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Twist-3 Contribution to the pion electromagnetic form factor
1. Introduction

There has been a lot of discussions about applying perturbative QCD (pQCD) to exclusive processes at large momentum transfer [1-15]. Although there is general agreement that pQCD can make successful predictions for the exclusive processes at asymptotic limit ($Q^2 \rightarrow \infty$), the applicability of pQCD to these processes at experimentally available $Q^2$ region has been being debated and attracted much of attention. The difficulties in practical calculation mainly come from the end-point singularity, i.e. in the end-point region ($x \rightarrow 0$), with $x$ being the fractional momentum carried by the parton) the virtuality of the intermediate states is small and the running couple constant $\alpha_s$ becomes large, thereby perturbation expansion might be illegal. However, perturbative calculation can be rescued with the help of some techniques to cure the end-point singularity [8-15], such as the incorporation of the transverse structure of the pion wave function [8-10], the introduction of an effective gluon mass [11] and a frozen running constant [11,12]. Recently, Li and Sterman [13,14] proposed a modified perturbation expression for the hadronic form factor by taking into account the renormalization group evolution of the pion's transverse momentum as well as Sudakov corrections. They point out that pQCD calculation for the pion form factor begins to be self-consistent at about $Q \approx 2\Lambda_{QCD}$, which is similar to the conclusions given in Ref. [8]. More recently, Ji, Pang and Szczepaniak [15] arrived at a similar conclusion as [8,13,14] by analyzing the factorization perturbation formalism for the pion form factor in the framework of light-cone time-order perturbative theory. These studies bring light for applying pQCD to exclusive processes at intermediate energy region.

However, there is still a crucial problem which has not been solved, that is although improved pQCD calculation for the exclusive processes is self-consistent at currently experimental accessible energy regions, the numerical predictions are general far smaller than the experimental data. For example, pQCD prediction for the pion form factor is about $1/2 \sim 1/3$ of the experimental data. There are two possible explanations: one is that non-perturbative contributions will dominate in this region; the other is that non-leading order contributions in perturbative expansions may be also important in this region. To make choice between the two possible explanations one needs to analyze all of the important non-leading contributions carefully. These contributions come from higher-twist effects, higher order in $\alpha_s$ and higher Fock states etc. Field, Gupta, Otto and Chang [16] pointed out that for the pion form factor the contribution from the next-leading order in $\alpha_s$ is about $1/2 \sim 30\%$. Employing the modified factorization expression for the pion form factor proposed by Li and Sterman [13,14], Refs. [17,18] considered the transverse momentum effect in the wave function and found that the transverse momentum dependence in the wave function plays the role to suppress perturbative prediction. Thus, it is necessary to study the other non-leading contributions such as from higher twist effects and higher Fock states.

It has been expected that the power corrections to the pion form factor ($\sim 1/Q^2$) which come from higher twist terms of pion wave function may be important in the intermediate energy region [19-23]. However, the calculations for these higher twist contributions are more difficult than that for the leading twist (2) because of the end-point singularity becoming more serious. The leading twist wave functions in the initial and final states being proportional to $2/3 \times (z_1$ and $z_2$ being the fractional momentum carried by the quark and anti-quark) and $y_1 y_2$ cancel the end-point divergent factor $1/2 Q^2$ in the hard-scattering amplitude. The asymptotic behavior of twist-3 wave function is $x$-independent, which has no help at all to cure the end-point singularity. In this case, Sudakov form factor is expected to be able to rescue the perturbative calculation. Unfortunately, the estimations for the twist-3 contribution in the medium energy region do not agree with each other [19-23] (see Fig. 1). Refs. [20] predicts that

$$F_2(x) = \frac{16\pi\alpha_s(Q^2)/4}{Q^2} \left[ 1 + \frac{m_0^2}{Q^2 Q^2 m_0^2} x^{\alpha_s/Q^2} \right] 3 (\ln(Q^2/\Lambda^2_{QCD}) + 1.5) \right),$$

where the first and second terms correspond to the leading twist (twist-2) and next-to-leading twist (twist-3) contributions respectively. $m_0 \approx 7$ MeV is the mean u- and d-quark masses. According to Eq. (1), the twist-3 contribution is larger than the asymptotic term (twist-2 contribution) in the region of $Q^2 \lesssim 30$ GeV$^2$. It is given in Ref. [21] that

$$F_2(Q^2) = \frac{16\pi\alpha_s(Q^2)/4}{Q^2} \left[ 1 + \frac{m_0^2}{Q^2 Q^2 m_0^2} 6\alpha_s(Q^2) \right] 3 (\ln(Q^2/\Lambda^2_{QCD}) + 1.5)$$

which predicts that the twist-3 contribution is larger than the twist-2 contribution at about $Q^2 \lesssim 15$ GeV$^2$. Ref. [22] gives another prediction

$$F_2(Q^2) = \frac{16\pi\alpha_s(Q^2)/4}{Q^2} \left[ 1 + m_0^2 (\ln(Q^2/\Lambda^2_{QCD}) + 1.5) \right]$$.  

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Eq. (3) tells us that the twist-3 contribution is about $2 \sim 0.6$ of the leading twist contribution at the energy region $2 \text{GeV}^2 \leq Q^2 \leq 10 \text{GeV}^2$. All of the above calculations (Eqs. (1), (2) and (3)) give correct power suppression ($\sim 1/Q^2$) behavior of the twist-3 contribution in the large $Q^2$ region, but their predictions for the dependence on $\ln Q^2$ are very different. The main reason for this difference is that Sudakov corrections are evaluated in different approximations in Refs. [20], [21] and [22]. In the modified hard-scattering approach (MHS(A)) for the exclusive processes proposed by Li and Sterman [13,14], the customarily neglected partonic transverse momentum is combined with Sudakov corrections, which provides a more reliable and systematic way to evaluate the Sudakov effect. Li and Sterman’s formalism is originally obtained for the hard twist-3 perturbation calculation. We point out that the MHS(A) can be extended readily to evaluate the non-leading twist perturbation contributions such as higher twist wave function contribution also. One manifest advantage of MHS(A) is that there is no other phenomenological parameter but the input wave function needed to be adjusted. This purpose of this letter is to analyse the twist-3 wave function contribution to the pion form factor in the framework of MHS(A).

2. Formalism

We first review the derivation of the modified hard-scattering formalism for the leading twist contribution to the pion form factor [13]. Taking into account the transverse momenta $k_1$ and $l_1$ that flow from the wave functions through the hard-scattering leads to a factorization form with two wave functions $\psi(x, k_1)$ and $\tilde{\psi}(y, l_1)$ corresponding to the external pions, combined with a hard-scattering function $T_H(x, y, Q, k_1, l_1)$, which depends in general on transverse as well as longitudinal momenta,

$$F_2(Q^2) = \int dx |\psi(x)|^2 \int dy |\tilde{\psi}(y)|^2 T_H(x, y, Q, k_1, l_1) \psi(x, k_1) \tilde{\psi}(y, l_1),$$

(4)

where $|dx| = dx_1 dx_2 2(1-x_1 - x_2)$, $|dy| = dy_1 dy_2 2(1 - y_1 - y_2)$, $Q^2 = Q^2 + P_1^2$, $P_3$ and $\mu$ is the renormalization and factorization scale. To the lowest order in perturbation theory, the hard-scattering amplitude $T_H$ is to be calculated from the one-gluon-exchange diagrams. Neglecting the transverse momentum dependence in the numerator one can obtain,

$$T_H(x, y, Q, k_1, l_1, \mu) = \frac{16\pi C_F \alpha_s(\mu) x y Q^2}{[x y Q^2 + k_1^2] [y^2 Q^2 + (k_1 - l_1)^2]},$$

(5)

where $C_F$ is the color factor. The first and the second terms in the denominator come from fermion and gluon propagators respectively.

Eq. (4) can be expressed in the $b$- and $h$-configuration via Fourier transformation

$$F_2(Q^2) = \int |db| \int |dh| \frac{\phi(b)}{(2\pi)^2} \frac{\bar{\phi}(h)}{(2\pi)^2} T_H(x, y, Q, b, h, \mu) \psi(x, b, P_1, \mu) \tilde{\psi}(y, h, P_3, \mu),$$

(6)

where wave functions $\psi(x, b, P_1, \mu)$ and $\tilde{\psi}(y, h, P_3, \mu)$ take into account an infinite summation of higher-order effects associated with the elastic scattering of the valence partons, which give Sudakov suppression to the large-$b$ and small-$x$ regions [13,24,25].

$$\phi(b) = \exp \left[-s(\xi, b, Q) - s(1 - \xi, b, Q) - 2 \int_{b_{1/2}}^{b} \frac{db'}{b'} \gamma_b(\mu(b')) \times \phi \left( \frac{b}{b'} \right) \right].$$

(7)

Here $\gamma_b = -\alpha_b/\pi$ is the quark anomalous dimension, $s(\xi, b, Q)$ is Sudakov exponent factor [13,24,25],

$$s(\xi, b, Q) = \frac{4\pi}{b_0^4} \ln \left( \frac{b_{1/2}^4}{b^4} + \frac{4\pi}{b_0^4} \left( \frac{1 + \xi}{4} - \frac{1}{4} \right) \right) - \frac{4\pi}{b_0^4} \ln \left( \frac{b_{1/2}^4}{b^4} + \frac{4\pi}{b_0^4} \left( \frac{1 - \xi}{4} - \frac{1}{4} \right) \right) - \frac{4\pi}{b_0^4} \ln \left( \frac{b_{1/2}^4}{b^4} + \frac{4\pi}{b_0^4} \left( \frac{1 + \xi}{4} - \frac{1}{4} \right) \right) \ln \left( \frac{4}{b_0^4} \right) + \frac{4\pi}{b_0^4} \ln \left( \frac{b_{1/2}^4}{b^4} + \frac{4\pi}{b_0^4} \left( \frac{1 - \xi}{4} - \frac{1}{4} \right) \right) \ln \left( \frac{4}{b_0^4} \right),$$

(8)

where

$$\xi = \ln [Q/(\sqrt{2} b_0)], \quad \beta = \ln (b_0),$$

$$a \xi^2 + b \xi + c = 0, \quad b_0 = \frac{53 - 2\beta_2}{12}, \quad \beta_2 = \frac{133 - 12\beta_1}{12},$$

$$A^{(2)} = \frac{23}{9} \text{F}_{1} = \ln \left( \frac{1}{\beta_2} \right)^{\frac{1}{2}},$$

(9)

$n_q$ is the number of quark flavors and $\gamma$ is the Euler constant.

Applying the renormalization group equation to $T_H$ and substituting the explicit expression for $T_H$, we have the following expression for the pion form factor

$$F_2(Q^2) = \int |db| \int |dh| \int |db'| \int |dh'| \int |db''| \int |dh''| T_H(x, y, Q, b, h, b', h', b'', h'') \psi(x, b, P_1, \mu) \tilde{\psi}(y, h, P_3, \mu),$$

(10)

where

$$S(x, y, Q, b, t) = \left[ \sum_{i=1}^{n_q} s(x, y, Q, b) + \sum_{i=1}^{n_q} s(x, y, Q, b) - \frac{1}{\beta_2} \ln \frac{1 - b}{1 - b'} \ln \frac{1 - b}{1 - b''} \right].$$

(11)

$K_0$ and $K_2$ are the modified Bessel functions of order zero. $t$ is the largest mass scale appearing in $T_H$,

$$t = \max (\sqrt{S} Q, 1/b, 1/b').$$

(12)

If $b$ is small, radiative corrections will be small regardless of the values of $x$ because of the small $\alpha_s$. When $b$ is large and $y$ is small, radiative corrections are still large in $T_H$, but $\varphi$ will suppress these regions. In Eq. (10), $\phi(x, 1/b)$ and $\phi(y, 1/b')$ are two input "wave functions" that respect the non-perturbative physics. In the large-$Q^2$ region, they can be taken as the standard distribution amplitudes [13,24,25].

$$\phi(x) = \sqrt{x} f(x, x_{1/2}), \quad \phi(y) = \sqrt{y} f(x, x_{1/2}).$$

(13)

In the above discussion, only the leading twist wave function is considered. Now, we address the contributions coming from the twist-3 wave functions. The operators which contribute to the twist-3 parts of the pion wave function include $\gamma_b$ and $\gamma_b \gamma_5$, and the two matrices might mix under the consideration of the evolution equation for two-quark state in the pseudoscalar channel. It is pointed out in Refs. [20,21] that the twist-3 wave function of pion can be expressed as

$$\phi^{(3)}(x, k_L) = \gamma_b \bar{\phi}_b \left[ 1 - \frac{2(1 - x) x}{Q^2} p^\mu p^\nu p^\rho p^\sigma - \frac{1}{2} \frac{1 + 1}{k_L^2} \right],$$

(14)

where $k_L$ is the partonic transverse momentum, and $\bar{\phi}_b$ is the distribution amplitude of twist-3 [19-22],

$$\bar{\phi}_b(x) = \frac{f_{\pi} m_{\pi}^2}{4\pi(m_{\pi}^2 + m_{\pi}(Q^2)^2)}$$

(15)

where $f_{\pi} = 92$ MeV is the decay constant of pion, $m_{\pi} = 139$ MeV is the pion meson mass and $m_{\pi}(Q^2)$ is the mean value of the $u$- and $d$-quarks masses at the scale $Q^2$,

$$m_{\pi}(Q^2) = \frac{\alpha_s(Q^2)}{\alpha_s(m_{\pi}^2)} \frac{1}{\sin \theta_C} m_{\pi}(Q^2),$$

(16)

with $\theta_C = 4/11 - 2b_0$, and $m_{\pi}(1 GeV^2) = \pm 2$ MeV.

The hard scattering amplitude for the twist-3 wave function differs from that for the twist-2 wave function, which turns out to be

$$T_H^{(3)}(x, y, Q, b, h, \mu) = \frac{8\pi C_F \alpha_s(\mu) x y}{[x y Q^2 + k_L^2]^2} [2 x y Q^2 + (k_L - 1)^2].$$

(17)
Following the derivation for the leading twist wave function we can obtain the twist-3 contribution to the pion form factor in the modified hard-scattering approach,

\[ F_3(Q^2) = \int d^2 p \int d^2 q \, \delta(2p + q) \, K_a(\sqrt{2} p Q) \, K_b(\sqrt{2} q Q) \, \gamma_5 (\bar{c} \gamma_5 q) \, \delta(p - q) \, \exp(-S(x, y, Q, k, l)) \]

(18)

It can be found that the hard scattering contributions \( T_{\pi}^{t-3} \) and \( T_{\pi}^{t-3} \) are divergent in the end-point region \( x, y \to 0 \), \( p_{\perp} \to 0 \). However, the twist-2 contribution to the pion form factor can be calculated readily because the twist-2 wave functions are proportional to \( x_{12} \) and \( y_{12} \) and cancel the divergent factor \( 1/\pi r_{12}^{Q^2} \) in the \( T_{\pi}^{t-2} \). Furthermore, the Sudakov corrections also suppress the contribution from the end point region. For the twist-3 contribution, the wave function is constant in the whole region of \( x \) (see Eq. (15)), which has no help at all to cure the divergent factor \( 1/\pi r_{12}^{Q^2} \) in the \( T_{\pi}^{t-3} \). In this case, the Sudakov form factor guarantees that the calculation is reliable since the factor \( e^{-Q^2} \) rapidly decreases to zero at the end-point region (see Eq. (11)).

3. Numerical result and discussion

We present the numerical evaluations for the twist-2 and twist-3 contributions to the pion form factor in Fig. 1. The thinner solid curve is the HISQ prediction for the twist-2 contribution (Eq. (10)). The thicker solid curve is twist-3 contribution to the pion form factor obtained in this work (Eq. (18), while the dash-dotted is the result of Refs. [20] (the second term in Eq. (11)). The dotted and dashed curves are the results given Refs. [21] (Eq. (22)) and [22] (Eq. (3)) respectively. All of the calculations given in this work, Refs. [20], [21], and [22] show that compared to the leading twist contribution, the twist-3 contributions are suppressed by the factor \( 1/Q^2 \) at asymptotic limit \( Q^2 \to 0 \). But the predictions are different in the medium and lower energy region. Our result is much larger than the result of Ref. [22] in the energy region of \( 2 \text{GeV}^2 < Q^2 < 40 \text{GeV}^2 \) and a little larger than the result of Ref. [21] as \( Q^2 > 5 \text{GeV}^2 \), Ref. [20] and this work give very similar results in the large-\( Q^2 \) region, although our prediction is a little smaller than the result of Ref. [20] at about \( Q^2 < 15 \text{GeV}^2 \). The Sudakov corrections are respected systematically in HISQ, while they are evaluated in various approximations in Refs. [20], 22], so the prediction in this work is more reliable. In Fig. 2, we include both twist-2 and twist-3 contributions (obtained in this work) to the pion form factor, and compare with the experimental data. The dotted and dashed curves are twist-2 and twist-3 contributions respectively, and the solid curve is the sum. Compared to the leading (twist-2) contribution, the twist-3 contribution is negligible at asymptotic limit since it is suppressed by the factor \( 1/Q^2 \). However, the twist-3 contribution is comparable with and even larger than the leading twist contribution at intermediate region of \( Q^2 \). Also it can be found that the perturbative calculations including both twist-2 and twist-3 contributions are larger than the experimental data at about \( Q^2 < 5 \text{GeV}^2 \). One can expects that the other nonleading contributions such as those coming from higher Fock states may be also important at lower energy regions.

In summary, we analysed the twist-3 contribution to the pion electromagnetic form factor in the modified hard scattering approach in which Sudakov corrections are respected systematically and compared to various approximations. It is found that the twist-3 contribution is enhanced significantly since the twist-2 wave functions are independent, while the twist-2 wave functions are proportional to \( x_{12} \) and \( y_{12} \) which cancel the end-point divergent factor \( 1/\pi r_{12}^{Q^2} \) in the hard-scattering amplitude. Thus, although it is suppressed by the factor \( 1/Q^2 \) as compared to the leading (twist-2) contribution, the twist-3 contribution is comparable with and even larger than the leading twist contribution at intermediate region of \( Q^2 \) (2 \to 40 \text{GeV}^2). The perturbative predictions including both twist-2 and twist-3 are larger than the experimental data at lower energy regions, which indicates the importance to study the other nonleading contributions at these energy regions.

Acknowledgments

This work was partially supported by the Postdoc Science Foundation of China and the National Natural Science Foundation of China.

Fig. 1

Fig. 2