) \right\}, \quad (1)$$

where $F_{1,2}$ are the amplitudes of proton and neutron Compton effect, respectively; $\varphi_d(\vec{r})$ is the spatial part of the deuteron wave function (we consider here the deuteron to be in a purely S-wave ground state); 3X its spin part (triplet); $\vec{Q} = \vec{k} - \vec{k}'$ the momentum transfer to the deuteron, and $\vec{r}$ is the relative co-ordinate of the two nucleons.

In isotopic spin space the amplitudes F_1 and F_2 are written, respectively

$$\left. \begin{aligned} F_1 &= F^S + F^V \\ F_2 &= F^S - F^V \end{aligned} \right\}, \quad (2)$$

where the indices s and v mean the isoscalar and isovector part of the general nucleon amplitude F , respectively. From Eqs. (1) and (2) we see that only that part of F_1 and F_2 contributes to F_d which connects the spin triplet, isospin singlet initial deuteron ground state to the final deuteron ground state. The amplitude F in the centre-of-mass system of $\gamma + N$ (where N stands for the nucleon which scatters the photon) can be written in the form¹⁶⁾

$$\begin{aligned} F &= R_1(\hat{e}, \hat{e}') + R_2(\hat{k}' \wedge \hat{e}')(\hat{k} \wedge \hat{e}) + iR_3(\vec{\sigma}, \hat{e}' \wedge \hat{e}) + iR_4(\vec{\sigma}[\hat{k}' \wedge \hat{e}'] \wedge [\hat{k} \wedge \hat{e}]) + \\ &+ iR_5[(\vec{\sigma} \cdot \hat{k})(\hat{k}' \wedge \hat{e}' \cdot \hat{e}) - (\vec{\sigma} \cdot \hat{k}')(\hat{k} \wedge \hat{e} \cdot \hat{e}')] + \\ &+ iR_6[(\vec{\sigma} \cdot \hat{k})(\hat{k}' \wedge \hat{e}' \cdot \hat{e}) - (\vec{\sigma} \cdot \hat{k})(\hat{k} \wedge \hat{e} \cdot \hat{e}')]. \end{aligned} \quad (3)$$

Now, if we want to calculate the cross-section of γ -d elastic scattering in the centre-of-mass system γ -d, we have to express all the quantities that appear in the amplitude (3) in the γ -d centre-of-mass system. That is, we have to perform a Lorentz transformation from the γ -N to the γ -d c.m. systems, which are related by a relative velocity $\vec{\beta}$ given by

$$\vec{\beta} = \frac{\vec{k} + 2\vec{p}}{2(E_N + k)}, \quad (4)$$

where $\vec{p}$ is the relative momentum of the two nucleons in deuterium and E_N the initial energy of the nucleon which scatters the photon.

With the simplifying assumption that the deuteron D-state contributions are negligible and that the amplitudes $F_{1,2}$ are linearly dependent on $\vec{p}$, one easily obtains for the amplitude (1)

$$\langle f | F_d | i \rangle = \left({}^3X \left[(L_1 + L_2) + \frac{\sigma_1 + \sigma_2}{2} (\vec{K}_1 + \vec{K}_2) \right] {}^3X \right) F_0(Q), \quad (5)$$

where

$$F_0(Q) = \int e^{\frac{i}{2} \vec{Q} \cdot \vec{r}} \phi_d^2(r) d\vec{r}, \quad (6)$$

and L_1 and $\vec{K}_1$ (L_2 and $\vec{K}_2$) mean the spin independent and spin dependent part of the proton (neutron) amplitude, respectively.

Now, introducing the appropriate kinematical factors, and averaging over initial and summing over final deuteron states, we find that the differential cross-sections of the elastic photon-deuteron scattering for photons polarized perpendicular $d\sigma_{\perp}/d\Omega$ and parallel $d\sigma_{\parallel}/d\Omega$ to the plane of scattering in the centre-of-mass system of γ -d to have the forms:

$$\begin{aligned} \left(\frac{d\sigma_{\perp}}{d\Omega} \right)_{\gamma+d \rightarrow \gamma+d}^{c.m.} &= S(Q) \left(\frac{e^2}{M} \right)^2 \left[\frac{E_d}{E_d + k_d} \right]^2 \left\{ |R_1|^2 + \frac{2}{3} |R_3|^2 + \frac{4}{3} |R_4|^2 + \frac{2}{3} |R_5|^2 + \right. \\ &\quad + \frac{2}{3} |R_6|^2 + \frac{4}{3} \operatorname{Re} (R_4 R_5^* + R_3 R_6^*) + 2 \operatorname{Re} (R_1 P_2^* + \frac{2}{3} R_3 R_4^* + \frac{4}{3} R_3 R_5^* + \\ &\quad + \frac{4}{3} R_4 R_6^* + 2 R_5 R_6^*) y + \\ &\quad + \left[|R_2|^2 - \frac{2}{3} |R_4|^2 + 2 |R_5|^2 + 2 |R_6|^2 + \frac{4}{3} \operatorname{Re} (R_4 R_5^* + R_3 R_6^*) \right] y^2 + \\ &\quad \left. + \frac{4}{3} \operatorname{Re} R_5 R_6^* y^3 \right\}. \quad (7) \end{aligned}$$

$$\begin{aligned}
\left(\frac{d\sigma_{II}}{d\Omega}\right)_{\gamma+d \rightarrow \gamma+d}^{c.m.} &= S(Q) \left(\frac{e^2}{M}\right)^2 \left[\frac{E_d}{E_d + k_d}\right]^2 \left\{ |R_2|^2 + \frac{4}{3} |R_3|^2 + \frac{2}{3} |R_4|^2 + \right. \\
&\quad + \frac{2}{3} |R_5|^2 + \frac{2}{3} |R_6|^2 + \frac{4}{3} \operatorname{Re} (R_4 R_5^* + R_3 R_6^*) + 2 \operatorname{Re} (R_1 R_2^* + \frac{2}{3} R_3 R_4^* + \\
&\quad + \frac{4}{3} R_3 R_5^* + \frac{4}{3} R_4 R_6^* + 2 R_5 R_6^*) y + \\
&\quad + \left[|R_1|^2 - \frac{2}{3} |R_3|^2 + 2 |R_5|^2 + 2 |R_6|^2 + \frac{4}{3} \operatorname{Re} (R_4 R_5^* + R_3 R_6^*) \right] y^2 + \\
&\quad \left. + \frac{4}{3} \operatorname{Re} R_3 R_6^* y^3 \right\}. \tag{8}
\end{aligned}$$

The differential cross-section for unpolarized photons is

$$\left(\frac{d\sigma}{d\Omega}\right)_{\gamma+d \rightarrow \gamma+d}^{c.m.} = \frac{1}{2} \left[\left(\frac{d\sigma_{\perp}}{d\Omega}\right) + \left(\frac{d\sigma_{\parallel}}{d\Omega}\right) \right]_{\gamma+d \rightarrow \gamma+d}^{c.m.} \tag{9}$$

where $y = \cos \vartheta$ and ϑ is photon scattering angle, E_d is the total deuteron energy, and k_d the incident photon energy. All these quantities are in the γ -d centre-of-mass system, and they are a function of the corresponding quantities in the γ -N c.m. system through a Lorentz transformation. M and e are the mass and charge of the photon.

$$S(Q) = F_0^2(Q)$$

is the so-called "sticking factor" and its value gives the probability that the deuteron remains bound after the scattering. Finally $R_i = R_i^{(1)} + R_i^{(2)}$ ($i = 1, \dots, 6$) where the upper indices (1) and (2) refer to proton and neutron respectively.

Now, the problem of calculation of $d\sigma_{\perp}/d\Omega$, $d\sigma_{\parallel}/d\Omega$ and $d\sigma/d\Omega$ consists of the following three points:

- i) find the values of $S(Q)$ for the range of energy we are interested in (up to 300 MeV);
- ii) calculate the E_d , k_d , and $\cos \theta$ through the Lorentz transformation from the corresponding quantities of γ -N c.m. system, and
- iii) find out the expressions of $R_i^{(1)}$ and $R_i^{(2)}$ ($i = 1, 2, \dots, 6$).

Since we do not know much about the "sticking factor" $S(Q)$, we treated the point (i) in two ways. In the first one, we take the values of $S(Q)$ directly from the experimental data on the electron-deuteron elastic scattering¹⁷⁾, and with these values of the $S(Q)$ we divide the experimental differential cross-section of γ -d scattering. [We do so instead of multiplying the $S(Q)$ with the "right-hand" parts of Eqs. (7) and (8) in order to avoid mixing later the predictions of the theory with the uncertainties of the $S(Q)$.] In the other case, we divide the experimental differential cross-section by the values of $S(Q) = F_0^2(Q)$ taken from the formula:

$$F_0(Q) = \frac{2\alpha\beta(\alpha+\beta)}{(\alpha-\beta)^2} \frac{2}{Q} \left\{ \operatorname{arctg} \left(\frac{Q}{4\alpha} \right) + \operatorname{arctg} \left(\frac{Q}{4\beta} \right) - 2 \operatorname{arctg} \left(\frac{Q}{2(\alpha+\beta)} \right) \right\} \quad (10)$$

which is obtained using the Hulthén wave function for the deuteron, in the ground state (we consider it to be in a pure s state).

In the above formula, $\alpha^2 = M\epsilon$, ϵ = the binding energy of deuteron and $\beta = 6.6\alpha$.

For the point (ii) we take in a first approximation that $\vec{\beta} = 0$.

Finally comes the point (iii). The evaluation of the $R_i^{(1)}$ (amplitude of the photon-proton elastic scattering) up to energies that correspond to the first resonance in photoproduction, has been carried out by many authors¹⁶⁾.

The steps in the estimation of these amplitudes in the framework of the dispersion relations are:

- i) to write down the relativistic invariant amplitude for the process $\gamma + p \rightarrow \gamma + p$ ¹⁶⁾;
- ii) to find the relation between this amplitude and the non-covariant one, Eq. (3);
- iii) to express the dynamical functions R_i of Eq. (3) in terms of a number of multiple amplitudes ¹⁶⁾, those, for example, up to $J = 3/2$;
- iv) to assume that the main contribution to the amplitudes comes from the one-nucleon and the pion-nucleon intermediate states, and to write down the imaginary part of them, as a function of the photoproduction multipole amplitudes, through unitarity. The real part will be given by expressing the amplitudes R_i in terms of the relativistic invariant amplitudes and using the dispersion relations satisfied by these.

The evaluation of the eventual subtraction constants may be done with the help of the low-energy amplitude.

Indications also exist that certain contributions of the exchange type as, for example, those of Figure 1, may be of significance ¹⁹⁾.

These contributions cannot be obtained from the above-mentioned procedure, and one has to evaluate them separately. [As we mentioned before, in the case of the deuteron, diagram 1 (a) does not contribute.]

In order to clarify our discussion of the amplitude $R_1^{(2)}$, we shall list here, without derivation, a few formulae involved in the above-mentioned steps. The expression for R_i in terms of multipole amplitudes including those up to $J = 3/2$ can be written

$$\begin{aligned}
 R_1 &= \mathcal{E}_1 + 2\mathcal{E}_3 + 2\mathcal{E}_2 \cos \vartheta - \mathcal{M}_2 \\
 R_3 &= \mathcal{E}_1 - \mathcal{E}_3 + 2\mathcal{E}_2 \cos \vartheta + \frac{1}{2}\mathcal{M}_2 + \sqrt{6} C'(\mathcal{E}_3, \mathcal{M}_2) \\
 R_5 &= -\mathcal{E}_2 - \sqrt{6} C'(\mathcal{M}_2, \mathcal{E}_2) .
 \end{aligned} \tag{11}$$

The corresponding expressions for R_2 , R_4 and R_6 may be obtained from those for R_1 , R_3 and R_5 , respectively, by the substitution $E_i \rightleftharpoons M_i$. (The pattern of magnetic-dipole scattering is identical with electric-dipole scattering, except that the role of the electric and magnetic vectors is interchanged.) The expression for $\text{Im}R_1$ in terms of the photo-production multipole amplitudes including those up to $J = 3/2$ can be written:

$$\begin{aligned}\text{Im}R_1^{(1)} &= k \left\{ |E_1|^2 + 2|E_3|^2 + \frac{1}{3}|E_2|^2 \cos \vartheta - \frac{1}{6}|M_2|^2 \right\} \\ \text{Im}R_3^{(1)} &= k \left\{ |E_1|^2 + \frac{1}{3}|E_2|^2 \cos \vartheta - |E_3|^2 + \frac{1}{12}|M_2|^2 + \text{Re}(E_3^* M_2) \right\} \\ \text{Im}R_5^{(1)} &= -k \left\{ \frac{1}{6}|E_2|^2 + \text{Re}(E_2^* M_3) \right\}.\end{aligned}\quad (12)$$

Again the corresponding expressions for $\text{Im}R_2^{(1)}$, $\text{Im}R_4^{(1)}$, $\text{Im}R_6^{(1)}$ may be obtained from those for $\text{Im}R_1^{(1)}$, $\text{Im}R_3^{(1)}$, $\text{Im}R_5^{(1)}$, respectively, by the substitution $E_i \rightleftharpoons M_i$. The notation is the same as in Ref. 16.

In principle, instead of writing the $\text{Im}R_i$ as a function of the electric and magnetic multipoles, we could write it also as a function of the total cross-section of the pion-photoproduction, taking into account the formula^{**}):

$$\sigma_{\text{tot}} = \frac{4\pi}{3} \left(3|E_1|^2 + 2|E_2|^2 + 3|M_1|^2 + 6|M_3|^2 \right). \quad (13)$$

*) By $|E_i|$ and $|M_i|$ we mean the sum of the π^0 and π^+ photoproduction multipole amplitudes.

**) We should mention that for any kind of analysis of deuteron Compton effect, the information about the pion photoproduction should be taken directly from the nucleon targets, and not from deuteron ones, because in the latter case, the differential cross-section of pion photoproduction is affected appreciably by the deuteron structure. For example, in coherent π^0 photoproduction, the effect of rescattering sometimes changes the differential cross-section by a factor two.

The form $R_i^{(1)}$ for low energies, on the basis of the single-nucleon terms is $^{20-22}$

$$\begin{aligned}
 R_1^{(1)} &= -\frac{e^2}{M} \left(1 - y \frac{k}{M}\right) & R_2^{(1)} &= -\frac{e^2}{M^2} k \\
 R_3^{(1)} &= -\frac{e^2}{2M^2} k & R_4^{(1)} &= -\frac{e^2(1+\lambda)^2}{2M^2} k \\
 R_5^{(1)} &= 0 & R_6^{(1)} &= \frac{e^2}{2M^2} (1+\lambda)k .
 \end{aligned} \tag{14}$$

The symbols have the usual significance. In these formulae we have neglected terms higher than k^2 . For higher terms see Ref. 23.

Continuing on the point (iii), we now discuss the amplitude $R_i^{(2)}$. In principle, one may apply exactly the same procedure as for $R_i^{(1)}$, taking into account that the intermediate pions will be π^0 and π^- coming from the process

$$\begin{cases} \gamma + n \rightarrow \pi^0 + n \\ \gamma + n \rightarrow \pi^- + p \end{cases} . \tag{15}$$

The corresponding expressions $^{(14)}$ for $R_i^{(2)}$ are all zero, since the charge is zero, except for $R_4^{(2)}$ which has the form

$$R_4^{(2)} = -\frac{e^2 \lambda'^2}{2M^2} k .$$

Unfortunately, we do not have available experimental data for the process (15). To get around this difficulty, authors make the assumption that the photoproductions (15) are equal to (16),

$$\begin{cases} \gamma + p \rightarrow \pi^0 + p \\ \gamma + p \rightarrow \pi^+ + n \end{cases} \tag{16}$$

and they predict a differential cross-section of γ -d elastic scattering^{10,13,15}).

With this assumption, and by using the work of Jacob and Mathews²⁴⁾ one obtains the results of Fig. 2. As we see from this figure, when we use the $S(Q)$, as it is given by the formula (10), the prediction is not so bad, while in the case where we take the $S(Q)$ from the e-d experiment, serious discrepancies appear, especially at low energies. However, we cannot argue that the electron-deuteron elastic scattering experiment is wrong, since we cannot say that our calculations, in the present status, are precise.

To evaluate these cross-sections we neglected the contributions coming from the diagrams 1(b) and 1(c). It is not clear yet how to take into account these contributions. For the diagram 1 (b) we do not know the coupling^{*)} of η to $N\bar{N}$, nor the lifetime for its decay into two γ rays. Furthermore, we are not sure of the relative sign of its contribution to the above process.

Also diagram 1(c) is essentially unknown, but we expect that it should not be of great importance since the intermediate state that it simulates cannot be in resonance^{**)} .

We estimate the contributions 1 (b) and 1 (c) by using the same parameters for them, as in Ref. 19, which can be obtained in fitting p- γ scattering. Then the predictions of Fig. 2 become those of Fig. 3. It is seen that these contributions increase the discrepancy.

*) In the unitary symmetry scheme, this coupling is critical to the ratio D/F as we may see from the formula²⁵⁾

$$G_{\eta N\bar{N}} = \frac{1}{\sqrt{3}} (3 - 4\alpha) G_{\pi N\bar{N}},$$

with $\alpha/(1 - \alpha) = D/F$.

**) In Ref. 19 an experiment was proposed that is now in progress at Frascati²⁶⁾, and which may assist us to understand the above uncertainties.

We underline that the existing discrepancies between experimental data and theoretical curves are opposite in the case of proton and deuteron, whatever $S(Q)$ we use. This is why, when the above contributions improve the prediction for the proton, the predictions for the deuteron become worse.

This situation may be explained in the following ways.

- i) The contributions of the exchange type, Figs. 1(b) and 1(c) are unimportant, and the discrepancies in proton and deuteron Compton effect are due to other reasons.
- ii) The contributions are important, but at least the Fig. 1 (b) has an opposite sign to that which has been taken in Ref. 19. In this case something has probably been ignored in the isovector part of the proton Compton amplitude by which we may fit the data in the proton Compton scattering, while it does not affect the deuteron since it does not contribute there. Consequently, the η contribution may slightly improve the prediction for the deuteron^{*)}.
- iii) The impulse approximation is completely wrong. Or, finally,
- iv) the neutron Compton amplitude is very large in comparison with the proton one.

We consider here the first case as the more probable. However, the other cases also do not affect our latter conclusions, except (iii) which, in any case, seems to be improbable. This being so, let us see if other approximations we have already made are not justified, and how sensitive the predictions are to them.

*) One should also keep in mind that such experiments as scattering of photons by protons and deuterons are very delicate, and do not exclude the possibility that some of the existing experimental points are not correct. So, the opposite character of the discrepancies in the case of protons and deuterons is only artificial.

III. First of all, let us examine the last assumption that the rates of process (15) are equal to those of (16). Indirect experimental data say that it is not true. In fact $d\sigma_{\pi^-}/d\sigma_{\pi^+} > 1$ at least up to the first photoproduction resonance for all angles²⁷⁾. A rough analysis of the data indicates that the inequality $d\sigma_{\pi^-}/d\sigma_{\pi^+} > 1$ is essentially due to inequality of the electric dipoles E_1 in the $d\sigma_{\pi^-}/d\Omega$ and $d\sigma_{\pi^+}/d\Omega$, with $|E_1^{\pi^-}|^2 > |E_1^{\pi^+}|^2$.

At threshold, we have $d\sigma_{\pi^-}/d\sigma_{\pi^+} \simeq 1.3$ and since the electric dipole E_1 predominates there, let us suppose that in the range of energy from threshold up to the first resonance

$$|E_1^{\pi^-}|^2 \simeq 1.3 |E_1^{\pi^+}|^2.$$

On the other hand, we take

$$|M_3^{\pi^-}|^2 = |M_3^{\pi^+}|^2$$

and keep in mind that this multipole predominates at the first resonance as is well known. It is trivial now from the formulae (12) how to carry out this correction to the $R_i^{(2)}$ amplitudes. We should mention that the contributions of the dispersion integrals to the amplitudes are strongly dependent on the values of the photoproduction multipole amplitudes in the low-energy region.

From Eq. (12), we see that only the amplitudes $R_1^{(2)}$ and amplitude $R_3^{(2)}$ will change.

In Fig. 4 we give the results. As we see, for both $S(Q)$ we now have a significantly better agreement between the theoretical and experimental curves. (In the following, we shall call this correction, "correction a".)

Another point that should be investigated is the influence of the exchange forces between neutron and proton on the scattering of photons by deuterons. The exchange force is known to increase the electric

dipole photodisintegration cross-section. Consequently, one should expect it to also affect deuteron Compton scattering. This is not accounted for in the impulse approximation, and has to be computed separately. In Refs. 28-29, it has been pointed out that the effect of exchange forces is of significance, especially at low energies. But it was not clear how they would influence the results, because in the experimental measurements at that time only the outgoing photon was observed, and there was no separation of elastic and inelastic processes. Consequently, it was concluded in Refs. 3, 29, that the impulse approximation was ambiguous.

Fortunately, in more recent experiments^{13,14}, the recoil deuteron has also been detected. This simplifies the evaluation of the effect of exchange forces, since we need not worry about the proton-neutron final-state interaction. We may make use of the arguments of Ref. 28, which shows, for pure elastic scattering, exchange forces affect essentially only the non-spin dependent electric dipole amplitude of the deuteron Compton processes which, in the approximation of Ref. 28, can be written in terms of the dipole photodisintegration cross-section as follows :

$$T_d^{ED} = - (\vec{e} \vec{e}') \frac{e^2}{M} + e^2 \langle \phi_d(\vec{r}) | xV(\vec{r})(\vec{e} \cdot \vec{r})(\vec{e}' \cdot \vec{r}) | \phi_d(\vec{r}) \rangle + \\ + \frac{(\vec{e} \vec{e}')}{2\pi^2} \int_0^\infty \frac{\sigma_{dis}^{ED} k_\ell'^2 dk_\ell'}{k_\ell'^2 - k_\ell^2} \quad (17)$$

where $xV(\vec{r})$ is the exchange part of the potential. The integral in the Eq. (17) is quite small for energies above 50 MeV and we may forget its contribution. The exchange force term may be estimated by using a Hulthén potential for $V(\vec{r})$, and the result is $\sim -0.09 e^2/M$ for all angles, and for energies in the range 50 - 100 MeV. For higher energies, it decreases, but to be sure that we have exhausted this effect, we extrapolate this value to higher energies too. (In the following, we shall call this correction, "correction b".) The results of the above two corrections

a and b together, are shown in Fig. 5. We see that the influence of the exchange potential affects our predictions, in a similar way as the previous correction for π^- photoproduction. We have expected it since these two inelastic processes, photodisintegration and photoproduction, are in competition.

As we now see from Fig. 5, our prediction has been improved, in such a way as to have a satisfactory agreement between calculated curves and experimental data when we use the "sticking factor" $S(Q)$ as it is given by the formula (10), while when we use an $S(Q)$ as it is given by the e-d experiment, the previous discrepancy has been reduced.

At any rate, there are still approximations, for example we have taken $\vec{\beta} = 0$ and we neglected the internal momentum of the deuteron. They have also affected the prediction, in the sense that the curves are (slightly) lower than they could have been without this approximation. On the other hand, to neglect the deuteron D state does not seem to introduce an error higher than 5%.

The previous discussion on the various approximations forces us to make no conclusions either on the question of whether discrepancy still exists between theory and experimental data on the γ -d scattering, or on the question of the correctness or otherwise of the e-d elastic scattering experiments.

Nevertheless, the above analysis convinces us that the corrections a and b, which we have previously performed, improve the agreement between calculated curves and experimental data for whatever $S(Q)$ we take. Since they are consistent with other independent experimental information, we may believe that the effects simulated by these corrections are of certain significance to the γ -d elastic scattering processes, and they should be taken into account.

IV. Now, it is interesting to know that from the above analysis we may learn about the polarizability of the nucleon. We follow Lapidus⁷⁾ who defines the electric polarizability α_e as the coefficient of the square of the radiation frequency in the amplitude R_1 , and the magnetic polarizability α_m as the coefficient of the square of the radiation frequency of the amplitude R_2 (these amplitudes have been defined above) and we adopt

$$\frac{e^2}{\hbar c} \left(\frac{\hbar}{Mc} \right)^3 = 6.8 \times 10^{-44} \text{ cm}^3$$

as the unit in which these quantities are measured.

With these definitions, and with the values of the amplitudes $R_i^{(1)}$ given by Eq. (14) and keeping in the differential cross-sections only terms up to k^2 , in the case of the proton we have:

$$\frac{d\sigma_{\perp}}{d\Omega} = \left(\frac{e^2}{M} \right)^2 \left\{ \left(1 - \frac{2k^2}{M^2} \alpha_e + \frac{k^2}{4M^2} \beta_1 \right) - 2 \left(\frac{k^2}{M^2} \alpha_m + \frac{k^2}{4M^2} \beta_2 \right) y - \frac{k^2}{4M^2} \beta_3 y^2 \right\} \quad (18)$$

$$\begin{aligned} \frac{d\sigma_{\parallel}}{d\Omega} = & \left(\frac{e^2}{M} \right)^2 \left\{ \frac{k^2}{4M^2} \beta_4 + 2 \left(\frac{k}{M} - \frac{k^2}{M^2} \alpha_m - \frac{k^2}{4M^2} \beta_2 \right) y + \right. \\ & \left. + \left(1 - \frac{2k^2}{M^2} \alpha_e + \frac{k^2}{4M^2} \beta_5 \right) y^2 - \frac{2k}{M} y^3 + \frac{k^2}{M^2} y^4 \right\}. \end{aligned} \quad (19)$$

where

$$\begin{aligned} \beta_1 &= 2(1+\lambda)^4 + \lambda^2 \\ \beta_2 &= 2\lambda^3 + 5\lambda^2 + 4\lambda + 1 \\ \beta_3 &= \lambda^4 + 4\lambda^3 + 3\lambda^2 \\ \beta_4 &= (1+\lambda)^4 + \lambda^2 + 5 \\ \beta_5 &= 3\lambda^2 + 4\lambda - 8 \end{aligned}$$

and the other symbols as before.

Basing themselves on a similar approximation, Goldanskii et al.⁴⁾ have been able to analyse their experimental data on the proton Compton effect with unpolarized γ 's at very low energies, and to estimate directly the electric and magnetic polarizability of the proton. They found

$$\left. \begin{aligned} \alpha_e &= (9 \pm 2) 10^{-43} \text{ cm} \\ \alpha_m &= (2 \pm 2) 10^{-43} \text{ cm} \end{aligned} \right\} . \quad (20)$$

The possibilities of writing down the corresponding formulae in the deuteron case in the impulse approximation scheme are dubious, because for very low energy (below 50 MeV) the contribution from the photodisintegration is large. For energies below 50 MeV the integral in Eq. (17) is no longer small. It very much complicates any attempt to determine the elastic and inelastic scattering of photons. Consequently, direct determination of the neutron polarizability is excluded.

Instead, the indirect method of dispersion relations for determining the polarizabilities seems to be fairly good.

Physically one expects that if a nucleon has a certain capability for absorption, it must be able to emit as well and the larger the capability for a certain kind of absorption (electric or magnetic) the larger the capability for emission (production) of the same kind. But larger capability for a certain kind of emission means greater polarizability of the same kind. This physical situation is describable in the scheme of dispersion relations for energies above the pion photoproduction as follows⁷⁾:

$$\alpha_e = \frac{2}{\pi} \int_{k_t}^{\infty} \frac{dk}{k^2} \left\{ |E_1|^2 + 2|E_3|^2 + \frac{1}{3}|E_2|^2 - \frac{1}{6}|M_2|^2 \right\} \quad (21)$$

and

$$\alpha_m = \frac{2}{\pi} \int_{k_t}^{\infty} \frac{dk}{k^2} \left\{ |M_1|^2 + 2|M_3|^2 + \frac{1}{3}|M_2|^2 - \frac{1}{6}|E_2|^2 \right\}, \quad (22)$$

where k_t is the threshold of the pion photoproduction, and the other symbols as before.

It is just a little unfortunate that adopting the previous definition for α_e and α_m , we are forced to mix states of opposite character, but both E_2 and M_2 happen to be small for the range of energy in which these dispersion integrals give the main contribution, and, consequently, they cannot alter the positive nature of α_e and α_m .

Now we may make an estimation of the neutron electric polarizability α_e^n by using the same values of the neutron electric multipole amplitudes as before, which, as we saw, are in satisfactory agreement with the deuteron Compton effect. If we adopt the approximation that the rates of the process (15) are equal to those of (16), then from Eqs. (21) and (22) we see that the electric and magnetic polarizabilities of the neutron are equal to those of the proton.

If we also take into account the "correction a" and admit that the value of the electric polarizability of the proton is thus given by Ref. 4, we find that the neutron one is of the order of $\sim 1.2 \times 10^{-42} \text{ cm}^3$.

This value is in agreement with an estimate made in Ref. 6 by using a completely different method. It is also remarkable that it does not disagree with the quite recent experimental results of small-angle elastic scattering of neutrons by heavy nuclei^{3c)}, if one analysed them by making use of an optical model with appropriate nuclear potential parameters.

It appears unlikely from the data we used here, that the electric polarizability of the neutron is greater than 40% of the proton one, whatever $S(Q)$ we use. The question of the exchange-type contributions, of course, remains open. If these contributions are important and contribute with the same sign as in Ref. 19, then the electric polarizability could be greater.

V. As shown in the previous sections, the present experimental information is insufficient for a complete and unambiguous analysis of γ -p and γ -d elastic scattering experiments.

Thus we cannot come to a definite conclusion concerning the significance of a number of effects in the above experiments.

Consequently, if we want to learn something more about these effects, more restrictive experimental information is necessary for their clarification. Here we want to point out some of the advantages of using polarized photons for the above processes^{*)}.

If the percentage of D deuteron state is not larger than 5%, the approximation of Eq. (5) is quite good. In that case, we can completely avoid the uncertainties of the form factors by studying the ratio $d\sigma_{||}/d\sigma_{\perp}$ of the differential cross-section of deuteron Compton effect with photons polarized, parallel and perpendicular to the scattering plane. In this ratio, the form-factor dependence is eliminated^{**)} .

On the other hand, we find that the ratio $d\sigma_{||}/d\sigma_{\perp}$ is not sensitive to the corrections "a" and "b", while it is sensitive if the exchange-type contributions of Fig. 1 (b), 1 (c) are important. In Figs. 6 and 7 we give an example.

Furthermore, with the polarized γ 's we may isolate the electric polarizability from the magnetic one very easily. In fact for both the proton and the deuteron, the differential cross-section $d\sigma_{\perp}/d\Omega$ at $\theta_{c.m.} = 90^\circ$ is independent of R_2 , while the $d\sigma_{||}/d\Omega$ is independent of R_1 . [For the proton

*) It is interesting to notice that with the help of polarized γ 's obtained by using crystals³¹⁾, we also simplify the experimental procedure in the case of Compton scattering. This is because the bremsstrahlung spectrum in crystals gives a peak at a certain energy value, which we may exploit; consequently we arrange our apparatus in such a way that, although we remain with a reasonable count-rate, we may reduce considerably (sometimes ~ 100 times) the contamination of our data from the γ 's coming from the π^0 which are photoproduced at the same time³²⁾.

**) This characteristic of the ratio $d\sigma_{||}/d\sigma_{\perp}$ also appears in the case of coherent photoproduction of π^0 from deuterium with polarized γ 's. Furthermore, this ratio is almost independent of the rescattering of the π^0 on the spectator nucleon of the deuteron. An effect, which, in the case of unpolarized γ 's scattering, prevents us from drawing conclusions about coherent π^0 photoproduction³³⁾.

case, see the last formula of Ref. 19, and for the deuteron, see the formulae (7) and (8) of the present note.] In addition, at the energy for which the formulae (18) and (19) are valid, at $\vartheta_{\text{c.m.}} = 90^\circ$ $d\sigma_{\parallel}/d\Omega$ is independent of both the electric and magnetic polarizability. This is interesting because eventual discrepancies in the $d\sigma_{\parallel}/d\Omega$ at low energies and at $\vartheta_{\text{c.m.}} = 90^\circ$ should not be attributed to the polarizabilities of the nucleon.

We also want to notice that with the new experimental techniques (for example, He^4 gas scintillation counter to be used as a target) one may realize experiments of elastic scattering of photons by He^4 . In this process, we have only two amplitudes corresponding to the previous R_1 , which as we showed, contains the electric polarizability, and to R_2 which contains the magnetic one. Now if we do this experiment using polarized photons, apart from the fact that we may fix the photon's initial energy, at $\vartheta_{\text{c.m.}} = 90^\circ$ for photons polarized perpendicular to the scattering plane we only have contributions corresponding to the amplitude R_1 , and for photons polarized parallel to the scattering plane, we only have contributions corresponding to the amplitude R_2 . In this way, the analysis of experimental data is simplified, and it may turn out that an experiment of this kind will help in the better determination of the neutron electric and magnetic polarizability.

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FIGURE CAPTIONS

- Fig. 1 Exchange-type contributions to γ -N scattering.
- Fig. 2 Differential cross-sections of γ -d elastic scattering for unpolarized photons at $\vartheta_{\text{c.m.}} = 90^\circ, 110^\circ$ and 140° , calculated without exchange-type contributions.
- The experimental data are taken from Refs. 13 and 14. The experimental points shown correspond to the following angles: squares ($\square$), $\vartheta_{\text{c.m.}} = 90^\circ$; circles ($\bigcirc$), $\vartheta_{\text{c.m.}} = 110^\circ$; triangles ($\triangle$), $\vartheta_{\text{c.m.}} = 140^\circ$. They have been normalized with $S(Q)$ obtained either from the Hulthén wave function (open symbols), or from e-d experiment (shaded symbols).
- Fig. 3 Differential cross-sections of γ -d elastic scattering for unpolarized photons at $\vartheta_{\text{c.m.}} = 90^\circ, 110^\circ$ and 140° , calculated with the addition of the contribution of diagrams 1 (b) and 1 (c) ($\tau_\eta = 10^{-20}$ sec). The experimental data are as in Fig. 2.
- Fig. 4 Differential cross-sections of γ -d elastic scattering for unpolarized photons at $\vartheta_{\text{c.m.}} = 90^\circ, 110^\circ$ and 140° , calculated taking into account the "correction a" and without exchange-type contributions. The experimental data are as in Fig. 2.
- Fig. 5 Differential cross-sections of γ -d elastic scattering for unpolarized photons at $\vartheta_{\text{c.m.}} = 90^\circ, 110^\circ$ and 140° , calculated taking into account the "correction a" and the "correction b" and without exchange-type contributions. The experimental data are as in Fig. 2.
- Fig. 6 The ratios $d\sigma_{\parallel}/d\sigma_{\perp}$ of the differential cross-sections of γ -d elastic scattering for polarized photons at $\vartheta_{\text{c.m.}} = 90^\circ$
- 1) calculated without exchange-type contributions;
 - 2) calculated taking into account the "correction a";
 - 3) calculated taking into account the "correction a" and "correction b";
 - 4) calculated without taking into account the "correction a" and the "correction b", but including the contribution of diagrams 1(b) and 1 (c).

- Fig. 7 The ratios $d\sigma_{||}/d\sigma_{\perp}$ of the differential cross-sections of γ -d elastic scattering for polarized photons at $\vartheta_{c.m.} = 140^\circ$,
- I) calculated with and without the "correction a" and the "correction b";
 - II) calculated without taking into account the "correction a" and the "correction b" but including the contribution of diagrams 1 (b) and 1 (c).

c.)

b.)

a.)

FIG. 1

FIG. 2

FIG. 3

FIG. 4

FIG. 5

FIG. 7