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IMAGINARY FERMI CONSTANT G AS A MODEL OF CP VIOLATION

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The observation ¹⁾ that K_2^0 decays into $\pi^+\pi^-$ is generally interpreted as indicating a violation of CP invariance. In this note we accept this interpretation in contrast to various suggestions which are consistent with CP invariance. Nearly all models of CP violation require the addition to the standard interaction Hamiltonian of elementary particles a new CP violating interaction with a new coupling constant ²⁾. We present here a model that involves only the standard forms of interaction and requires no new coupling constant. Furthermore, the Hamiltonian of the model appears to involve a maximal violation of CP invariance.

We write the standard interaction Hamiltonian as

$$H = H_{00} + [c_L H_L + c_{10} H_{10} + c_{11} H_{11} + c_{01} H_{01} + h.c.] \quad (1)$$

where H_{00} is the usual strong (plus electromagnetic) interaction and H_L , H_{10} , H_{11} , and H_{01} represent the usual weak interactions responsible for mu-meson decay, neutron β decay and muon capture, strange particle $\Delta Q = \Delta S$ leptonic decays, and strange particle non-leptonic decays, respectively. The notation H_{qs} implies that the strongly-interacting particles involved satisfy $\Delta Q = q$, $\Delta S = s$. Using the standard forms ³⁾ for H_L , H_{10} , H_{11} , and H_{01} the coupling constants are $C_L = C_{10} = G$ and $C_{11} = C_{01} = G \tan \theta$, where $\tan \theta \simeq \frac{1}{4}$ and $G \simeq 10^{-5}/m_p^2$ with m_p the proton mass. The model consists solely in replacing G by iG .

It is clear that this model can be given in a great variety of alternative forms. It is possible to change the phase of the weak coupling constants in Eq. (1) without changing any physical consequence by gauge transformations that leave H_{00} invariant. There are three of these:

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$e^{-i\alpha e}$, $e^{-i\beta\mu}$, and $e^{-i\gamma S}$ where e is the ν_e number, μ is the ν_μ number, and S is the strangeness of the strongly interacting particles. It follows that the phase of c_L has no significance and that if we write

$$c_{qs} = |c_{qs}| \exp(i\delta_{qs})$$

there exists only one observable phase factor given by

$$\Delta = \delta_{10} + \delta_{01} - \delta_{11}$$

Our model corresponds to choosing $\Delta = \pi/2$. It is completely equivalent to choosing any two of the c_{qs} real and the other purely imaginary.

The basic feature of this model is that CP violating matrix elements are higher order in the weak interaction than CP conserving ones. Thus the CP violation in the K^0 decay amplitude comes from the interference between the contribution of the second-order diagram of Figure 1 and the usual first-order contribution from H_{01} . In Fig. 1 X may be a strong system with $S = 1$ and $Q = 1$ in which case the vertex a is due to H_{10} and b to $H_{11}^\dagger$ or X may have $S = 0$, $Q = -1$ in which case the vertex a is due to $H_{11}^\dagger$ and b to H_{10} . In either case the relative phase of the second-order contribution to the first order ⁴⁾ is essentially Δ . If we set the order of magnitude ⁵⁾ of the first-order contribution as $G \tan \theta m_p^2 / 4\pi$ then the second-order has the magnitude $G^2 \tan \theta [m_p^4 / (4\pi)^2] (\Lambda / m_p)^2$ and the ratio of second to first order is $G \Lambda^2 / 4\pi$. Here Λ represents the "weak interaction cut-off" associated with the integration over the internal momentum of the lepton system in the intermediate state ⁶⁾. The lack of observed cut-off effects in any known processes suggests $\Lambda > 1$ GeV while the success of weak interaction theory as a perturbation theory requires $\Lambda < 300$ GeV. We obtain the

experimental ratio ¹⁾ (10^{-2} to 10^{-3}) of CP violating to CP conserving effects by choosing Λ between 30 and 100 GeV. Similar arguments hold for the K^0 mass matrix for which the normal contribution is second order proportional to C_{01}^2 and the CP violating contribution, an example of which is shown in Fig. 2, is third order proportional to $C_{10}^* C_{01} C_{11}$. It should be emphasized that this discussion is purely phenomenological; no understanding is claimed for the value chosen for Λ or for a consistent method of calculating higher-order weak interactions.

The consequences of this model are as follows:

1. CP or T violating effects of the order 10^{-2} to 10^{-3} are expected generally in the non-leptonic decays of strange particles.
2. CP or T violating effects in leptonic decays of strange particles are expected to be of the order $|C_{10}^* C_{11}/C_{01}| \sim G m_p^2/4\pi \sim 10^{-6}$ and of non-strange particles $|C_{11} C_{01}^*/C_{10}| \sim G m_p^2 \tan^2\theta/4\pi \sim 10^{-7}$. An exception is the case of K^0 leptonic decays where CP violating effects (but not obvious T violating effects) occur because of the CP violation in the mass matrix itself. It is not, however, possible to deduce quantitatively these effects in K^0 decay from the observation of Ref. ¹⁾ alone as in the case of a superweak interaction ⁷⁾ with CPT invariance, since in the present model the observations of Ref. ¹⁾ do not determine the mass matrix uniquely.

The main disadvantage of this model may lie in the necessity to abandon the current-current form of weak interaction; the usual current J is the sum of four terms (two lepton terms and the $\Delta S = 0$ and $\Delta S = 1$ non-leptonic), the relative phases of which are arbitrary because of the gauge transformations discussed above and the over-all phase of which cancels out in $J^\dagger J$.

The Hamiltonian of this model has the appearance of maximal CP violation in the sense that the weak interaction is 90° out of phase with the strong. However, even though no new coupling is added, its predictable consequences are the same (neglecting effects of order 10^{-6}) as a phenomenological model in which CP violation is introduced solely by including in H_{01} a CP violating term with a coupling of 10^{-2} to 10^{-3} . Therefore we can think of no practical method of testing the most distinctive feature of this model. It is, however, distinguished from theories in the current-current framework in which the CP violation is introduced in the $\Delta S = 1$ and/or $\Delta S = 0$ currents⁸⁾ as well as from theories such as that of Ref. 2) by the negligible amount ($\sim 10^{-6}$ to 10^{-7} in the present model) of CP or T violation in all leptonic decays other than that of the K^0 . On the other hand it is distinguished from the theories of superweak⁷⁾ interaction by the non-negligible ($\sim 10^{-2}$ to 10^{-3}) CP or T violation in non-leptonic strange-particle decays in the present model.

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Phys. Rev. Letters 13, 138 (1964).
- 2) A recent suggestion by J. Prentki and M. Veltman (to be published) on the other hand identifies CP violation with the semi-strong interactions responsible for the breakdown of SU_3 symmetry. CP violation in $K^0 \rightarrow 2\pi$ occurs because of the complex phases with which other states are admixed with the state vector of K^0 (or of 2π). It is hard to understand in this model why the CP violating effect is so small since the K^0 by its large mass is likely to have relatively large admixtures; furthermore, SU_3 arguments (N. Cabibbo, Phys. Rev. Letters 12, 62 (1964), M. Gell-Mann, Phys. Rev. Letters 12, 155 (1964)) suggest that the $K^0 \rightarrow 2\pi$ amplitude is inhibited for the pure K^0 state in which case the admixed states should make a relatively larger rather than smaller contribution to the decay amplitude.
- 3) The "standard forms" are those derived from the current-current theory. Since we abandon this theory our designation of $C_{01} = G \tan \theta$ is clearly quite arbitrary but plays no essential role in what follows.
- 4) This is true only if we make the approximation of ignoring the energy-conserving intermediate states in Fig. 1. As indicated below the importance of Fig. 1 derives from very high effective masses of the lepton pair so that this approximation is satisfactory. In any case, the CP violation is proportional to $\sin \Delta$.

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- 5) All order-of-magnitude arguments made here and elsewhere relating K^0 decays and mass matrix to weak interaction coupling constants are unfortunately of dubious reliability. Our inclusions of the factors 4π is motivated by the analogy between $G/4\pi$ and the fine structure constant $e^2/4\pi$ in the intermediate boson model. Also the $K_1^0-K_2^0$ mass difference Δm has the correct order of magnitude when 4π is included: $\frac{\Delta m}{m_K} \sim G^2 m_p^4 \tan^2 \theta / (4\pi)^2$. I am indebted to Professor T.D. Lee for a discussion of this point.
- 6) The discussion of "weak interaction cut-offs" in diagrams such as those of Figures 1 and 2 as well as possible limits on the value of Λ have been given by B.L. Ioffe, Zh. Eksperim. i Teor. Fiz. 42, 1411 (1962), 38, 1608 (1960) [translations: Soviet Physics - JETP 15, 978 (1962), 11, 1158 (1960)]. A survey of the upper limits on Λ has been given by J. Nilsson and R.E. Marshak, Proceedings of the International Conference on Theoretical Aspects of Very High Energy Phenomena, CERN (1961). Unfortunately, the only significant limits found were based on the hypothesis that there was only one kind of neutrino.
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FIG. 1

FIG. 2