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SHADOW MODEL FOR THE HIGH-ENERGY PION-PROTON ELASTIC
BACKWARD PEAK

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ABSTRACT

We propose a model in which the elastic backward peak for high-energy pion-nucleon scattering is explained as the shadow of the production processes. The small height of the backward peak as compared to that of the forward one is due to the forward-backward asymmetries in the angular distributions of the secondaries in πp collisions. The application of the model to $\pi^+ p$ scattering at 8 GeV/c gives the right order of magnitude for the height of the backward peak and a width approximately equal to that of the forward one.

1.

INTRODUCTION

In the last few months, evidence for a very sharp peak in the backward elastic $\pi^\pm p$ scattering in the GeV region has been accumulating ¹⁾⁻⁵⁾. The most characteristic features of this peak are :

- a) The value of the differential cross-section at $\theta_{c.m.} = 180^\circ$ is much smaller than at $\theta_{c.m.} = 0^\circ$; for instance, for $\pi^+ p$ scattering at 8 GeV/c ²⁾

$$\left(\frac{d\sigma}{d\Omega}\right)_{180^\circ} / \left(\frac{d\sigma}{d\Omega}\right)_{0^\circ} \simeq \frac{1}{2000} .$$

- b) There is a marked difference between the backward cross-sections for $\pi^+ p$ and $\pi^- p$ scattering, the latter being consistently lower by a factor of the order of $10^2, 3, 5$).

- c) At not too high energies (incident momentum of the order of few GeV/c), the 180° differential cross-section shows a series of maxima and minima, whose positions correspond to the masses of the excited states of the nucleon. These bumps are superimposed on a background, which seems to decrease exponentially with the energy ⁴⁾. These features suggest that two different mechanisms are effective in the backward direction, one connected with the nucleon resonances, which produces the maxima and minima, and is more effective at lower energies, and the other one, responsible for the background.

- d) As far as the form and width of the peaks are concerned, the experimental situation is less clear.

Some experiments ²⁾ give indications for a backward $\pi^+ p$ peak which depends exponentially on the momentum transfer, much steeper than the forward diffraction peak, and for a backward $\pi^- p$ peak broader than the forward one.

2.

Other experimental indications ^{1),3)} favour an exponential backward peak for both $\pi^{\pm}p$ scattering, with a slope very similar to that of the forward one.

Other experiments ⁵⁾ are less conclusive, since the explored range of momentum transfer is too far from the backward direction. It is probably not unreasonable to conclude that, as a first approximation, both $\pi^{\pm}p$ backward peaks have an exponential form with a slope not too different from that of the forward diffraction peak, i.e.,

$$\left(\frac{d\sigma}{d\Omega}\right)_{\theta \sim 180^\circ} = \left(\frac{d\sigma}{d\Omega}\right)_{180^\circ} \exp\left\{b_B(t_{\max} - t)\right\},$$

where $t_{\max}$ is the value of the momentum transfer for $\theta = 180^\circ$.

In the past, three main kinds of models have been proposed to account for the backward elastic peaks.

The first one consists in applying near the backward direction the optical model ^{6),7)}, much in the same way as one does for the forward peak. In this way one gets a peak which has a width of the same order of magnitude as that of the forward one (or also narrower). The ratio between the forward and backward cross-sections is of the order of $k^2 L^2$ (L is the maximum value of the angular momentum ⁷⁾). The difficulties in such a simple model are that it does not allow any simple explanation of the different values of the backward cross-sections of π^+p and π^-p scattering. Moreover it gives a backward cross-section which decreases only as $1/k^2$.

A second way is to assume that the backward scattering is dominated by the nearby singularities in the u channel ⁷⁾⁻¹⁰⁾. If one uses the pole approximation (neutron or N^{*0} exchange for π^+p , N^{*++} exchange for π^-p scattering), the Born approximation gives a

result in complete disagreement with experiment for both the magnitude of the backward cross-section (2 orders of magnitude too large) and its energy dependence (the N^* exchange contribution increases with the energy). The inclusion of absorption and form factor corrections could improve these results ¹⁰⁾; but we think that it would be difficult to get a peak as steep as is found experimentally.

A third possible explanation is given in terms of Regge poles ¹¹⁾, where the neutron and the N^* are considered as reggeized particles. This is possibly a good description, but, in the same way as in forward scattering, it is not a complete dynamical model, in the sense that its parameters are phenomenological functions to be determined from the fits of the experiments.

The similarities of the backward and forward elastic peaks can be taken as an indication of a common dynamical origin. Indeed Byers and Yang ¹²⁾ have put forward the idea that all the small momentum transfer peaks (forward elastic and charge exchange scattering, quasi-two-body scattering, backward elastic scattering) are due to some coherent phenomenon.

It is therefore interesting to explore the possibility of explaining the elastic backward πp peak (or, at least, the non-resonant background) in the same way as the forward diffraction peak. As it is well known, the general features of the forward diffraction scattering can be correctly accounted for as due to the shadow of the production processes through the unitarity of the S matrix ^{13),14)}. The sharp forward peaking of the elastic angular distribution is explained in this way as due to the jet structure of the secondary particles produced in the inelastic collisions.

In this paper we try to apply the same idea to backward elastic scattering ^{*)}. In Section 2 we shall discuss the basic assumptions and general features of such a model. The fact that the backward peak is much smaller than the forward one is shown to be due to the forward-backward asymmetry of the c.m.s. angular distribution of the secondary particles.

Section 3 will be devoted to the application of the model to π^+p scattering at 8 GeV/c. We obtain the right order of magnitude for the backward differential cross-section. The width of the backward peak turns out to be similar to that of the forward one.

In the concluding section we give a qualitative explanation for the different magnitudes of the π^+p and π^-p backward cross-sections. We also discuss the problem of the energy dependence of the backward cross-section.

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^{*)} This kind of explanation is not in contradiction with the Regge pole description. In fact, as in forward diffraction scattering, it can be considered as a way to compute the parameters of the Regge trajectories from the shadow of the production processes ¹⁵⁾.

2. THE GENERAL FEATURES OF THE MODEL

The forward diffraction peak for the high energy elastic differential cross-section is usually explained as mainly due to the imaginary part of the scattering amplitude. This assumption can be experimentally tested only in the forward direction, where both the imaginary part (through the optical theorem) and the real part of the scattering amplitude are measured. For any other angles within the forward peak the dominance of the imaginary over the real part is a plausible assumption, since in this way one can account for the observed exponential behaviour of the differential cross-section.

As far as the backward peak is concerned, even at 180° one cannot get from the experiments the ratio of the real over the imaginary part ^{*)}, due to the lack of an optical theorem ^{**)}. Nevertheless, the fact that the momentum transfer dependence of the backward peak is very similar to that of the forward one, suggests that also the former could be mainly due to the imaginary part of the scattering amplitude. Moreover the unitarity condition, as we shall see, can account for a steep variation of the imaginary part near both the forward and backward directions, whereas it is not so easy to conceive of a simple mechanism, providing such a variation for the real part.

In any way, we take as our main assumption that also the backward peak is mostly due to the imaginary part. Therefore we will take from now on the scattering amplitude as purely imaginary within the forward and backward peaks. This physically means that also the backward peak is considered to be the shadow effect of the production processes.

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^{*)} This can be done only at lower energies, where a phase shift analysis can be performed.

^{**)} Except when, as in pn charge exchange, charge independence expresses the backward imaginary amplitude as a combination of total cross-sections.

This model can be applied to any elastic backward peak ; for sake of definiteness we refer in what follows to elastic pion-proton scattering, neglecting the spin of the proton.

The unitarity condition for the elastic scattering amplitude is

$$A(k, \theta) = \gamma_m \langle \vec{k} | T | \vec{k}' \rangle \propto \sum_n \langle \vec{k} | T | n \rangle \langle n | T^* | \vec{k}' \rangle, \quad (1)$$

where $\vec{k}$ and $\vec{k}'$ are the c.m.s. momenta of the initial and final pions, θ is the c.m.s. scattering angle, and the sum is taken over all the states $|n\rangle$, which can be reached both from the initial and final state. The matrix elements $\langle \vec{k} | T | n \rangle$ are related to the production processes ; more precisely $|\langle \vec{k} | T | n \rangle|^2$ is proportional to the cross-section for the inelastic scattering of the initial state into the channel $|n\rangle$.

The inelastic processes at high energies are known to be characterized by some general features, which allow a qualitative understanding of the steep behaviour of the forward diffraction peak^{16), 19)} :

- a) in the c.m.s. the secondary particles are concentrated into two narrow cones around the forward and backward directions ;
- b) the transverse momentum of the secondaries is small (of the order of 400 MeV/c), and constant with the energy ;
- c) a large fraction (80%) of the secondaries is constituted by pions of not too high energies whose angular distribution in the c.m.s. is almost forward-backward symmetrical ;
- d) a large fraction of the energy is taken by a few energetic particles (leading particles), the final nucleon and a few others, whose angular distribution is strongly asymmetrical in the c.m.s. .

We can now introduce this information in the unitarity condition (1). In the forward direction ($\vec{k} = \vec{k}'$) each term of the sum (being a modulus squared), is positive definite, and therefore all the contributions add up in a coherent way to build up the forward coherent imaginary amplitude (remember the optical theorem). Remembering now properties a) and b), we can say that $\langle \vec{k} | T | n \rangle$ will be different from zero only when the angles between the initial direction and those of the particles in the intermediate states $|n\rangle$ are small, (or near to π); since the same property is also valid for $\langle n | T^* | \vec{k}' \rangle$, each product $\langle \vec{k} | T | n \rangle \langle n | T^* | \vec{k}' \rangle$ [and also therefore $A(k, \theta)$] will vanish very rapidly when the scattering angle θ increases from 0. The decrease law for $A(k, \theta)$ as a function of θ depends, moreover, also on the angular dependence of the phase of the production amplitude $\langle \vec{k} | T | n \rangle$. The actual angular dependence of the shadow scattering depends therefore on the detailed properties of the production mechanism, but its gross features, namely the existence of a forward peak, can be deduced from the cone structure of the production cross-sections.

This is essentially the mechanism proposed by Van Hove^{13),14)} to explain the forward elastic peak; moreover it has been shown that the width of the forward peak depends mainly on the transverse and longitudinal momentum distribution of the (large number of) produced pions, whereas the leading particles, under plausible assumptions, have no effect on it¹⁷⁾.

The same reasoning can now be applied also near the backward direction. Let us take the unitarity condition for $\vec{k}' = -\vec{k}$

$$A(k, \theta = \pi) = \sum_n \langle \vec{k} | T | n \rangle \langle n | T | -\vec{k} \rangle \propto \sum_n |\langle \vec{k} | T | n \rangle| |\langle n | T | -\vec{k} \rangle| \cos[\delta_n(\vec{k}) - \delta_n(-\vec{k})]$$

$$(\langle \vec{k} | T | n \rangle = |\langle \vec{k} | T | n \rangle| e^{i\delta_n(\vec{k})}) \quad (2)$$

8.

Owing to the forward-backward cone structure of the secondaries, the product of the production amplitudes will be different from zero only when the angles between the initial and intermediate directions are near to zero (and those between the final direction and the intermediate ones are near to π), or in the symmetrical situation, when the angles between the initial and the intermediate directions are near to π (and the angles final-intermediate are near to zero).

Therefore when the scattering angle θ decreases from 180° , again $A(k, \theta)$ will rapidly decrease (unless the very unlikely situation happens, in which the decrease of $|\langle \vec{k} | T | n \rangle| |\langle \vec{k}' | T | n \rangle|$ is compensated by an increase of the phase factor).

One can then conclude that the cone structure of the production processes predicts, via the unitarity condition, the existence of a peak for the imaginary part of the scattering amplitude near both the forward and backward directions.

In order to discuss the magnitude of the backward differential cross-section and the width of the backward peak, we shall work, for simplicity, in the framework of the uncorrelated particle model ¹³⁾.

This model is characterized by the absence of correlations among the secondaries (except those given by the over-all energy momentum conservation). With this hypothesis the unitarity condition reads

$$A(k, \theta) = \frac{k}{4\pi} \int d\Omega' A(k, \theta') A(k, \theta'') + F(k, \theta) \quad (3)$$

where the overlap function $F(k, \theta)$ can be written as ¹³⁾

$$F(k, \theta) = \sum_N \int d^3q_1 \dots d^3q_N \left[\prod_{i=1}^N \psi_i^*(\vec{q}_i) \psi_i(\vec{q}_i) \right] \delta^{(4)} \left(\sum_{i=1}^N q_i^\mu - P_0^\mu \right). \quad (4)$$

Here N is a label for the inelastic channels (specifying also the number of particles), P_0 is the total energy momentum four-vector in the c.m.s., $\psi_i(\vec{q}_i)$ is the i^{th} particle wave function in momentum space ($|\psi_i(\vec{q}_i)|^2 d^3q_i$ is proportional to the probability of finding the i^{th} produced particle in the interval between $\vec{q}_i$ and $\vec{q}_i + d\vec{q}_i$), and $\vec{q}'_i$ is obtained from $\vec{q}_i$ with a rotation of an angle θ around an arbitrary axis perpendicular to $\vec{k}$.

Taking as variables the longitudinal and transverse momenta q_L and q_T , one gets for the imaginary part of the scattering amplitude in the forward and backward directions ^{*})

$$A(k,0) = \frac{k}{4\pi} \left\{ d\Omega A^2(k,\theta) + \sum_N c_N (2\pi)^N \int_{-\infty}^{+\infty} dq_{L1} \dots dq_{LN} \int_0^\infty q_{T1} dq_{T1} \dots q_{TN} dq_{TN} \right. \\ \left. \left[\prod_{i=1}^N |\psi_i(q_{Li}, q_{Ti})|^2 \right] \delta^{(4)}\left(\sum_{i=1}^N q_i^\mu - P_0^\mu\right) \right\}, \quad (5)$$

$$A(k,\pi) = \frac{k}{4\pi} \left\{ d\Omega A(k,\theta) A(k,\pi-\theta) + \sum_N c_N (2\pi)^N \int_{-\infty}^{+\infty} dq_{L1} \dots dq_{LN} \int_0^\infty q_{T1} dq_{T1} \dots q_{TN} dq_{TN} \right. \\ \left. \left[\prod_{i=1}^N |\psi_i(q_{Li}, q_{Ti})| |\psi_i(-q_{Li}, q_{Ti})| \cos\{\delta_i(q_{Li}, q_{Ti}) - \delta_i(-q_{Li}, q_{Ti})\} \right] \right. \\ \left. \delta^{(4)}\left(\sum_{i=1}^N q_i^\mu - P_0^\mu\right) \right\}. \quad (5')$$

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^{*}) The elastic amplitude is normalized in such a way that the optical theorem reads $A(k,0) = k \sigma_T / 4\pi$. In the elastic contribution to the unitarity condition we have replaced the scattering amplitude by its imaginary part not only inside the forward and backward peaks (where it has been assumed to be purely imaginary), but everywhere. Neglecting the real part outside the peaks gives a negligible error, since almost all the contributions to the integral come from the peaks.

where c_N are normalization constants giving the relative weight of the channels

$$\frac{R}{4\pi} \sigma_N = c_N (2\pi)^N \int dq_{l1} \dots dq_{lN} \int dq_{t1} dq_{t1} \dots dq_{tN} dq_{tN} \left[\prod_{i=1}^N |\varphi_i(q_{li}, q_{ti})|^2 \right] \delta^{(4)}\left(\sum_i q_i^\mu - P_0^\mu\right). \quad (6)$$

As it can be seen from Eqs. (5) and (5'), there are two mechanisms which tend to reduce the height of the backward peak as compared to that of the forward one.

The first one (amplitude effect) comes from the obvious inequality

$$|\varphi_i(q_{li}, q_{ti})|^2 + |\varphi_i(-q_{li}, q_{ti})|^2 \geq 2 |\varphi_i(q_{li}, q_{ti})| |\varphi_i(-q_{li}, q_{ti})|. \quad (7)$$

The equality holds only when the angular distribution of the i^{th} particle is forward-backward symmetric. Therefore any forward-backward asymmetry in the angular distribution of any secondary particle has the effect of reducing the magnitude of the backward peak as compared to that of the forward one. The main contribution to this amplitude effect comes obviously from the (strongly asymmetrical) leading particles.

The second mechanism (phase effect) comes from the variation of the relative phase $\delta_i(q_{li}, q_{ti}) - \delta_i(-q_{li}, q_{ti})$ with the longitudinal momentum.

Let us now turn to the problem of the width of the peaks. The simplest way to obtain an exponential form for the forward peak is to assume, following Van Hove¹³⁾, that the intermediate state in the overlap function consists of a fixed number $N \gg 1$ of identical particles^{*)}.

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*) For a more sophisticated treatment of the width of the forward peak see Refs. 14), 17).

Further simplifications are introduced when one adopts for the one-particle wave function the factorized form

$$\Psi(q_e, q_t) = f_e(q_e) f_t(q_t) \exp \{ i [S_e(q_e) + S_t(q_t)] \}. \quad (8)$$

(f is a real function)

The basis of this factorization, as far as the modulus of $\Psi(q_e, q_t)$ is concerned, lies in the experimental evidence of the constancy of the mean transverse momentum of the secondaries ; for the phase it is just a simplifying hypothesis.

With these hypotheses the form of the elastic scattering amplitude at small angles is ¹³⁾

$$A(k, \theta) = A(k, 0) e^{-i a_F N \theta^2}, \quad (9)$$

where

$$a_F = a_t \cdot \frac{\int_{-\infty}^{+\infty} dq_e q_e^2 f_e^2(q_e)}{\int_{-\infty}^{+\infty} dq_e f_e^2(q_e)},$$

$$a_t = \frac{1}{4} \int_0^\infty q_t dq_t \left[\left(\frac{df_t}{dq_t} \right)^2 + f_t^2 \left(\frac{dS_t}{dq_t} \right)^2 \right] / \int_0^\infty q_t dq_t f_t^2. \quad (9')$$

In writing (9') use has been made of the experimental evidence that the average transverse momentum of the secondaries is much smaller than their average longitudinal momentum ^{*)}.

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*) Recently Fukuda and Iso ¹⁸⁾ have pointed out the possible importance of the contribution of the phase derivative $dS_t(q_t)/dq_t$ on the actual value of the width.

For the backward peak, with the same approximations, one gets

$$A(k, \theta = \pi - \theta') = A(k, \pi) e^{-a_B N \theta'^2} \quad (10)$$

where

$$a_B = a_t \cdot \frac{\int_{-\infty}^{+\infty} dq_e q_e^2 f_e(q_e) f_e(-q_e) \cos[\delta_e(q_e) - \delta_e(-q_e)]}{\int_{-\infty}^{+\infty} dq_e f_e(q_e) f_e(-q_e) \cos[\delta_e(q_e) - \delta_e(-q_e)]} \quad (10')$$

Comparing formulae (9') and (10') one sees that the ratio a_F/a_B depends only on the longitudinal momentum distribution and on the variation of the phase with q_e .

3. APPLICATION OF THE MODEL TO π^+p SCATTERING AT 8 GeV/c

As we have seen, in order to compute the height and width of the backward peak from formulae (5), (5') and (10), (10'), one has to know the wave functions of the secondary particles.

In the absence of a reliable theory for the production processes at high energies, we shall rely as much as possible on the experimental data.

In this framework the actual computation of the backward peak parameters can be performed only when the angular distributions of all the secondaries have been experimentally measured with good statistics, at least for those channels having an appreciable cross-section. For obvious reasons, there are inelastic channels for which the angular distribution of all the produced particles cannot be measured (e.g., when more than one neutral particle is present). We shall discuss later on how to overcome this difficulty.

In order to perform the actual computation we shall introduce two further assumptions :

- i) we neglect the dependence of the relative phase $\delta_i(q_{\ell i}, q_{ti}) - \delta_i(-q_{\ell i}, q_{ti})$ on the longitudinal momentum, i.e., we assume $\cos[\delta_i(q_{\ell i}, q_{ti}) - \delta_i(-q_{\ell i}, q_{ti})] = 1$.
- ii) we neglect the correlations between the secondaries which arise from energy and momentum conservation.

The first hypothesis is needed since experiments give no information on the phases of the production amplitudes ; as we have already said, it amounts to computing an upper limit for the backward differential cross-section [see Eq. (5')]. If the relative phase $\delta_i(q_{\ell i}, q_{ti}) - \delta_i(-q_{\ell i}, q_{ti})$ does not vary too much with the longitudinal momentum, one can hope to get the order of magnitude of the backward peak from the amplitude effect alone.

The second assumption is introduced to reduce the great computational complexity connected with the kinematical correlations. It allows all the particles to be treated as completely independent. One can expect this hypothesis to be better the larger the multiplicity and, as we shall see, the reactions with low multiplicity contribute only a small amount to the height of the backward peak, since the angular distributions of all the particles are strongly asymmetrical. Furthermore, it is reasonable to assume that the neglect of the kinematical correlations will affect in a similar way the computation of both $A(k,0)$ and $A(k,\pi)$, and therefore that the ratio $A(k,\pi)/A(k,0)$ will not be strongly affected by this assumption.

With these two last hypotheses, the ratio of the backward to the forward amplitude can be written as follows

$$C(k) \equiv \frac{A(k,\pi)}{A(k,0)} = \sum_N \frac{\sigma_N R_N}{\sigma_{tot.}} + C(k) \frac{\sigma_{el}}{\sigma_{tot.}} \frac{4 b_F}{b_F + b_B}, \quad (11)$$

where σ_N is the total cross-section in the channel N and the asymmetry factors R_N are given by

$$R_N = \frac{|\langle \vec{R} | T | N \rangle| |\langle N | T | -\vec{R} \rangle|}{|\langle \vec{R} | T | N \rangle|^2} = \prod_{i=1}^N \frac{\int_{-\infty}^{+\infty} dq_e \int_0^{\infty} q_t dq_t f_i(q_e, q_t) f_i(-q_e, q_t)}{\int_{-\infty}^{+\infty} dq_e \int_0^{\infty} q_t dq_t f_i^2(q_e, q_t)} \equiv \prod_{i=1}^N R_i. \quad (12)$$

The second equality follows from Eqs. (5) and (5'). In order to find the second term in Eq. (11) we have used an exponential form for the forward and backward peaks $A_F(k,t) = A_F(k,0) \exp \{b_F t/2\}$, $A_B(k,t) = A_B(k, t_{max}) \exp \{b_B(t_{max} - t)/2\}$. Equation (12) can be rewritten as

$$C(k) = \frac{\sum_N \sigma_N R_N}{\sigma_{tot.} - 4 \frac{b_F}{b_F + b_B} \sigma_{el}} \quad (13)$$

The 2 and 4 prong reactions for π^+p collisions at 8 GeV/c incident momentum have been recently analyzed with very good statistics by the Aachen-Berlin-CERN collaboration¹⁹⁾. We have evaluated the asymmetry factors R_N using the values of $f_i(q_{li}, q_{ti})$ obtained from the experimental $p_k^* - p_t$ distributions (Peyrou plots) for each particle in the reactions corresponding to 2 and 4 prongs (with no more than one neutral particle). The results are given in Table I.

It is worth noting that the main contribution to the asymmetry factor in any reaction comes from the nucleon. Moreover the asymmetry factor increases with the number of secondary pions : among the measured reactions, those with 4 pions in the final state give the largest contribution to the backward cross-section.

The other reactions which could not be fully analyzed and which had a non-negligible cross-section, are listed in Table II. The corresponding asymmetry factors have been estimated in the following way.

For the strange particles, the main part of the cross-section comes from reactions containing a small number of particles with strongly peaked angular distributions, and therefore we assume that the asymmetry factor is negligible.

For the reactions $\pi^+p \rightarrow p \pi^+ Z^0$, $\pi^+p \rightarrow \pi^+ \pi^+ Z^0$, (Z^0 means 2 or more neutral particles) most of the cross-sections come from the reactions $\pi^+p \rightarrow \pi^+ p \pi^0 \pi^0$, $\pi^+p \rightarrow \pi^+ \pi^+ \pi^0 n$ respectively, where always three pions having only two different charges are present in the final state. We can then assume that their asymmetry factors do not differ very much from those of reaction $p \pi^+ \rightarrow p \pi^+ \pi^+ \pi^-$. With a similar argument, we can try to ascribe to reactions $\pi^+p \rightarrow p \pi^+ \pi^+ \pi^- Z^0$, $\pi^+p \rightarrow \pi^+ \pi^+ \pi^+ \pi^- Z^0$ and to the six prong reactions, asymmetry factors of the same order as those of the reactions $p \pi^+ \rightarrow p \pi^+ \pi^+ \pi^- \pi^0$, $p \pi^+ \rightarrow n \pi^+ \pi^+ \pi^+ \pi^-$, where 4 pions are present.

Of course, all these arguments are very rough and can be modified by the fact that the angular distribution of pions in the different reactions can be affected in a different way by the formation of nucleon or boson resonances. Nevertheless it is reasonable to expect that the uncertainties in the asymmetry factors in Table II do not change the order of magnitude of the final result.

Inserting in formula (13) the values given in Tables I and II and the experimental values for the widths of the forward and backward peaks ²⁾ ($b_B \simeq 2b_F$), one gets

$$c(8 \text{ GeV}/c) = \frac{1}{22}$$

to be compared with the experimental value ²⁾ :

$$c(8 \text{ GeV}/c) \simeq \frac{1}{\sqrt{2000}} \simeq \frac{1}{45}$$

The agreement between the measured and predicted backward cross-section, although not accurate, is satisfactory and means that the backward peak can be qualitatively understood through the mechanism we have proposed. It should also be noticed that the number we have obtained is larger than the experimental one ; this is what one expects from neglecting the phase effect in Eq. (5'). Another important source of the difference between the measured and predicted values of $c(k)$ can be found in the fact that we have neglected all the correlations between the secondaries. It is well known experimentally that strong correlations actually exist, since about one half of the inelastic cross-section comes from the quasi-two-body channels. Our theoretical value of $c(k)$ would be smaller if these correlations were taken into account. This is due to both the peaked production of the resonances and the phases induced by the resonances on the production amplitudes.

As far as the widths of the peaks are concerned, formulae (10) and (10') are perhaps too naive to be directly compared with experiment at not too high energy, as in our case. Indeed they assume that only a fixed number $N \gg 1$ of identical particles are produced. Nevertheless, if the widths of the peaks are related mainly to the properties of the secondary pions, it is likely that the ratio of the backward to the forward width will not be affected very much by this assumption.

From (10) and (10') it follows

$$\frac{a_B}{a_F} = \frac{\int_{-\infty}^{+\infty} dq_e q_e^2 f_e(q_e) f_e(-q_e)}{\int_{-\infty}^{+\infty} dq_e q_e^2 f_e^2(q_e)} \cdot \frac{\int_{-\infty}^{+\infty} dq_e f_e^2(q_e)}{\int_{-\infty}^{+\infty} dq_e f_e(q_e) f_e(-q_e)}, \quad (14)$$

where, accordingly to assumption 1, we have neglected the phase effect.

Using in (14) the values of $f_e(q_e)$ coming from the pion distributions for reactions $\pi^+ p \rightarrow p \pi^+ \pi^+ \pi^- \pi^0$ and $p \pi^+ \rightarrow n \pi^+ \pi^+ \pi^- \pi^-$ (the ones with the highest multiplicity) one gets a mean value for the ratio a_B/a_F approximately equal to one.

This result is probably not much affected by having neglected the phase effect, since the latter tends to reduce both the numerator and the denominator in the ratio a_B/a_F [see (10')]. It is worth noting that this situation is different from that of the ratio $A(k, \pi)/A(k, 0)$, where the phase effect only affects the numerator.

Therefore one concludes that in first approximation the forward and backward peaks have the same width *).

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*) This result is not in very good agreement with the experiment of Frisken et al. ²⁾, which gives at 8 GeV/c a backward peak twice as narrow as the forward one. However, as we have said in the Introduction, the experimental information on the width of the backward peak is rather contradictory. If in formula (13) one puts $b_B = b_F$, the value of $c(8\text{GeV}/c)$ is changed to $c(8\text{GeV}/c) \simeq 1/18$.

4. CONCLUSIONS

We shall now briefly discuss the results we have obtained and some possible generalizations.

The diffraction model described in Section 2, when applied to the elastic backward peak for π^+p scattering at 8 GeV/c, gives a reasonably good agreement with experiment, at least for the order of magnitude of the scattering amplitude at 180° ; this in spite of the very crude approximations needed for the actual computations.

The computed value for the cross-section at 180° is larger than the experimental value, as it was to be expected from the neglect of the phase effect in Eq. (5'). If the other approximations do not affect significantly our final result, the contribution of this phase reduces by a factor of about 2 the computed value of the backward scattering amplitude.

The width we have computed is about the same as that of the forward peak. Of course, a more refined model than that discussed in Section 2, which would take into account more detailed effects, could give a backward peak broader or narrower than the forward one, but it is unlikely that the difference would be very large.

The root of the near equality of the forward and backward widths can be traced to the universality of the transverse momentum distribution of the secondary pions, and more precisely to the fact that the mean value of the transverse momentum is the same within the forward and backward cone of the produced particles. In this spirit, the similarity of the forward and backward width has to persist also at different energies.

It would be of the greatest importance to apply our analysis to π^+p scattering at different energies, in order to see whether the energy dependence of the parameters of the backward peak agrees with experiment. Work in this direction is in progress, but one can give a qualitative argument, according to which the cross-section of the backward

peak has to decrease with the energy. Indeed, in our model the value of the elastic cross-section at 180° is closely related to the probability that, in each inelastic channel, the secondary nucleon is produced with the maximum possible momentum transfer with respect to the initial proton. This probability has to decrease with the primary energy ; indeed, since the inelasticity is nearly constant, the maximum possible value of the momentum transfer increases with the primary energy, and the experimental evidence (coming also from cosmic rays physics) indicates that the high momentum transfers are strongly disfavoured. Of course, the actual energy variation of the height of the backward peak depends on the energy variation of the whole large momentum transfer distribution of the secondary nucleon. It is amusing to note that if one assumes a sort of universality for the energy dependence of the large momentum transfer distribution of the nucleons, taking it to be exponentially decreasing (as found experimentally in the large angle elastic πp ²⁰⁾ and pp scattering ²¹⁾), the height of the backward peak will also decrease exponentially.

It would also be very important to apply the model to π^-p scattering. Also in this direction work is in progress, but the experimental material at our disposal has not enough statistics to give definite results. However a qualitative argument shows that the backward peak for π^-p scattering has to be smaller than the corresponding one for π^+p scattering. The argument goes as follows : the production experiments in πp reactions show that the mean charge in the cone of the secondaries (without taking into account the leading particles) depends on the charge of the incident particles. For π^+p scattering both the forward and backward cone have a mean positive charge, whereas for π^-p scattering the mean charge of the forward cone has a negative sign and that of the backward cone a positive sign. This effect is illustrated in the Figure, where the data from the reaction $\pi^-p \rightarrow 3 \pi^- + 2 \pi^+ + p$ at 10 GeV/c are reported ²²⁾.

Therefore in π^-p reactions there is less coherence in the charge between the forward and backward cones, and this feature has the effect of reducing further the height of the elastic backward peak ^{*)}.

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REACTION	PARTIAL CROSS-SECTION (mb)	R_i				R_N
$\pi^+ p \rightarrow n \pi^+ \pi^+$	0.70	neutron 0.043	π^+ 0.28			0.012
$\pi^+ p \rightarrow p \pi^+ \pi^0$	0.91	proton 0.088	π^+ 0.15	π^0 0.29		0.0038
$\pi^+ p \rightarrow p \pi^+ \pi^+ \pi^-$	1.98	proton 0.087	π^+ 0.54	π^- 0.51		0.024
$\pi^+ p \rightarrow n \pi^+ \pi^+ \pi^+ \pi^-$	0.75	neutron 0.20	π^+ 0.64	π^- 0.52		0.066
$\pi^+ p \rightarrow p \pi^+ \pi^+ \pi^- \pi^0$	2.09	proton 0.25	π^+ 0.71	π^- 0.64	π^0 0.59	0.067
Elastic	4.89					

T A B L E I

REACTION	PARTIAL CROSS-SECTION (mb)	R_N
$\pi^+ p \rightarrow$ strange particles	1.44	~ 0
$\pi^+ p \rightarrow p \pi^+ Z^0$	2.13	0.024
$\pi^+ p \rightarrow \pi^+ \pi^+ Z^0$	2.17	0.024
$\pi^+ p \rightarrow p \pi^+ \pi^+ \pi^- Z^0$	2.35	0.066
$\pi^+ p \rightarrow \pi^+ \pi^+ \pi^+ \pi^- Z^0$	2.50	0.066
$\pi^+ p \rightarrow 6$ prongs	2.87	0.066

(Z^0 means more than one-neutral particle)

T A B L E II

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FIGURE   CAPTION

The difference $N_+ - N_-$ between the number of positively and negatively charged particles in the reaction

$$\pi^- p \rightarrow p \pi^+ \pi^+ \pi^- \pi^- \pi^- \quad (10 \text{ GeV}/c)$$

is plotted versus the longitudinal momentum in the c.m.s.

The forward-backward asymmetry of the charge is clearly shown.

$^{-}N - ^{+}N$

Fig. 1