KAON-NUCLEON SCATTERING AND THRESHOLD K^* PRODUCTION

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A B S T R A C T

Using the boundary condition model, it is shown that the rise of the $I=0$ S state KN phase shift near P_L (kaon laboratory momentum) = 800 MeV/c can be attributed to the effect of the K^*N threshold. Approximately 2.8 mb of K^* production at 1140 MeV/c is predicted for that channel. If the interchannel coupling is reduced for the $I=1$ channel in the ratio given by isovector exchange, the experimental production cross-section is obtained and the earlier fit to the $I=1$ S state phase shift altered negligibly. Elastic boundary condition fits are given for the P and D waves. The boundary radius, consistent with theory, is $0.45\mu^{-1}$ except for one D state. The $0.7\mu^{-1}$ radius required in the latter case indicates a significant repulsive potential tail.

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1. INTRODUCTION

This report exhibits a boundary condition model ¹⁾ (B.C.M.) description of the kaon-nucleon interaction up to and above the threshold for K^* production. This application of the B.C.M. is mainly phenomenological, but the boundary radius, r_0 , conforms to the theoretical expectation ¹⁾ that a simple boundary condition behaviour should be approached at a range near or a little less than $\frac{1}{2}\mu^{-1}$ (μ is the charged meson mass, $\hbar = c = 1$). The absence of strong $\bar{K}\pi K$ vertices, because of parity and angular momentum conservation, implies that the longest range elastic processes are those of Figs. 1.a and 1.b. We may expect that Fig. 1.b is effective in producing a boundary condition behaviour, because the initial barycentric energy is greater than the intermediate state mass for most of our energy range. Figure 1.c should certainly be effective, implying $\frac{1}{4}\mu^{-1}$ as a minimum value of r_0 . Figures 1.a and 1.b will also imply potential type interaction. But, as their maximum radius of decay is $\frac{1}{2}\mu^{-1}$, they may have little effect outside r_0 . The absence of a single pion exchange contribution gives some reason to hope that the amplitudes will be well approximated by a "pure" B.C.M., i.e., one with no potential tails. The longest range K^* production diagram (Fig. 1.d) supplies an off-diagonal potential contribution of $1\mu^{-1}$ range. We neglect this potential effect also and for simplicity use an off-diagonal boundary condition at the same r_0 as for the diagonal processes.

In the next Section we fit the $I=0$ S state phase shift and show that the apparent rise in this phase shift ²⁾ near P_L (kaon laboratory momentum) = 800 MeV/c can be attributed to the effect of the K^*N threshold which is still 200 MeV/c above. The required interchannel coupling predicts a reasonable amount of K^* production in the $I=0$ S channel, which should not be difficult to check experimentally. In Section 3 we assume that the interchannel coupling in the $I=1$ state is related to that of the $I=0$ state as if caused by the exchange of an isovector particle, and obtain results consistent with K^+_p elastic ^{3),4)} and production ⁵⁾ data.

2.

Production may also take place in this energy region from the P and D states. This is in fact probable for the observed $N^*(\frac{3}{2}, \frac{3}{2})$ production (which can only take place in the $I=1$ channel) as the spin and parity of the N^* require the KN^* channel to be in a D state for an incoming S state KN system. However, the accuracy of the experimental phase shifts ^{2),4)} only warrant a simple fit. In Section 4 we show that an uncoupled, "pure" B.C.M. adequately describes the P and D state phase shifts up to the K^* threshold. One of the $I=0$ D states requires a repulsive interaction of large radius indicating that the longest range potential tail is significant.

Section 5 discusses these results and the relevance of N^* production, two pion "background", the width of the K^* and N^* and the possible constraint of "MacDowell symmetry" ⁶⁾.

2. THE T=0 S STATE

The analysis of the $T=0$ S state data ²⁾ indicates that the rate of increase of the attraction increases with the momentum of the incoming kaon from 300-800 MeV/c. The errors are still too large to conclude that the above effect is real; but if it is, then a likely interpretation is that it is an effect of the K^*N threshold near 1000 MeV/c. It is readily seen by examining any form of interaction in perturbation theory that the effect of going into a higher mass intermediate state and back to the elastic channel always adds an attraction which increases as one approaches the inelastic threshold from below; one obtains absolute squares of matrix elements (because one returns to the initial state) and the decreasing energy denominators have a unique sign while the intermediate states lie above. The exact solution of our model reproduces the same behaviour ^{1),7)}.

In the absence of a potential tail the external wave functions are the $L = 0$ spherical Hankel functions and the basic two-channel boundary condition ¹⁾ reduces to

$$\begin{pmatrix} \frac{1}{k^{1/2}} \frac{d}{dr} (e^{-ikr} - S_e^0 e^{ikr}) \\ \frac{1}{K^{1/2}} \frac{d}{dr} S_i^0 e^{iKr} \end{pmatrix}_{r_0} = \begin{pmatrix} f^0 & f_c^0 \\ f_c^0 & f_*^0 \end{pmatrix} \begin{pmatrix} \frac{1}{k^{1/2}} (e^{-ikr} - S_e^0 e^{ikr}) \\ \frac{1}{K^{1/2}} S_i^0 e^{iKr} \end{pmatrix}_{r_0} \quad (1)$$

where k and K are the relativistic centre-of-mass momenta of the kaon and the K^* respectively. The reality of f^0 , f_*^0 and f_c^0 guarantees unitarity, i.e.,

$$|S_e^0|^2 + |S_i^0|^2 = 1 \quad \dots \quad (2)$$

where $S_c^0 \equiv \eta_{00} e^{2i\delta_{00}}$ and S_i^0 are, respectively, the elastic and inelastic components of the S matrix in the $I=0, L=0$ channel. For that channel f^0 and f_*^0 denote the diagonal boundary condition for the KN and K^*N respectively, and f_c^0 the interchannel coupling. The solution of Eq. (1) below threshold ($\eta_{00}=1$) leads to ^{1),7)}

$$\tan \delta_{00} = \frac{kr_0 - f_{eff}^0 \tan kr_0}{kr_0 \tan kr_0 + f_{eff}^0} \quad \dots \quad (3)$$

with

$$f_{eff}^0 = f^0 - \frac{(f_c^0)^2}{Kr_0 + f_*^0} \quad \dots \quad (4)$$

replacing the constant of the single channel case. We have replaced $K=i\chi$, as χ is real below threshold. Equation (4) displays the behaviour discussed above. If $f_*^0 > 0$ the addition due to interchannel coupling is always attractive (negative) and becomes increasingly so as we approach inelastic threshold from below ($\chi \rightarrow 0_+$). If $f_*^0 < 0$, the strongest attraction is below threshold and a resonance appears due to the decay of a quasi-bound state of the second channel.

In Figure 2 we show the effect of the interchannel coupling. When $f_c^0 = 0$ the fit fails completely for $r_0 = 0.5 \mu^{-1}$. An acceptable fit under that condition is only obtained for $r_0 \leq 0.125 \mu^{-1}$. It would be difficult to justify the lack of interaction down to such a small distance. A static potential would lose attraction less quickly, but would also require a small radius or non-monotonicity.

A good fit is obtained, as shown, for $r_0 = 0.45 \mu^{-1}$ (taken to agree with the earlier $I=1$ fit ¹⁾) and with $f^0 = 3.89$, $f_*^0 = 1.20$ and $f_c^0 = 3.00$. The scattering length predicted by these parameters is $0.04 \mu^{-1}$, in agreement with experiment ²⁾. The variation of the effective scattering length up to 230 MeV/c (the experimental value was analyzed from data in this range) is much less than the quoted experimental error ²⁾.

The inelastic cross-section (K real)

$$\sigma_i^e = \frac{\pi}{k^2} |S_i^e|^2 \quad \dots \quad (5)$$

could be obtained by solving Eq. (1) for S_i^0

$$|S_i^0|^2 = \frac{(f_c^0)^2}{(f_*^0)^2 + K^2 r_0^2} \frac{K}{k} \left\{ 1 - 2\eta_{co} \cos 2(\delta_{co} + kr_0) + \eta_o^2 \right\} \dots (6)$$

but is more readily obtained from unitarity [Eq. (2)] :

$$|S_i^0|^2 = 1 - |S_e^0|^2 = 1 - \eta_{00}^2 = \frac{-4Kr_0 f_i^0}{(f_r^0)^2 + (f_i^0)^2 + (Kr_0)^2 - 2Kr_0 f_i^0} \quad (7)$$

where

$$f_r^0 = \text{Re } f_{\text{eff}}^0 = f^0 - \frac{(f_c^0)^2 f_*^0}{(f_*^0)^2 + K^2 r_0^2}$$

and

$$f_i^0 = \text{Im } f_{\text{eff}}^0 = - \frac{(f_c^0)^2 Kr_0}{(f_*^0)^2 + K^2 r_0^2}$$

Unitarity ($0 \leq |S_i^0|^2 \leq 1$) is automatically preserved by the negativity of f_i^0 .

We can also obtain the real phase shift above threshold, simply by replacing, in Eq. (3), f_{eff}^0 by

$$\bar{f}^0 = (2f_r)^{-1} \left\{ [f_r^2 + f_i^2 - (Kr_0)^2] + [(f_r^2 + f_i^2 + (Kr_0)^2)^2 - 4f_i^2 (Kr_0)^2]^{\frac{1}{2}} \right\}. \quad (8)$$

With the above parameters, fitted to the phase shifts below inelastic threshold, we predict $\sigma_i^0 = 2.8 \text{ mb}$ for $P_L = 1140 \text{ MeV/c}$. The $K^+p \rightarrow K^{*+}p \rightarrow K^0 + \pi^+ + p$ cross-section at this momentum is known to be $0.9 \pm 0.3 \text{ mb}$ ⁵⁾. We will show below that this $I=1$ result is consistent with our $I=0$ prediction if we further assume that the two isotopic states are connected by the peripheral diagram. Of course, our prediction for K^* production in the $I=0$ state can be checked experimentally in K^+d or K_2^0p reactions. With more difficulty our prediction for the phase shift $\delta_{00} = 53.2^\circ$ at 1140 MeV/c can also be checked.

The above fit was obtained by choosing f_*^0 and then adjusting f_c^0 and f^0 to fit the phase shifts at $P_L = 350$ and 812 MeV/c. The fit is not very sensitive to f_*^0 , but the range of this parameter is limited by the amount of curvature permitted in the phase shift curve. A smaller f_*^0 implies more curvature, and for $f_*^0 \lesssim 0.5$ the increase in $d \int_{00}/dP_L$ above $P_L = 6$ leads to a resonance before the K^* threshold. On the other hand, $f_*^0 \gtrsim 2$ would produce a much more linear phase shift curve than we have presented. It would also reduce the imaginary part of f_{eff}^0 above threshold and make the production cross-section more linear in momentum. The present large errors on the phase shift and unknown production behaviour permit this. However, the present choice of f_*^0 leads, as presented below, to a simple relation between the $I=0$ and $I=1$ phase shift and production data.

3. THE T=1 S STATE

The phase shift in the $T=1$ S state is strongly repulsive and has the peculiar feature of fitting an effective range curve up to $P_L = 810$ MeV/c. A hard core, fitted to the scattering length, is not sufficiently repulsive at the largest momentum, and a "soft" monotonic potential would do worse. The same difficulty besets dispersion fits ⁸⁾. A multiparameter non-monotonic dispersion fit is undoubtedly possible; but it has been shown that a two parameter pure B.C.M. fits the data well ¹⁾. The large rapidly increasing repulsion is typical of the elastic B.C.M. with its saturation of the Wigner condition on $d\delta/dk$. Indeed the boundary radius found, $r_0 = 0.45 \mu^{-1}$ is close to that expected theoretically as discussed in Section 1, and only $f^1 = 1.885$ was a completely free parameter. In this one channel problem $f_{\text{eff}}^1 \equiv f^1$ in the application of Eq. (3) for δ_{10} .

However, inelasticity has been observed in the $I=1$ channel ^{5),9)} and we must enquire as to its effect on the phase shift below threshold. If the additional attraction engendered would be as large as for $I=0$,

the fit would be destroyed. We have found that a simple relation between the $I=1$ and $I=0$ interchannel coupling strengths at once predicts the experimental production cross-section and alters only slightly the phase shifts below threshold.

Although the vector exchange part of the production process seems to be dominantly isoscalar¹⁰⁾ the pseudoscalar exchange is important at low energies and is presumably isovector^{9),11)}, being pion exchange. We will assume that the isovector behaviour dominates at threshold. The resulting $\vec{T}_K \cdot \vec{T}_N$ dependence ($\vec{T}$ is the isospin operator) gives $f_c^1 = -\frac{1}{3}f_c^0$. We assume $f_*^1 = f_*^0$ for simplicity and because of the relative unimportance of this parameter. Some approximate justification for that assumption may be obtained from A conservation¹²⁾. Because of the negative value of the pion A quantum number, single pion exchange would be inhibited in K^* elastic scattering by this new selection rule. Consequently a minimum of two pions would be involved in the strong part of the forces. An important contribution from the $I=0$ two-pion state may be expected at long range, because of the apparent low mass enhancement in this state. Thus the diagonal part of the K^*N interaction may be largely isospin independent.

We retain $r_0 = 0.45 \mu^{-1}$ in the expectation that a fit to the phase shifts will require the same range dependence as in the one-channel case. This leaves f^1 to be adjusted so that $f_{\text{eff}}^1 = 1.855$ in the low energy range, i.e., the same value as f^1 in the one-channel case. In this case, choosing $f^1 = 2.203$ makes $f_{\text{eff}}^1 = 1.855$ at $P_L = 350 \text{ MeV/c}$. This makes the agreement with experiment as good as before up to and beyond that momentum. The qualitative similarity of f^1 and f^0 is to be expected on the basis of the absence of single pion exchange. At $P_L = 810 \text{ MeV/c}$ Eq. (4) gives $f_{\text{eff}}^1 = 1.762$, a small decrease from the low energy value. The resulting phase shift $\delta_{10} = -43.2^\circ$ does not depart seriously from the experimental value. At that momentum⁴⁾ the phase shift analysis yields $-47^\circ \pm 1^\circ$ or -45° if the inelasticity is assumed to be in the S state or in the $P_{\frac{1}{2}}$ state, respectively. The value of δ_{10} could be decreased, if necessary, by varying f_c^1 and f_*^1 .

At $P_L = 1140 \text{ MeV/c}$ we obtain $f_{\text{eff}}^1 = 1.51 - 0.314 i$ yielding $\sigma_i^1 = 1.2 \text{ mb}$. This is in good agreement with the experiment ⁵⁾ quoted in Section 2. Their cross-section must be multiplied by the isotopic ratio of $\frac{3}{2}$, to take into account the unobserved decay of $K^{*+} \rightarrow \pi^0 + K^+$, giving $\sigma_i^1 = 1.35 \pm 0.5 \text{ mb}$ to be compared with our prediction. Similar results have been obtained at somewhat higher energies ⁹⁾. The phase shift predicted at this momentum is $\delta_{10} = -27.0^\circ$, considerably less repulsive than a simple extrapolation of the data below threshold.

The substantial production obtained in spite of the decrease of f_i by a factor of 9 from its value in the $I=0$ channel, is due to the decreased competition from the elastic channel (smaller f_r).

4. THE P AND D STATES

The analysis of the $L > 0$ phase shifts is more ambiguous than for the S states, because of the possibility of S or S and P state fits, and because of the Fermi-Yang ambiguity ^{2),3),4)}. For the most part the errors are as large as the phase shifts. Nevertheless, we here display simple boundary condition fits for P and D states. It may be useful to have a simple representation for kaon nucleon amplitudes in analyzing more complicated phenomena, e.g., final state interactions in multiparticle final states. Also some indication of the nature of the interaction may even be obtained from a preliminary fit.

As explained in the introduction, the observed N^* production, from the K^+p system, seems to imply inelasticity for $L > 0$. Because of the angular momentum barrier in the final state, it is easier to produce N^* near threshold from the K^+p P state or D state — going to the KN^* P state or S state, respectively — than from the K^+p S state which can only go to the KN^* D state. But the $I=1$ channel has negligible P and D states, which do not seem to warrant elaborate fitting. For the moment we give only one channel fits for $L > 0$ and leave N^* production for a forthcoming analysis ¹³⁾. The replacement for Eq. (3), when $L \neq 0$, is easily obtained from Eq. (4.14) of Ref. 7), with $f_{\text{eff}} = f_L(\text{constant})$.

As the $I=1$, P and D phases are consistent with zero over the whole energy range, a trivial fit is obtained with $r_0=0$. As it is unrealistic to have a radius of interaction much less than $\frac{1}{2}\mu^{-1}$, we prefer instead to show that a fit can be obtained with $r_0=0.45\mu^{-1}$. This implies (for constant f) that the phase shifts do not vanish identically, but indeed they remain sufficiently small for a reasonable range of f , even at the radius used. "Sufficiently small" is here taken as given by the quoted errors at 810 MeV/c⁴⁾ or the errors of the $I=0$ analysis²⁾, i.e., approximately $\pm 2^\circ$. The curves shown in Fig. 3 show that this is easily achieved for P waves. With $1.5 < f_P < 1.75$ the phase shift starts weakly attractive and then (as is characteristic of constant f) becomes less attractive, finally becoming repulsive in the vicinity of 800 MeV/c. Under these conditions $|\delta_P(\max)| < 2^\circ$. The larger centrifugal barrier makes it easier to keep δ_D small. For $2.5 < f_D < 3$ it follows that $|\delta_D| < 0.2^\circ$ for the whole energy range. We do not bother to present curves for this range of f_D .

Our fit to both types of $I=0$ P wave phase shifts²⁾ is displayed in Fig. 3 and is seen to be good. The fit to the solution a) values of $\int_{01}(\frac{3}{2})$ [better defined by the data than the other phase shifts] can be improved by decreasing r_0 to $0.40\mu^{-1}$ [with $f_0=0.25$ one obtains chi-squared = 0.4]. The present value of chi-squared = 2 is satisfactory however. A weakly repulsive potential tail, opposing the attractive boundary condition, would have the same effect as decreasing r_0 .

In Figure 4 we displayed our fit to the $I=0$ D waves. As the $J=\frac{3}{2}$ and $J=\frac{5}{2}$ waves are nearly exchanged by the Fermi-Yang ambiguity, we display only the solution a) analysis of the data. The small D state phase shifts, with large assigned errors, are easily fit with $r_0=0.45\mu^{-1}$ and $1 < f < \infty$.

The comparatively large (negative) phase shifts at 350 and 530 MeV/c demand a larger radius of interaction than $0.45\mu^{-1}$. For $kr_0 \ll \frac{\pi}{2}$ the B.C.M. gives :

$$\tan \delta_D = \frac{(kr_0)^5 [1 - \frac{1}{2}(f_D + 2)]}{9(f_D + 2)} \dots (9)$$

The maximum repulsive phase shift is seen to be that of the hard core ($f \rightarrow \infty$), giving $\delta_D = -0.2^\circ$ at 350 MeV/c for $r_0 = 0.45\mu^{-1}$. Because of the r_0^5 dependence, a hard core at $r_0 = 0.7\mu^{-1}$ already increases the repulsion to $\delta_D = -1.9^\circ$, consistent with experiment. The fit for $r_0 = 0.7\mu^{-1}$, $f \rightarrow \infty$, is shown in Fig. 4. While this "pure" B.C.M. fit is good, both the theory and the tendency of the data to show a "softening" of the repulsion at higher energies encourage the belief that a repulsive tail outside a hard core at $0.45\mu^{-1}$ would be a better representation. Improved data may warrant such a fit.

5. DISCUSSION AND CONCLUSIONS

In the above analysis several possible components of the interaction were ignored. Several of these will need inclusion when details of the production data and more precise scattering data are to be described. Potential tails have been neglected altogether, and even the present analysis indicates the need of a repulsive tail in one D state, and perhaps also in one P state. It would be appropriate to examine the effect of the long range parts of the potential given by the unphysical cuts in a dispersion theory approach.

We have also ignored the effects of all inelastic channels except the K^*N . The KN^* channel couples only to $I=1$ and is then probably unimportant in the K^+p state because of the large angular momentum barrier ($L=2$), implied for the low momentum KN^* state. The effect of the known N^* production near threshold on P waves is now being analyzed¹³⁾.

The neglect of the width of the K^* has been unimportant for our present investigation. It becomes very important when examining K^* production near or below the threshold for producing K^* at its central mass. The same is true for N^* production at low energy. An investigation of such data ¹⁴⁾ is now under way ¹³⁾ including the particle widths by a distribution of channels ^{7),15)}. If a K meson proves necessary, its channel will be added.

This still leaves unexamined the effect of the "background" pion production which has been observed ¹⁶⁾ for P_L as low as 642 MeV/c. This continuum of production channels can contribute to all the states we have considered. The additional energy dependence so given to f_{eff} will be slow, because of the smooth continuum of thresholds. Consequently it is hoped that the effect of the background is adequately taken into account by a "renormalization" of the two-body channel boundary condition parameters, over the physical energy range.

For the future, one would like to eliminate phenomenological aspects of the above representation by imposing crossing symmetry between these channels and related reactions. We have already mentioned the possibility of adding the long range interaction derived from the unphysical cut. In addition, one may require that the f 's satisfy conditions imposed by crossing symmetry. As extensive analytic continuation is implied by such conditions, the effect of singularities that have here been absorbed into other constants will show up. MacDowell symmetry is easily tested with our B.C.M. amplitudes ⁶⁾. In this case the S state channels would be connected to the P state channels of the same isotopic spin. For our present model we find that the $I=0$, P state threshold condition, $f_{\text{eff}}^0(-K_0) = 1$ [where K_0 is the imaginary K^* channel momentum at elastic threshold], is badly violated. If we combine that condition with the phase shift requirements $f_{\text{eff}}^0(P_L=350 \text{ MeV/c}) = 0.76$ and $f_{\text{eff}}^0(P_L=835 \text{ MeV/c}) = -0.18$ we can solve for $f_*^0 \approx -1$, $f_c^0 \approx 0.4$ and $f_o^0 \approx 0.95$. These parameters (the negative value of f_*^0) imply a resonance for $P_L < 800 \text{ MeV/c}$, which is not present in the data.

The addition of the "background" production cuts and perhaps the potential tail cuts may permit the satisfaction of the symmetry requirement, the new coupling parameters being determined by the theoretical condition. For the $I=1$ channel the N^* cut would also be important in extensive analytic continuations.

In conclusion the above multichannel B.C.M. without potential tails represents the two-body data well. The theoretical conditions which should apply in the physical region, i.e., the radius of interaction and some isospin relations between strengths are satisfied. To discuss three-body final state data the widths of the K^* and N^* must be accounted for and the calculation can be extended to do so ¹³⁾. The imposition of crossing symmetry conditions awaits the addition to the model of potential tails and perhaps "background" production.

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FIGURE CAPTIONS

Figure 1 Diagrams determining the long range potential and the boundary condition radius r_0 . The particles are represented as follows: — nucleon; — pion; ~~~ kaon; and ----- K^* meson. The direction of time is upward.

Figure 2 The $I=0$ S state phase shifts. The experimental values are from Ref. 2), including the scattering length given by The theoretical curves are as follows:

—	$r_0 = 0.45\mu^{-1}$	$f^0 = 3.89$	$f_c^0 = 3.0$	$f_*^0 = 1.2$
-----	$r_0 = 0.5\mu^{-1}$	$f^0 = 0.6$	$f_c^0 = 0$	
-.-.-	$r_0 = 0.125\mu^{-1}$	$f^0 = 0.72$	$f_c^0 = 0$	

Figure 3 a) The $I=0$ P state phase shifts, solution a). The experimental values are from Ref. 2). $\frac{1}{2}$ $J=\frac{1}{2}$ data; $\frac{3}{2}$ $J=\frac{3}{2}$ data; — $r_0 = 0.45\mu^{-1}$, $f_P = 2.6$; ----- $r_0 = 0.45\mu^{-1}$, $f_P = 0.4$.

b) The $I=0$ P state phase shifts, solution b), and the $I=1$ P state phase shifts. The experimental values are from Refs. 2) and 4). $\frac{1}{2}$ $I=1$ data; $\frac{1}{2}$ $J=\frac{1}{2}$ data; $\frac{3}{2}$ $J=\frac{3}{2}$ data; — $r_0 = 0.45\mu^{-1}$, $f_P = 0.1$; ----- $r_0 = 0.45\mu^{-1}$, $f_P = 1.5$; -.-.- $r_0 = 0.45\mu^{-1}$, $f_P = 1.75$. The last two curves are consistent with the vanishing of δ_P for $P_L < 810$ MeV/c, within experimental error [Ref. 3].

Figure 4 The $I=0$ D state phase shifts. The experimental values are from Ref. 2), solution a). $\frac{3}{2}$ $J=\frac{3}{2}$ data; $\frac{5}{2}$ $J=\frac{5}{2}$ data: — $r_0 = 0.7\mu^{-1}$, $f = \infty$ [for $J=\frac{5}{2}$ solution a), or $J=\frac{3}{2}$ solution b)]; ----- $r_0 = 0.45\mu^{-1}$, $f = 4$ [for $J=\frac{3}{2}$ solution a), or $J=\frac{5}{2}$ solution b)]. The data are fitted satisfactorily for $1 < f < \infty$.

(a)

(b)

(c)

(d)

Fig.1

Fig. 2

P_L - kaon laboratory momentum (MeV/c)

Fig. 3a

Fia. 3b

Fig. 4