The phase of $K_L - K_S$ regeneration is evaluated in a simple Regge pole model. The result is compared with data on the interference of $K_S$, $K_L \rightarrow 2\pi$ and found compatible with the Wolfenstein prediction if $M_L > M_S$. 

66/844/5 - TH. 678

21 June 1966
An interference effect between the $\pi^+\pi^-$ decays of $K_{S1}^0$ and $K_L^0$ has been recently demonstrated in a few independent experiments.

In these experiments a material regenerator is used to obtain - from a beam of $K_L^0$ - a coherent superposition of $K_L^0$ and $K_S^0$, with a relative phase which is determined - apart from other factors - by the phase of the regeneration amplitude in the forward direction. As long as this latter phase is not known, the experiments quoted above are not able to determine the phase of $\eta$, the ratio of the amplitudes

$$\eta = \frac{K_L \rightarrow \pi^+\pi^-}{K_S \rightarrow \pi^+\pi^-}$$  \hspace{1cm} (1)$$

In the "superweak" theory of Wolfenstein $5^1$, the phase of $\eta$ is predicted to be

$$\phi_\eta = \phi_\eta^{\psi} = \tan^{-1} \frac{2 \Delta M}{\Gamma_S}$$  \hspace{1cm} (2)$$

where $\Delta M = M_L - M_S$. The same prediction is approximately true in a large class of theories in which the $2\pi^0$ decay of $K_L^0$ happens through the $I=0$ channel and where the $\Delta S=\Delta Q$ rule is respected in $K_L^0$ decays and the $\Delta I=\frac{1}{2}$ rule is respected in $K^0 \rightarrow 3\pi^-$. The more accurate results so far have been obtained by the groups of Refs. 2) and 3). Their results are comparable in giving:

$$\phi_\eta - \phi_\pi = \pm(1.53\pm0.09)$$ \hspace{1cm} Refs. 2) and 6), carbon regenerator, average $K_L^0$ momentum : $p = 4.5$ GeV/c

$$\phi_\eta - \phi_\pi = \pm(1.41\pm0.18)$$ \hspace{1cm} Ref. 3), copper regenerator, $\overline{p} = 2.8$ GeV/c
The sign is the same as that of $\Delta M$, and

$$\phi_f = \arg \left[ i \left( \frac{f}{\bar{f}} \right) \right]$$

(4)

is the argument of the regeneration amplitude.

These results \(^6\) are to be compared with the prediction of Eq. (2), which \(^6\), assuming $2 \Delta M/\Gamma_S = \pm(0.960 \pm 0.044)$ give $\psi_\eta^W = \pm(0.78 \pm 0.03)$, the sign being again that of $\Delta M$. One concludes \(^3\) that either $\psi_\eta^\eta \neq \psi_\eta^W$, or that the phase of the regeneration amplitude is quite sizable:

$$\phi_f = \mp(0.75 \pm 0.12)$$

from Eq. (3), carbon

$$\bar{p} = 4.5 \text{ GeV/c}$$

(5)

Whichever is the correct alternative, $\phi_f$ seems not to be violently dependent upon energy and the details of nuclear structure.

In the present note we wish to point out that the regeneration phase $\phi_f$ can be obtained in a Regge pole model of $K-N$ scattering, and that the result indicates an agreement between Eqs. (2) and (3), if $\Delta M$ is positive \(^7\), i.e., $M_L \geq M_S$.

A remarkable success has been lately obtained by different authors \(^8\) in fitting meson-nucleon and nucleon-nucleon scattering with a restricted family of Regge poles: a nonet of trajectories of even signature \(^9\) and a nonet of odd signature corresponding to the reggeization of the familiar $1^-$ mesons. By use of these poles, it has been possible to obtain a good fit of total cross-sections above $\sim 3$ GeV, as well as of the real parts of the forward scattering amplitude, where the relevant information is available.

Let us concentrate on the case of regeneration in carbon at relatively high energies. Since we have to do with a nucleus with $I = 0$, the regeneration amplitude per nucleon, uncorrected for nuclear absorption, will be \(^10\) :
\[ f - \bar{f} = \frac{1}{2} \left[ \bar{f}(K^0P) + \bar{f}(\bar{K}^0N) - f(K^0P) - f(\bar{K}^0N) \right] \tag{6} \]

We are therefore restricted to the exchange of poles of negative C and \( I=0 \), i.e., to the singlet and eight-component of the nonet with odd signature, \( v_0 \) and \( v_8 \). Furthermore, in the SU(3) limit \( v_0 \) will not couple with the pseudoscalar mesons, so only \( v_8 \) remains.

Neglecting for a moment the complications arising from \( \phi - \omega \) mixing, we can then write:

\[ f - \bar{f} = -\left( \frac{-1 + \pi v_8}{\lambda_{IM} \pi x_8} \right) \times \text{real positive number} \tag{7} \]

The sign is fixed by the requirement that the exchange of \( v_8 \) gives rise to a repulsive force between \( K^0 \) and \( P \), which have the same hypercharge, and as a consequence we have the prediction [see Eq. (4)], uncorrected for nuclear absorption,

\[ f f = -\frac{\pi}{2} x_8 \tag{8} \]

In a more realistic model, singlet and eight-components would be mixed, although the mixing parameter at \( t=0 \) could differ from the one which prevails for physical \( \omega \) and \( \phi \). One would, in general, have two poles, \( v_\omega \) and \( v_\phi \), contributing to Eq. (6) and very different values of \( f f \) could be obtained. If, however, the residues of the two poles are both positive, as is the case of the fits obtained by Barger and Olsson \(^8\) with \( \beta \approx 0 \) [see Ref. 8], one still finds \( f f \) to be contained in the range \((-\frac{\pi}{2} \omega, -\frac{\pi}{2} \phi)\). Since the intercepts of the vector trajectories, as proposed by different authors \(^8\), lie in the general range \( \chi(0) = 0.3-0.5 \), the above argument strongly suggests that
\[ 0.4 < -\varphi_f < 0.8 \]  

in complete agreement with Eq. (5), if \( M_L > M_S \).

We do not believe that the nuclear corrections will drastically change the picture in the case of a light nucleus at relatively high energy, but this problem, as well as that of the \( \varphi - \omega \) mixing, deserves a more careful investigation \(^{11}\).

Although this note strongly suggests that Eq. (2) may be indeed correct, it cannot substitute for an independent measurement of \( \varphi_\pi \), or for a direct measurement of \( \varphi_\eta \) which avoids the use of a material regenerator.

I am grateful to the authors of References 2) and 3), for making their new data available, and in particular to Dr. M. Vivargent, Dr. C. Rubbia and Professor J. Steinberger. I am also grateful to Dr. J.S. Bell for a discussion of the possible corrections from nuclear absorption.
REFERENCES


6) M. Bott-Bodenhausen et al., Ref. 2), private communication. These data are also compatible with those from Ref. 3), obtained at a somewhat lower energy on a copper regenerator.

7) Recent experimental results reported by O. Piccioni at the 1966 Washington meeting of the American Physical Society favour a positive sign of $\Delta M$.

8) See, V. Barger and M. Olsson - "Forward Elastic Scattering at High Energy", University of Wisconsin preprint:


T. Binford and B. Desai, Phys.Rev. 138, B1167 (1965);


9) In the work of Barger and Olsson there are ten even trajectories, the Pomeranchuk pole being added as a singlet.

10) The argument should apply with excellent approximation to nuclei with a small neutron excess, including copper.

11) J.S. Bell suggests that the effects of nuclear absorption on $\phi_f$ will be small if $f+\bar{f}$ is mainly absorptive.