Duality without Supersymmetry¹

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Abstract:

Most recent developments in string duality have focused on string theories with spacetime supersymmetry. However, it has not been known whether string duality is a more general phenomenon, and might hold even without supersymmetry. In this talk, I will discuss the methods that can be used for extending duality beyond supersymmetry, and I will present the first known examples of strong/weak coupling duality relations between non-supersymmetric tachyon-free string theories. I will largely focus on the case of the tachyon-free $SO(16) \times SO(16)$ string, but I will also consider additional string models as well. Using these duals, I will then propose a phase diagram describing the stability of these strings as functions of their couplings and radii. This talk reports on work [1, 2, 3] performed in collaboration with Julie Blum.

1 Introduction: Why study non-supersymmetric dualities?

During the past several years, significant advances have taken place in our understanding of the strong-coupling behavior of heterotic string theories. For example, it has been conjectured that the strong-coupling behavior of the ten-dimensional supersymmetric $SO(32)$ heterotic string can be described as the weak-coupling behavior of the $SO(32)$ Type I string; likewise, the strong-coupling behavior of the supersymmetric $E_8 \times E_8$ heterotic string can be described as the compactification of a certain eleven-dimensional “M-theory” on a line segment. The existence of these strong-coupling dual relations has made it possible to address numerous non-perturbative issues pertaining to supersymmetric string theories.

But these are not the only heterotic strings in ten dimensions. In particular, there also exist seven *non-supersymmetric* string solutions [4] which satisfy all of the properties (such as conformal invariance, modular invariance, Lorentz invariance, proper spin-statistics relations, and physically consistent GSO projections) that we demand for *bona-fide* string solutions. The gauge groups of these additional string models are $SO(16) \times SO(16)$; $SO(32)$; $SO(8) \times SO(24)$; $U(16)$; $SO(16) \times E_8$; $(E_7)^2 \times SU(2)^2$; and E_8 . Although the latter six theories are tachyonic, the first one, with gauge group $SO(16) \times SO(16)$, is tachyon-free. All of these strings can be realized as $\mathbb{Z}_2$ orbifolds of either the supersymmetric $SO(32)$ or $E_8 \times E_8$ heterotic strings, and in each case the orbifold element has the form $(-1)^F W$ where F is the spacetime fermion number and W is an appropriate Wilson line. The effect of the $(-1)^F$ factor is to break spacetime supersymmetry, while the effect of the W factor is to break the gauge group.

Given the existence of these additional theories, the natural question arises: What are the strong-coupling duals of these *non-supersymmetric* strings? There are several reasons why this is an important

¹Invited talk given at the Conference *Quantum Aspects of Gauge Theories, Supersymmetry, and Unification*, held at the University of Neuchâtel, Neuchâtel, Switzerland, 18–23 September 1997. To appear in Proceedings published in *Fortschritte der Physik*.

question. First, knowing the answer might enable us to better understand the nature of string duality: Is duality a property of supersymmetry or supersymmetric strings, or is it, more generally, a property of *string theory itself*, independently of supersymmetry? Second, of course, knowing the answer would enable us to understand the non-perturbative behavior of the non-supersymmetric strings. Are these strings stabilized non-perturbatively? If not, how (and where) do they flow in moduli space? If so, what stringy mechanisms manage to introduce stability *without* supersymmetry? Such mechanisms could have important phenomenological implications. Finally, we could even imagine that such a study might yield a potential new method of supersymmetry-breaking. Indeed, if somehow the duals of the non-supersymmetric strings were found to be supersymmetric, then viewing the situation in reverse, we would have found a supersymmetric theory in which supersymmetry is broken at strong coupling.

2 Our method

How then can we approach the task of deriving non-supersymmetric duals? It seems that we are immediately confronted with a variety of problems: we have no supersymmetry algebra to help us; we have no BPS states whose masses and charges are protected at strong coupling; we have no non-renormalization theorems at our disposal; and the moduli spaces of such non-supersymmetric strings are much more complicated than those of supersymmetric strings, and are not understood. Therefore, we require a different approach. Our method [1, 2] will therefore be to *use known supersymmetric duality relations in order to generate non-supersymmetric duality relations*.

There are several ways in which one might imagine implementing this idea. One option, which is analogous to taking an airplane, would be to start the known supersymmetric $SO(32)$ or $E_8 \times E_8$ dual pairs, and then to duplicate the supersymmetry-breaking $\mathbb{Z}_2$ orbifold action on both sides of the duality relation. However, this procedure has a number of problems. Since such orbifolds break supersymmetry, they may not commute with the duality relation. Moreover, since orbifolding is a *discrete* operation (wherein we jump directly from the original theory to the final orbifolded theory), it is hard to study if, how, and why the duality might fail in passing from the initial to the final theory. Indeed, in terms of our airplane analogy, we only get to see the airports.

Given that we cannot take the plane, we will instead take the train. Specifically, we shall use *continuous deformations*: we shall connect non-supersymmetric strings to supersymmetric strings through continuous deformations in moduli space. The advantages of such an approach are that we can *generate* duality relations as a function of the deformation parameter, and thereby study *how far* from the supersymmetric limit the duality relation continues to hold. We would then learn *where* it fails (if at all); we would learn *how* it fails; and we would thereby map out the phase structure of non-supersymmetric strings. In the language of our travel analogy, we would be seeing the landscape.

Of course, there do not exist any such continuous deformations in ten dimensions. We shall therefore go to nine dimensions by compactifying one dimension. Specifically, we shall construct what we will refer to as *nine-dimensional interpolating models* by compactifying the supersymmetric $SO(32)$ heterotic string on a circle of radius R along with a twist. We shall carefully choose this twist so that as $R \rightarrow \infty$, the original supersymmetric $SO(32)$ model is reproduced, but as $R \rightarrow 0$, the ten-dimensional $SO(16) \times SO(16)$ model is reproduced. Such nine-dimensional interpolating models are completely smooth and continuous as functions of their radius R , and thereby *interpolate* between ten-dimensional supersymmetric and non-supersymmetric string models.

It is clear how to construct such interpolating models on the heterotic side, because modular invariance tells us what sorts of new “twisted” sectors must be introduced when we implement our carefully chosen twists. However, it is less clear how the analogous twisting will act on the Type I side, because we do not have modular invariance to guide us. Indeed, instead of modular invariance constraints we have tadpole anomaly cancellation constraints, and it is not clear whether such tadpole constraints alone are sufficient to uniquely determine the possible self-consistent Type I interpolations. We shall therefore circumvent this problem by realizing that since the $SO(32)$ Type I theory can be realized as an orientifold of the Type IIB theory, we might as well *interpolate before orientifolding*. Indeed, it turns out that there exists

Figure 1: Proposed method for deriving the dual of the heterotic $SO(16) \times SO(16)$ string model through a continuous deformation away from the supersymmetric $SO(32)$ string model. Analogous deformations exist on both the heterotic and the Type II sides, from which the corresponding Type I deformation can be obtained through orientifolding.

a unique nine-dimensional interpolating Type II model that connects to the ten-dimensional Type IIB model and breaks supersymmetry: this model connects the Type IIB theory as $R \rightarrow \infty$ to the non-supersymmetric so-called Type 0B model as $R \rightarrow 0$.

Putting the pieces together (as illustrated in Fig. 1), it would therefore seem that the dual for the heterotic $SO(16) \times SO(16)$ model must be the orientifold of the Type 0B model (or more precisely, the Type 0A model to which it is T -dual). Unfortunately, all of the possible orientifolds of the Type 0B model are found [5] to have gauge groups with rank 32 rather than 16. Moreover, all but one of these models has tachyons. Similar problems afflict the orientifolds of the Type 0A model. Therefore, we conclude that these cannot be the strong-coupling duals of the ten-dimensional heterotic $SO(16) \times SO(16)$ model, and that the attempted duality must *fail somewhere* in passing from $R = \infty$ to $R = 0$. Therefore, in order to study this issue (and ultimately to generate non-supersymmetric duality relations), we shall first interpolate our Type II model to *finite* radius R , and then take the orientifold. This procedure then yields an open-string model at arbitrary radius R , and we can study the behavior of this model as a function of this radius.

3 Our results: The $SO(16) \times SO(16)$ case

It is straightforward to carry out the procedure we have outlined. We shall first describe the heterotic interpolating model, then the Type II interpolating model, and finally the Type I interpolating model.

We begin by constructing the nine-dimensional interpolating model that connects the ten-dimensional supersymmetric $SO(32)$ heterotic model to the ten-dimensional non-supersymmetric $SO(16) \times SO(16)$ model. Our procedure for doing this rests upon the observation [2] that if any two ten-dimensional models M_1 and M_2 are related to each other via a $\mathbb{Z}_2$ orbifold relation, so that M_2 is the Q -orbifold of M_1 for some $\mathbb{Z}_2$ operator Q , then there exists a nine-dimensional interpolating model M such that $M \rightarrow M_1$ as $R \rightarrow \infty$, while $M \rightarrow M_2$ as $R \rightarrow 0$. Such a nine-dimensional model M is constructed by compactifying M_1 on a circle of radius R , and then orbifolding the resulting model by the element $\mathcal{T}Q$ where Q is the above $\mathbb{Z}_2$ operator and where $\mathcal{T}$ represents a half-rotation around the compactified dimension X :

$$\mathcal{T}: \quad X \rightarrow X + \pi R. \quad (1)$$

While the operator Q by itself would take us directly from M_1 to M_2 in ten dimensions, the effect of multiplying Q by $\mathcal{T}$ in nine dimensions is to ensure that the effect of the Q -orbifold is *introduced gradually* between $R = \infty$ and $R = 0$.

Note that this procedure is valid for any closed string models, whether Type II or heterotic, and does not rely on the precise form of the $\mathbb{Z}_2$ orbifold operator Q . Thus, any two ten-dimensional closed-string

models which are connected discretely through a $\mathbb{Z}_2$ orbifold relation in ten dimensions can be connected continuously through an interpolating model in nine dimensions.

Using this procedure, it is a simple matter to construct the nine-dimensional heterotic interpolating model [1, 2] that connects the supersymmetric $SO(32)$ model to the non-supersymmetric $SO(16) \times SO(16)$ model. This heterotic interpolating model, which we shall call Model B, is non-supersymmetric for generic radii $0 < R_H < \infty$ and has a massless spectrum consisting of the standard nine-dimensional gravity multiplet; vector gauge bosons in the adjoint of $SO(16) \times SO(16) \times U(1)^2$; and a positive-chirality spinor in the $(\mathbf{16}, \mathbf{16})$ representation with no $U(1)$ charges. The rest of the spectrum and masses are R_H -dependent, and there are no tachyons for any radius. As $R_H \rightarrow \infty$, certain extra vectors in the $(\mathbf{16}, \mathbf{16})$ representation become massless, enhancing the gauge symmetry to $SO(32)$; extra positive-chirality spinors in the adjoint become massless, forming the superpartners of the vectors and restoring supersymmetry; the $U(1)^2$ gauge bosons augment the gravity multiplet to ten dimensions; and extra light singlet fermions become massless to complete the supergravity multiplet. Together, this reproduces the ten-dimensional supersymmetric $SO(32)$ string model. Likewise, at the other ($R_H \rightarrow 0$) limit, the $U(1)^2$ gauge bosons augment the gravity multiplet to $D = 10$ and an extra negative-chirality spinor in the $(\mathbf{128}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{128})$ representation becomes massless. This reproduces the ten-dimensional non-supersymmetric $SO(16) \times SO(16)$ model.

We can also illustrate the continuity of our interpolating model by calculating its one-loop cosmological constant Λ as a function of its radius R_H . Here we define

$$\Lambda \equiv - \frac{R_H}{\sqrt{\alpha'}} \int_{\mathcal{F}} \frac{d^2 \tau}{(\text{Im } \tau)^2} Z(\tau, R_H) \quad (2)$$

where $Z(\tau, R_H)$ is the partition function of our model and $\mathcal{F}$ is the fundamental domain of the modular group. The result is shown in Fig. 2(a). We see that $\Lambda \rightarrow 0$ as $R_H \rightarrow \infty$, reflecting the emerging supersymmetry in this limit, while as $R_H \rightarrow 0$, Λ approaches the cosmological constant of the $SO(16) \times SO(16)$ model. The perturbative interpretation of Fig. 2(a) is as follows. Since Λ is proportional to the one-loop dilaton tadpole, the fact that $\Lambda > 0$ implies that our nine-dimensional interpolating model tends to flow in the direction of weak coupling. Likewise, since Λ also serves as the potential for the radius, we see that our model tends to flow in the direction of larger radius. Thus, we see that our heterotic interpolating model tends to flow back to the weakly coupled supersymmetric $SO(32)$ string.

Figure 2: The one-loop cosmological constants of the (a) heterotic, (b) Type II, and (c) Type I interpolating models as functions of radius R , plotted in units of $\frac{1}{2}(4\pi^2\alpha')^{-5}$.

We now turn to the construction of our Type II interpolating model. Once again, the procedure is completely analogous to that for the heterotic case: smooth interpolations will exist that connect models related by $\mathbb{Z}_2$ orbifolds. In this case, the supersymmetric Type IIB string and the non-supersymmetric

Type 0B string are related by a $\mathbb{Z}_2$ orbifold. Thus there exists a nine-dimensional interpolating model, which we shall call Model B', that connects them to each other. In fact, Model B' turns out to be the unique supersymmetry-breaking deformation away from the Type IIB string. Unlike the heterotic case, however, our Type II interpolating Model B' is found [1, 2] to be tachyon-free only in the restricted range $R \geq R^*$ where $R^* \equiv 2\sqrt{2\alpha'}$. The appearance of such a tachyon is to be expected, since we know that the Type 0B model, unlike the analogous $SO(16) \times SO(16)$ model on the heterotic side, is tachyonic. However, the appearance of this tachyon implies that there is a *discontinuity* in the cosmological constant of this model, with Λ dropping discontinuously to $-\infty$ for $R < R^*$. This behavior suggests the existence of some sort of phase transition at $R = R^*$. Above this radius, however, our interpolating model is completely smooth and tachyon-free, and its cosmological constant is shown in Fig. 2(b). From this result, we see that unlike the heterotic case, the Type II interpolating model tends to flow towards *strong* coupling and towards *small* radius — *i.e.*, in the direction of the tachyon. Thus, unlike the heterotic interpolating model, this Type II interpolating model is deeply unstable.

Finally, we turn to our Type I interpolating model, which we construct by taking the worldsheet parity-reversing orientifold of our Type II interpolating model. The procedure for calculating such an orientifold is standard, and details can be found in Ref. [2]. We impose only the most elementary requirements: cancellation of NS-NS and Ramond-Ramond tadpole anomalies; cancellation of tree-channel open-string tachyons; and $SO(n)$ open-string gauge groups. The result we find, however, is rather surprising. First, we find that there is only one possible gauge group, $SO(16) \times SO(16) \times U(1)^2$. This is the same as that of the heterotic interpolating Model B. In fact, we find that the *entire* massless spectra of these two models agree exactly. Moreover, we can examine the D-string soliton of our open-string interpolating model, and we find [2] that it reproduces the worldsheet theory of the $SO(16) \times SO(16)$ heterotic string. All of these facts strongly suggest that **our Type I interpolating model is the strong-coupling dual of our heterotic interpolating model**. Further evidence comes from the fact that we can take the $R \rightarrow \infty$ limit on both sides in order to smoothly reproduce a known supersymmetric duality relation. **Therefore, if this interpretation is correct, this would be the first known example of a strong/weak coupling duality relation between non-supersymmetric tachyon-free string theories.**

We can also analyze the stability of our Type I interpolating model. In general, the one-loop cosmological constant of an open-string model receives four contributions: torus, Klein bottle, cylinder, and Möbius strip. The torus contribution is always half that of the Type II model from which the Type I model is derived via orientifolding. Thus, in our case, the torus contribution is half that of Model B', which was shown in Fig. 2(b). It also turns out that in the present case, the Klein-bottle and cylinder contributions vanish identically. Thus, we are left with a Möbius contribution to add to the torus contribution. The result is shown in Fig. 2(c). Remarkably, we see that the Möbius contribution completely *compensates* for the torus contribution, producing a stable theory! Specifically, our Type I interpolating model now flows towards weak coupling and large radius, *towards the supersymmetric $SO(32)$ limit and away from the tachyonic regions*. This behavior exactly mimics the behavior on the heterotic side.

Using this non-supersymmetric dual, we can now map out a phase diagram [1, 2] of our nine-dimensional non-supersymmetric $SO(16) \times SO(16)$ heterotic interpolating model as a function of its heterotic ten-dimensional coupling $\lambda_H^{(10)}$ and its radius R_H of compactification. We have already seen that when our model is perturbative on the heterotic side (which corresponds to $\lambda_H^{(9)} \ll 1$, or $\lambda_H^{(10)} \ll \sqrt{R_H}$), it flows back to the supersymmetric $SO(32)$ string. Likewise, when the string is perturbative on the Type I side (corresponding to $\lambda_I^{(9)} \ll 1$, or $\lambda_H^{(10)} \gg R_H^{-2/3}$), it again flows back to the supersymmetric $SO(32)$ string. These two regions are shown schematically as the shaded regions in Fig. 3.

It is also possible to make some conjectures [1, 2] about the behavior of the string in other regions of parameter space. In particular, the existence of our *continuous deformations* in the tachyon-free region $R_I > R^*$ suggests that our interpolating model should continue to flow to the supersymmetric $SO(32)$ string provided that $R_I > R^*$, or equivalently $\lambda_H^{(10)} < (R_H/R^*)^2$. This region is indicated as “Phase I” in Fig. 3. Finally, we can even make a conjecture about the region *beyond* the phase boundary, *i.e.*, $R_I < R^*$, which we shall denote as “Phase II”. In this region, an M-theoretic description is valid because the constraint $R_I < R^*$ translates into a constraint on the length of the eleventh dimension,

bounding it from below. This suggests that the behavior of our string should be describable in terms of compactifications of M-theory. However, there are no stable nine- or ten-dimensional non-supersymmetric compactifications of M-theory [6], so we expect that our string must flow to a supersymmetric limit. The discontinuity relative to Phase I suggests that this limit will *not* be the $SO(32)$ theory. Thus the only possible option is that our string flows to the $E_8 \times E_8$ theory. This conjecture receives further support from the observation that the D-string soliton of our Type I interpolating string model behaves as an $E_8 \times E_8$ heterotic string in this phase, and also from the observation that there indeed exist continuous heterotic interpolations that connect the $SO(16) \times SO(16)$ string to the $E_8 \times E_8$ string.

Figure 3: Proposed phase diagram for the $SO(16) \times SO(16)$ interpolating model as a function of its coupling $\lambda_H^{(10)}$ and radius R_H . In Phase I, the string flows back to the supersymmetric $SO(32)$ theory, while in Phase II, the string flows to the $E_8 \times E_8$ theory. The phase boundary occurs at $\lambda_H^{(10)} = (R_H / R^*)^2$ where $R^* = 2\sqrt{2\alpha'}$ is the critical radius.

4 Other non-supersymmetric cases

What about the *other* non-supersymmetric heterotic string theories in ten dimensions? All of these theories are tachyonic. It is therefore important to understand how, and to what extent, the presence of tachyons affects the duality analysis.

For the $SO(8) \times SO(24)$, $SO(32)$, and $U(16)$ models, we can essentially repeat the same analysis as for the $SO(16) \times SO(16)$ case. First, we construct nine-dimensional heterotic interpolating models that connect these models to the supersymmetric $SO(32)$ model. However, because these non-supersymmetric endpoints are tachyonic, we find that our heterotic interpolating models develop tachyons at the same critical radius R^* . Indeed, the presence of these tachyons causes the plots of the cosmological constants of these models to resemble Fig. 2(b) rather than Fig. 2(a). In particular, these models are deeply unstable. We therefore focus on only the tachyon-free regions $R \geq R^*$. Next, since our previous Model B' is the unique supersymmetry-breaking deformation of the Type IIB string, we see that once again we must take the orientifold of this same model. However, choosing different Wilson lines for the orientifold enables us to exactly duplicate each of the required gauge groups. Moreover, in each case, we also find [2] that the entire massless spectra agree exactly, and that the D-string soliton of the open-string interpolation precisely reproduces the worldsheet theory of the heterotic side. Thus, in the tachyon-free range $R > R^*$, we see that even these other non-supersymmetric heterotic strings have duals! Furthermore, in the $SO(8) \times SO(24)$ and $SO(32)$ cases, we find that the cosmological constants of our open-string models now receive large, negative cylinder contributions in addition to their previous torus and Möbius contributions.

These cylinder contributions thus drive these open-string theories towards strong coupling and towards the tachyon. Once again, this exactly mimics the behavior of the heterotic interpolating models. Even the $U(16)$ case is similar: in this case we have no cylinder contribution, but the Möbius contribution switches sign. Thus, even though all of these theories are tachyon-free in the region $R \geq R^*$ where they continue to have duals, we see that both the original and dual theories “feel” the tachyon at $R = 0$ and are unstable. The $SO(16) \times SO(16)$ case is truly unique.

Finally, we comment on the cases of the $SO(16) \times E_8$, $(E_7)^2 \times SU(2)^2$, and E_8 strings. Since these heterotic strings are not $\mathbb{Z}_2$ orbifolds of the $SO(32)$ string, there do not exist smooth nine-dimensional interpolations that connect these models to the supersymmetric $SO(32)$ string. Thus, they are not expected to have Type I duals. Of course, this is not surprising, since we know that we cannot realize exceptional gauge groups from perturbative open-string theories. However, since these strings are $\mathbb{Z}_2$ orbifolds of the $E_8 \times E_8$ string, there exist heterotic interpolating models that connect these strings smoothly to the $E_8 \times E_8$ string. This suggests that duals for these theories might be constructed via smooth supersymmetry-breaking interpolations or deformations of M-theory.

5 Conclusions

So what have we learned? First, we have seen that like their supersymmetric counterparts, non-supersymmetric string theories also have duals. Thus, duality appears to be a general property of string theory, independently of supersymmetry. We have also seen that non-supersymmetric strings have a very rich phase structure, and that non-supersymmetric duals can be used to map it.

This work also raises a number of outstanding questions. First, how can we find more evidence for non-supersymmetric dualities? Are there analogues of BPS-states for non-supersymmetric strings, or hidden non-renormalization theorems or finiteness properties that underlie such duality relations? The notion of “misaligned supersymmetry” [7] may play a crucial role in this regard. Second, are there any *stable* non-supersymmetric strings? In the ten-dimensional case we analyzed, our non-supersymmetric string always flowed to a supersymmetric point, but we conjecture that stable non-supersymmetric strings might exist in lower dimensions at the non-trivial intersections of phase boundaries. Third, there are also numerous applications of our results that one might imagine: supersymmetry-breaking interpolations of M-theory; possible relations to supersymmetry-breaking in M-theory and/or Matrix theory; derivations of non-supersymmetric dualities in field theory; and applications to non-supersymmetric black-hole physics. Indeed, it turns out that these interpolation techniques can even be used in a purely *supersymmetric* setting in order to realize a new continuous connection between the Type I string and M-theory [3].

But perhaps most importantly, our results suggest that duality is not crucially tied to the existence of supersymmetry. This in turn suggests that it might be profitable to reformulate the ideas and techniques for studying duality in string theory so that they do not rest so directly on the existence of supersymmetry.

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