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SATURATION OF CHIRAL $U(3) \times U(3)$ ALGEBRA

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The $SU(3) \times SU(3)$ chiral algebra ¹⁾ has been successfully applied (together with the PCAC hypothesis) by Adler and Weisberger ²⁾ to determine the axial vector coupling constant g_A . The algebra in question is generated by the integrals of the fourth components of vector and axial vector currents, F^i and F_5^i :

$$[F^i, F^j] = i f^{ijk} F^k \quad (1)$$

$$[F^i, F_5^j] = i f^{ijk} F_5^k \quad (2)$$

$$[F_5^i, F_5^j] = i f^{ijk} F^k \quad (3)$$

The Adler-Weisberger result was obtained by considering a particular case of Eq. (3):

$$[F_5^+, F_5^-] = 2 F^3 \quad (4)$$

and taking the matrix element of the two sides between proton states of infinite momentum. The left-hand side is then computed by inserting a complete set of intermediate states, and using the PCAC ³⁾ hypothesis to express the contribution of states other than a single nucleon in terms of pion-nucleon total cross-sections.

It is interesting to ask whether it is possible to reduce the sum over intermediate states to a small number of discrete contributions. As a further step one could ask whether it is possible to find a set of particles which - in the limit of infinite momentum - saturate the algebra. By this we mean that taking the matrix element of the two sides of one of Eqs. (1)-(3) between two states belonging to the set, the result is accurately given by inserting on the left-hand side only states belonging to the same set.

The first attempt in this direction is due to Gerstein ⁴⁾, who tried saturation in a set corresponding to the 56 of SU(6), namely the octet with $J^P = \frac{1}{2}^+$ and the decuplet $\frac{3}{2}^+$, and obtained the same predictions of SU(6), in particular $g_A = 5/3$. More satisfactory results can clearly only be obtained by enlarging the set of particles. This has been recently tried by Gatto, Maiani and Preparata ⁵⁾, who added to the states of the 56 of SU(6) those contained in a 20 with ℓ excitation.

In the quark model these states could be described as built with three quarks with one unit of orbital angular momentum and in a completely antisymmetric state of spin and unitary spin. In this enlarged set, the solution is not unique, but they find two different solutions, containing some free parameter. In the one which the authors consider as more interesting, g_A is related to the D/F ratio of axial couplings :

$$g_A = \frac{D+F}{3(D-F)} \quad (5)$$

This relation is well satisfied by the experimental values of g_A and D/F : for $g_A = 1.2$ it gives $D/F = 1.75$ ⁶⁾.

The aim of the present paper is to see whether different choices of the additive set of states, corresponding to different classifications of the higher resonances, lead again to this relation or to others which are not in agreement with data. This could be used as an argument in order to discriminate between different possible classifications.

We shall restrict the problem by assuming that :

- i) the matrix elements of F^1 and F_5^1 between states of infinite momentum are related by the Clebsch-Gordan coefficients of SU(3), i.e., exact SU(3) relations for the matrix elements.

The matrix elements between states of two particles of the same mass are, given the relative parity, a unique function of helicity ^{5),7)}. Given assumption i), this is also true of matrix elements between particles which belong to the same SU(3) multiplet, even if not degenerate in mass.

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All such matrix elements can, in fact, be expressed in terms of diagonal ones. Although this does not cover the case of matrix elements between different $SU(3)$ multiplets having both spin greater than $\frac{1}{2}$ ⁸⁾, we will further restrict the class of solutions by assuming that :

- ii) the matrix elements of the F^i , F_5^i are unique functions of the helicity, as specified, e.g., in Ref. ⁵⁾.

As noted above, the second assumption may be derived from the first one in simple cases, like the one treated by Gerstein ⁴⁾.

Since the generators of the algebra commute with helicity, the system of equations implied by saturation of (1)-(3) within a certain set of particles at infinite momentum breaks into subsystems corresponding to the different values of the helicity.

Given a solution of the saturation equations for a certain set of particles, divided in $SU(3)$ multiplets of given spin and parity, the matrices of F^i and F_5^i within the set give a representation of the $SU(3) \times SU(3)$ algebra. So the problem is reduced to that of arranging the states of given helicity in one or more representations of $SU(3) \times SU(3)$. The representations chosen for different helicities form a family of representations, which we will require to satisfy the hypotheses i) and ii). A class of such families can be found in the following way : given assumptions i) and ii), the masses do not play any role in the problem, which is then formally identical with that of saturating the algebra in a completely degenerate set of particles. If these particles have all the same parity this latter problem is reduced to that of saturating the collinear algebra $SU(3) \times SU(3)$ at zero momentum ⁹⁾. A solution of this problem, which automatically satisfies i) and ii) is obtained by embedding the collinear $SU(3) \times SU(3)$ into a larger algebra, which is transformed into itself by rotations. The states at rest should be arranged - if possible - into a representation of the larger algebra which is then reduced into representations of $SU(3) \times SU(3)$.

4.

The most appealing choices for this larger algebra, and the ones suggested by the extended current algebra ¹⁰⁾ are $SU(6)$ or $SU(6) \times O(3)$.

The solution given by Gerstein can be obtained in this way from the 56 representation of $SU(6)$. The inclusion of more complicated sets of particles, like the one considered in Ref. ⁵⁾, can be taken into account by starting from two or more independent families of representations, corresponding to different multiplets of $SU(6) \times O(3)$ and allowing for some mixing of the physical states. The mixing should be compatible with our assumptions i) and ii).

If the set of particles under consideration contains n octets with spin $\frac{1}{2}$, in the helicity $\frac{1}{2}$ members of the family of representations, these octets will be contained in certain representations $R_1, \dots, R_n$ of $SU(3) \times SU(3)$. Each of these will imply values $d_1, \dots, d_n$ and $f_1, \dots, f_n$ for the F and D reduced matrix elements of $F_5^{\frac{1}{2}}$. The baryon octet B can in general be a linear superposition of $R_1, \dots, R_n$:

$$B = \sum_i c_i R_i$$

$$\sum_i |c_i|^2 = 1$$

Correspondingly, we have :

$$F = \sum_i f_i |c_i|^2$$

$$D = \sum_i d_i |c_i|^2$$

$$g_A = F + D = \sum_i (f_i + d_i) |c_i|^2$$

If $n = 1$, as in the case studied by Gerstein ⁴⁾, there is a unique prediction for F , D , g_A . If $n = 2$ there may be a linear relation between F and D , of the form

$$\alpha F + \gamma D = 1$$

This leads to a relation similar to that in Eq. (5) :

$$g_A = \frac{D + F}{\alpha F + \gamma D}$$

For $n > 2$ and in some cases of $n = 2$, there are only inequalities for F , D and g_A .

In Table I we list different families of representations obtained from $SU(6) \times O(3)$ multiplets. For each of them, in the second column, we specify the $SU(3) \times SU(3)$ representation, with helicity $+\frac{1}{2}$ which contains a spin $\frac{1}{2}$ octet. For each of the representations, we give the F and D in the next two columns. If we adjoin a 56 with one of the other representations listed, the baryon octet can be a mixture, so that at most only a linear relation between F and D survives. This linear relation is listed in the fifth column, on the line of the representation which is added to the 56. If more than one representation of $SU(6)$ is added to the 56, there is in general no relation of kind (5). The same is true if we add a 56, $\ell = 1$ to the 56. In this case one predicts $D/F = \frac{5}{2}$, $g_A < 5/3$.

$SU(6) \times O(3)$	$SU(3) \times SU(3)$	F	D	Linear relation
56	$(6, 3)$	$\frac{2}{3}$	1	
20 $\ell = 0$	$(3^*, 3)$	0	-1	$3F - D = 1$
20 $\ell = 1$	$\sqrt{\frac{2}{3}}(3, 3^*) - \frac{1}{\sqrt{3}}(3^*, 3)$	0	$\frac{1}{3}$	$3D - 3F = 1$
56 $\ell = 1$	$\sqrt{\frac{2}{3}}(3, 6) - \frac{1}{\sqrt{3}}(6, 3)$	$-\frac{4}{9}$	$-\frac{1}{3}$	$2D - 3F = 0$
70 $\ell = 0$	$\frac{1}{\sqrt{2}}(6, 3) - \frac{1}{\sqrt{2}}(3^*, 3)$	$\frac{1}{3}$	0	$3F - D = 1$

TABLE I

No relation is obtained in the case of 70, $\ell = 1$, not listed in the Table since it contains two spin $\frac{1}{2}$ octets.

It is seen that if the $SU(3) \times SU(3)$ algebra is to be saturated by a 56 plus one of the other representations of $SU(6) \times O(3)$ listed in Table I, the only possible choice which gives rise to relation (5) is the $20, \ell = 1$, representation ¹⁰).

Finally we notice that the class of solutions obtained by embedding $SU(3) \times SU(3)$ into $SU(6) \times O(3)$ does not exhaust all possibilities. As an example, let us consider some different families of representations which saturate the algebra (1)-(3) and obey i) and ii), containing the same set of particles

$$(8, \frac{1}{2}^-) + (8, \frac{3}{2}^-) + (1, \frac{1}{2}^-) + (1, \frac{3}{2}^-) + (1, \frac{5}{2}^-)$$

which enters the $20, \ell = 1$.

Besides the solution suggested by $SU(6) \times O(3)$, called I., we have :

- II) a family of representations obtained by changing each representation (a,b) in I. into (b,a) [this is the second solution of Ref. ⁵];
- III) put all $SU(3)$ singlets in $(1,1)$, all octets in $(8,1)$ and $(1,8)$;
- IV) change (a,b) into (b,a) in solution III.

Let us give the explicit form of solution III. :

Helicity	$(8, \frac{3}{2})$	$(8, \frac{1}{2})$
$+\frac{3}{2}$	$(8,1)$	—
$+\frac{1}{2}$	$\sqrt{\frac{2}{3}}(8,1) + \sqrt{\frac{1}{3}}(1,8)$	$\sqrt{\frac{1}{3}}(8,1) - \sqrt{\frac{2}{3}}(1,8)$

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This solution (contrary to solutions II. and IV.) gives rise to the relation (5), and would be even considered better than I. in the sense that the mixing necessary to obtain $g_A = 1.2$ is of only 29° , while solution I. requires a mixing of 36° .

The existence of other solutions, besides those obtained by embedding $SU(3) \times SU(3)$ in $SU(6) \times O(3)$ suggests that the requirement of saturation of $SU(3) \times SU(3)$ at infinite momentum is not restrictive enough, and should be implemented by other and more stringent requirements, perhaps by extending the algebra to $SU(6)_W$ by including tensor densities.

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