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THE DASHEN-FRAUTSCHI METHOD AND PARITY-CONSERVING
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The Dashen-Frautschi method ¹⁾, which was successfully applied in particular to the mass-differences among the baryon octet and decuplet ²⁾, is here used to treat parity-conserving SU_3 non-invariant baryon-meson couplings (the method being "universal", the same arguments hold for medium strong SU_3 breaking and for parity-conserving weak interactions). The results are not very gratifying :

1) Both octet and 27 - like coupling shifts are enhanced by comparable factors, which means in particular that the $\Delta I = \frac{1}{2}$ rule for P - wave non-leptonic hyperon decays does not come out as a consequence of a dynamical enhancement, as it did in the S wave (parity violating) case.

2) Furthermore, if one nevertheless tries the hypothesis of octet dominance, the model investigated is incapable of fitting the experimental results concerning the relevant hyperon decays. As for medium - strong interactions, the situation is better : the calculated widths for the $\frac{5}{2}^+$ resonances are reasonable ; however, the over - all magnitude of the SU_3 breaking constants needed is so large as to make the use of first order perturbation theory quite questionable.

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I. INTRODUCTION

Using their S - matrix perturbation technique ¹⁾, Dashen and Frautschi have studied the mass differences among the members of the baryon octet and decuplet ²⁾, and succeeded in showing that mass-shifts transforming like the centre of an octet are dynamically enhanced, which gives an explanation for the Gell-Mann - Okubo mass formula. Furthermore, they obtained ratios among the three octet mass-shifts (δM^{8s} and δM^{8a} for the baryon octet and δM^{Δ} for the decuplet) which are in fair agreement with experiment.

An important point, which makes their calculation relatively simple, is that they were able to treat mass-shifts by themselves, since it seems that coupling shifts in the crossed channel have little influence on the positions of the poles in the direct one (see especially Appendix B of Ref. ²⁾).

It is our purpose here to try to go one step further, and to see whether parity-conserving baryon-meson couplings can also be explained by similar arguments.

The starting point is always the same : one postulates that, in exact SU_3 symmetry, the baryon octet B and decuplet Δ are dynamical poles in the corresponding amplitudes of P + B scattering (P being the pseudoscalar octet). This constitutes the unperturbed solution of the problem.

One then switches on a parity conserving, but SU_3 violating, perturbation. This may be due either to the SU_3 non-conserving medium-strong interactions or to the parity-conserving weak ones : as emphasized in Refs. ¹⁾ and ²⁾, the enhancement arguments are "universal", since they only depend on SU_3 invariant factors.

2.

After the perturbation is taken into account, the degenerate poles of the unperturbed problem are split, and their residues altered. These modifications are treated to first order.

To calculate them, use is made of the static model for $P + B$ scattering, which enables one to evaluate most of the integrals appearing in the formulae of Ref ¹⁾, despite the fact that the unperturbed solution mentioned above is actually unknown.

The main point is then that the B and Δ multiplets which one wants to study in the direct channel also appear as intermediate states in the crossed one, and hopefully give the most important contributions to the left-hand cuts ³⁾.

Let us symbolically denote by δM and δG the mass and coupling shifts (each of them is actually a vector with several components). We will have to solve a system of equations of the form ⁵⁾:

$$\begin{pmatrix} \delta M \\ \delta G \end{pmatrix} = \begin{pmatrix} A & 0 \\ X & B \end{pmatrix} \begin{pmatrix} \delta M \\ \delta G \end{pmatrix} + \begin{pmatrix} R_M \\ R_G \end{pmatrix} \quad (1)$$

R_M and R_G are due to all the singularities given by other exchanges than B and Δ , and contain in particular the "driving terms" (see Refs. ¹⁾ and ²⁾) if they exist. As recalled above, the influence of δG on δM is negligible in this scheme, giving one vanishing off-diagonal element. However, the converse is far from true: the octet B is one of the external particles considered, and the corresponding mass-shifts will strongly react on δG .

Dashen and Frautschi have studied the first equation of system (1) :

$$\delta M = A \delta M + R_M \quad (2)$$

To solve it approximately, it is convenient to look first for the eigenvalues α_i and eigenvectors U_i of the matrix A . One expands δM and R_M on these eigenvectors :

$$\begin{cases} \delta M = \sum_i \delta m_i U_i \\ R_M = \sum_i r_i U_i \end{cases} \quad (3),$$

which gives

$$\delta m_i = \frac{r_i}{1 - \alpha_i} \quad (4)$$

If one of the eigenvalues, say α_1 , is near to 1, δm_1 will be enhanced, i.e., much greater than r_1 . If all r_i 's are of the same order, the other δm_i 's, corresponding to eigenvalues far from 1, will be negligible against δm_1 , and δM will be approximately collinear to U_1 :

$$\delta M \simeq \delta m_1 U_1 \quad (5)$$

This is what happens for δM^8 , transforming like $N = 8$, which is then likely to be much greater than δM^{27} , since the eigenvalues of A^{27} are all far from 1²).

This argument is then able to predict first that the mass differences will behave essentially like an octet, and second that the ratios between the various components of δM^8 will be those given by U_1 . Provided all the r_i 's, for $N = 8$ and 27, are of the same order of magnitude, a precise knowledge of them is not required.

Notice, however, that this very convenient situation (it is hard to actually evaluate R_M , while the matrix A can be calculated approximately) has an important drawback : if we knew R_M , the fact

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that α_1 is near to 1 means that a reasonable uncertainty on its value would result in a tremendous one for δm_1 ⁶⁾. This remark will be needed below.

Having an approximate solution of equation (2), we now consider the second equation of system (1) :

$$\delta G = B\delta G + X\delta M + R_G \quad (6)$$

and will try to solve it using the same method.

II. EVALUATION OF THE MATRIX CONNECTING CROSSED TO DIRECT COUPLING SHIFTS

A - Definition and expansion of the coupling shifts

Let δf_{ijk} and δg_{ijk} be the SU_3 non-invariant parts of the coupling constants at the vertices $B_j + P_k \rightarrow B_i$ and $B_j + P_k \rightarrow \Delta_i$ respectively (i, j, k are SU_3 indices).

Due to the presence of 3 indices, these coupling-shifts may be expanded in several ways on irreducible representations of SU_3 : one combines first a pair of indices to make an intermediate representation, which is then coupled to the third index to give the final transformation properties. We will have to use two of these "bases" in what follows.

In the first one, the intermediate representation β is made up of the last two indices (corresponding to a baryon and a pseudoscalar meson) ⁷⁾:

$$\delta f_{ijk} = \sum_{\beta \gamma y; N \Gamma n} \begin{pmatrix} 8 & 8 & \beta \gamma \\ j & k & y \end{pmatrix} (-1)^i \begin{pmatrix} 8 & \beta & N \Gamma \\ -i & y & n \end{pmatrix} \delta f_{\beta \gamma}^{N \Gamma n} \quad (7-a)$$

$$\delta g_{ijk} = \sum_{\beta \gamma y; N n} \begin{pmatrix} 8 & 8 & \beta \gamma \\ j & k & y \end{pmatrix} (-1)^i \begin{pmatrix} 10 & \beta & N \\ -i & y & n \end{pmatrix} \delta g_{\beta \gamma}^{N, n} \quad (7-b)$$

The second one will be considered only for the δf 's. It is built up by combining first the two baryons:

$$\delta f_{ijk} = \sum_{\alpha \gamma \beta; N \Gamma n} (-1)^i \begin{pmatrix} 8 & 8 & \alpha \gamma \\ -i & j & \beta \end{pmatrix} \begin{pmatrix} \alpha & 8 & N \Gamma \\ \beta & k & n \end{pmatrix} \delta f_{\alpha \gamma}^{N \Gamma n} \quad (8)$$

6.

Of course, the $\delta f_{\beta_\gamma}^{N_\Gamma, n}$ and $\delta \bar{f}_{\alpha_{\gamma'}}^{N_{\Gamma'}, n}$ are not independent. The properties of Clebsch-Gordan coefficients enable one to write :

$$\delta f_{\beta_\gamma}^{N_\Gamma, n} = \sum_{\alpha_{\gamma'}, \Gamma'} V_{\beta_\gamma; \alpha_{\gamma'}}^{N_\Gamma, N_{\Gamma'}} \delta \bar{f}_{\alpha_{\gamma'}}^{N_{\Gamma'}, n} \quad (9)$$

with

$$V_{\beta_\gamma; \alpha_{\gamma'}}^{N_\Gamma, N_{\Gamma'}} = \sum_{ij, k, y, z} \begin{pmatrix} 8 & 8 & \beta_\gamma \\ j & k & y \end{pmatrix} \begin{pmatrix} 8 & \beta & N_\Gamma \\ -i & y & n \end{pmatrix} \begin{pmatrix} 8 & 8 & \alpha_{\gamma'} \\ -i & j & z \end{pmatrix} \begin{pmatrix} \alpha & 8 & N_{\Gamma'} \\ z & k & n \end{pmatrix} \quad (10)$$

It is in fact easy to show (see Appendix I) that the matrix V relates only $(N_{\Gamma'}, n)$ to (N_Γ, n) and is independent of n .

The matrices V for $N = 8$ and $N = 27$ are given in Tables I and II respectively. Notice that they are not quite symmetric, in the bases just defined ; more precisely :

$$V_{\alpha_{\gamma'}, \beta_\gamma}^{N_{\Gamma'}, N_\Gamma} = \xi_1(8 \otimes 8; \alpha_{\gamma'}) \xi_1(8 \otimes 8; \beta_\gamma) \xi_1(8 \otimes \alpha; N_{\Gamma'}) \xi_1(8 \otimes \beta; N_\Gamma) \times \\ \times V_{\beta_\gamma; \alpha_{\gamma'}}^{N_\Gamma, N_{\Gamma'}} \quad (11)$$

B - Method of calculation of the matrix elements

We will give here only one example of such a calculation since the method is always the same. To simplify the expressions, we will take the case of semi-strong interactions, which conserve isospin and strangeness. This means that $n = (0, 0, 0)$. As recalled

above, the results are independent of n and "universal" ; i.e., the matrix we will find will also be valid for parity-conserving weak interactions, where one is more interested by terms where, for instance, $n = (1, \frac{1}{2}, -\frac{1}{2})$.

Let us consider the scattering amplitude for :

$$(B + P)_{\beta_Y, i; J^P = \frac{3}{2}^+} \longrightarrow (B + P)_{10, i; J^P = \frac{3}{2}^+} \quad (12)$$

To first order in the perturbation, this amplitude has a pole at $W = M^*$ (where W is the centre-of-mass energy and M^* the unperturbed decuplet mass), whose residue is:

$$\delta R_{i(\beta_Y, i)} = \begin{cases} -g \delta g_{i(\beta_Y, i)} & \text{if } \beta \neq 10 \\ -2g \delta g_{i(10, i)} & \text{if } \beta = 10 \end{cases} \quad (13)$$

where

$$\delta g_{i(\beta_Y, i)} = \sum_{j, k} \begin{pmatrix} 8 & 8 & \beta_Y \\ j & k & i \end{pmatrix} \delta g_{ijk} \quad (14),$$

and g is the unperturbed $\bar{\Delta}BP$ coupling constant, normalized in such a way that the SU_3 conserving part of the vertex $B_j + P_k \rightarrow \Delta_i$ is equal to

$$g_{ijk} = \begin{pmatrix} 8 & 8 & 10 \\ j & k & i \end{pmatrix} g \quad (15)$$

Now the formulae of Ref.¹⁾ give the following expression for $\delta R_{i(\beta_Y, i)}$:

$$\delta R_{i(\beta_Y, i)} = \left[\tilde{\Delta} J(M^*) \Delta + \tilde{\Delta} J'(M^*) \Delta + \tilde{\Delta} J(M^*) \Delta' \right]_{10, i; \beta_Y, i} \quad (16),$$

8.

$$\Delta = \lim_{W \rightarrow M^*} [(W - M^*) D^{-1}(W)] \quad (17)$$

$$\Delta' = \lim_{W \rightarrow M^*} \frac{d}{dW} [(W - M^*) D^{-1}(W)] \quad (18)$$

$$J(W) = \frac{1}{\pi} \int_L \frac{dW'}{W' - W} \tilde{D}(W') \text{Im} \delta T(W') D(W') + \\ + \frac{1}{\pi} \int_R \frac{dW'}{W' - W} \tilde{N}(W') \delta \rho(W') N(W') \quad (19)$$

$$J'(W) = \frac{d}{dW} J(W) \quad (20)$$

(N and D are the unperturbed matrices for B + P scattering in the state $J^P = \frac{3}{2}^+$, and $\delta \rho$ is a purely kinematical factor, influenced only by external mass-shifts).

Now, apart from the case of scattering in octet states, the N and D matrices are diagonal. The only non-vanishing element of Δ is then :

$$\Delta_{10,i;10,i} = 1 / D'_{10}(M^*) \quad (21)$$

where $D'_{10}(W)$ is the derivative of the D function for scattering in the representation 10.

Similarly, the non-vanishing elements of Δ' are :

$$\Delta'_{\alpha_{\gamma,i};\alpha_{\gamma',i}} = \mathbb{I}_{\alpha_{\gamma},\alpha_{\gamma'}}^{-1}(M^*) \quad \text{for } \alpha \neq 10 \quad (22-a)$$

$$\Delta'_{10,i;10,i} = -\frac{1}{2} \frac{\mathbb{I}_{10}''(M^*)}{[\mathbb{I}_{10}'(M^*)]^2} \quad (22-b)$$

Formulae (13) and (16) then give :

$$\begin{aligned} -g \delta g_{i(\beta_{\gamma,i})} &= \frac{1}{\mathbb{I}_{10}'(M^*)} \sum_{\gamma' \gamma''} \mathbb{I}_{\beta_{\gamma'},\beta_{\gamma''}}^{-1}(M^*) \times \\ &\times \left\{ \frac{1}{\pi} \int_L \frac{dW'}{W'-M^*} \mathbb{I}_{10}(W') \operatorname{Im} \delta T_{10,i;\beta_{\gamma'',i}}(W') \mathbb{I}_{\beta_{\gamma''},\beta_{\gamma'}}(W') + \right. \\ &\left. + \frac{1}{\pi} \int_R \frac{dW'}{W'-M^*} N_{10}(W') \delta \rho_{10,i;\beta_{\gamma'',i}}(W') N_{\beta_{\gamma''},\beta_{\gamma'}}(W') \right\} \quad (23-a) \end{aligned}$$

for $\beta \neq 10$, and :

$$\begin{aligned} -2g \delta g_{i(10,i)} &= \frac{1}{[\mathbb{I}_{10}'(M^*)]^2} \left\{ \frac{1}{\pi} \int_L \frac{dW'}{(W'-M^*)^2} [\mathbb{I}_{10}(W')]^2 \operatorname{Im} \delta T_{10,i;10,i}(W') + \right. \\ &\quad \left. + \frac{1}{\pi} \int_R \frac{dW'}{(W'-M^*)^2} [N_{10}(W')]^2 \delta \rho_{10,i;10,i}(W') \right\} - \\ &- \frac{\mathbb{I}_{10}''(M^*)}{[\mathbb{I}_{10}'(M^*)]^3} \left\{ \frac{1}{\pi} \int_L \frac{dW'}{W'-M^*} [\mathbb{I}_{10}(W')]^2 \operatorname{Im} \delta T_{10,i;10,i}(W') + \right. \\ &\quad \left. + \frac{1}{\pi} \int_R \frac{dW'}{W'-M^*} [N_{10}(W')]^2 \delta \rho_{10,i;10,i}(W') \right\} \quad (23-b) \end{aligned}$$

As mentioned above, the integrals over the right-hand cuts give the influence of external mass-shifts on the δg 's. We are more interested here in the contribution of the left-hand cut, since it will contain terms proportional to δg and δf .

To evaluate $\text{Im } \delta T$, let us consider the scattering :

$$B_j + P_k \longrightarrow B_\ell + P_m \quad (24)$$

in the state $J^P = \frac{3}{2}^+$. The first order modification of the corresponding scattering amplitude will be called $\delta T_{lm;jk}$.

Clearly :

$$\delta T_{10,i;\beta_\gamma,i} = \sum_{lmjk} \begin{pmatrix} 8 & 8 & 10 \\ \ell & m & i \end{pmatrix} \begin{pmatrix} 8 & 8 & \beta_\gamma \\ j & k & i \end{pmatrix} \delta T_{lm;jk} \quad (25)$$

Now we want to single out, in $\text{Im } \delta T$, the terms due to B and Δ exchange, represented in Figures 1 and 2.

In the crossed channel :

$$B_j + \overline{P}_m \longrightarrow B_\ell + \overline{P}_k \quad (26),$$

the $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$ amplitudes have poles at $W_c = M$ and $W_c = M^*$ respectively (W_c being the energy in the crossed channel). Their first order residues are (Figure 1) :

$$\delta R_{\ell \bar{k}, j \bar{m}}^{(B)} = - \sum_{\nu} (-1)^{m+k} \left[f_{\nu j-m} \delta f_{\nu \ell-k} + f_{\nu \ell-k} \delta f_{\nu j-m} \right] \quad (27-a)$$

and (Figure 2) :

$$\delta R_{\ell \bar{k}, j \bar{m}}^{(\Delta)} = - \sum_{\nu} (-1)^{m+k} \left[g_{\nu j-m} \delta g_{\nu \ell-k} + g_{\nu \ell-k} \delta g_{\nu j-m} \right] \quad (27-b)$$

(f_{ijk} is the unperturbed coupling constant at the vertex $B_j + P_k \rightarrow B_i$).

In the static model :

$$W_c = 2M - W \quad (28)$$

and the angular momentum crossing matrix reads :

$$\begin{array}{cc} J^P = \frac{1}{2}^+ & J^P = \frac{3}{2}^+ \\ J^P = \frac{1}{2}^+ & \begin{pmatrix} -\frac{1}{3} & \frac{4}{3} \\ \frac{2}{3} & \frac{1}{3} \end{pmatrix} \\ J^P = \frac{3}{2}^+ & \end{array} \quad (29)$$

The diagrams of Figures 1 and 2 then contribute to $\delta T_{lm; jk}$ the terms :

$$\delta T_{lm; jk}^{(B+\Delta)} = - \frac{2}{3} \frac{1}{W-M} \delta R_{\ell \bar{k}, j \bar{m}}^{(B)} - \frac{1}{3} \frac{1}{W-M^*} \delta R_{\ell \bar{k}, j \bar{m}}^{(\Delta)} \quad (30),$$

where :

$$M^* = 2M - M^* \quad (31)$$

Let us provisionally ignore the Clebsch-Gordan coefficients and indices, for simplicity. Im δT appearing in Equations (23-a) and (23-b) will have a term proportional to $-\frac{2}{3}\pi \delta(W-M)$ coming from B exchange and another one proportional to $-\frac{1}{3}\pi \delta(W-M^*)$ due to Δ exchange (where we have already taken the minus sign of $\delta R_{lk;j\bar{m}}^{(B)}$ and $\delta R_{lk;j\bar{m}}^{(\Delta)}$).

The first term will give :

$$-\frac{2}{3} \frac{1}{D'_{10}(M^*)} \sum_{\gamma'\gamma''} D_{\beta_{\gamma'}\beta_{\gamma''}}^{-1}(M^*) \frac{D_{10}(M)}{M-M^*} D_{\beta_{\gamma''}\beta_{\gamma'}}(M) \quad (32-a)$$

in equation (23-a), and :

$$-\frac{2}{3} \frac{1}{[D'_{10}(M^*)]^2} \left[\frac{D_{10}(M)}{M-M^*} \right]^2 + \frac{2}{3} \frac{D''_{10}(M^*)}{[D'_{10}(M^*)]^3} \frac{[D_{10}(M)]^2}{M-M^*} \quad (32-b)$$

in Equation (23-b).

As in Ref.²⁾, we expand the D functions around $W = M^*$ and keep only the leading term as a rough approximation⁹⁾. Expressions (32) then become both simply $-\frac{2}{3}$.

Now the unperturbed coupling constants appearing in (27) are proportional to Clebsch-Gordan coefficients : g_{ijk} is defined by formula (15), and f_{ijk} is taken as :

$$f_{ijk} = \frac{f}{\sqrt{5+9\lambda^2}} \left\{ \sqrt{5} \begin{pmatrix} 8 & 8 & 8_a \\ j & k & i \end{pmatrix} + 3\lambda \begin{pmatrix} 8 & 8 & 8_a \\ j & k & i \end{pmatrix} \right\} \quad (33)$$

(where λ is the F/D ratio for SU_3 invariant BP couplings), which we will write more simply :

$$f_{ijk} = \begin{pmatrix} 8 & 8 & 8_\lambda \\ j & k & i \end{pmatrix} f \quad (34)$$

Putting back the indices and Clebsch-Gordan coefficients, formula (23-a) becomes :

$$\begin{aligned} \delta g_{i(\beta_\gamma, i)} = & \sum_{l m j k v} (-1)^{m+k} \begin{pmatrix} 8 & 8 & 10 \\ l & m & i \end{pmatrix} \begin{pmatrix} 8 & 8 & \beta_\gamma \\ j & k & i \end{pmatrix} \times \\ & \times \left\{ \frac{2}{3} \frac{f}{g} \left[\begin{pmatrix} 8 & 8 & 8_\lambda \\ j & -m & v \end{pmatrix} \delta_{vl-k} + \begin{pmatrix} 8 & 8 & 8_\lambda \\ l & -k & v \end{pmatrix} \delta_{vj-m} \right] + \right. \\ & \left. + \frac{1}{3} \left[\begin{pmatrix} 8 & 8 & 10 \\ j & -m & v \end{pmatrix} \delta_{gl-k} + \begin{pmatrix} 8 & 8 & 10 \\ l & -k & v \end{pmatrix} \delta_{gj-m} \right] \right\} + \\ & + R_{i(\beta_\gamma, i)}^{(g)} \quad (35) \end{aligned}$$

while (23-b) takes on the same form (β_γ being replaced by 10), except for a factor of $\frac{1}{2}$.

A similar expression can be obtained for $\delta f_{i(\beta_\gamma, i)}$. A small complication arises here from the fact that $8 \otimes 8$ contains 8 twice. Instead of the symmetric and antisymmetric 8_s and 8_a , one has to take the combinations :

$$(8_\lambda) = \frac{1}{\sqrt{5+9\lambda^2}} \left[\sqrt{5} (8_s) + 3\lambda (8_a) \right] \quad (36)$$

$$(8_{\bar{\lambda}}) = \frac{1}{\sqrt{5+9\lambda^2}} \left[-3\lambda(8_s) + \sqrt{5}(8_a) \right] \quad (37)$$

Only the state 8_{λ} has a direct pole at $W = M$ in the $J^P = \frac{1}{2}^+$ wave, and $\delta f_{i(8_{\lambda}, i)}$ has a factor of $\frac{1}{2}$ compared to the others $\delta f_{i(\beta_{\gamma}, i)}$.

However, one has to postulate that the D matrix, which is diagonal, in the representation $(8_{\lambda}, 8_{\bar{\lambda}})$, at the pole, remains approximately so in a certain energy region around $W = M$ (see Ref.²).

But, from another point of view, the situation is better than in the example just treated in detail, since the crossed B pole lies in the static model at the same position as the direct one. Consequently, when calculating the influence of the diagram of figure 1 on δf , one need not expand the D functions around the pole: the approximation mentioned above in the evaluation of (32) becomes "exact".

The rest of the work is straightforward in principle but rather long and tedious in practice. It consists in plugging the expansions (7) into (35) and the similar expression for δf and calculating the corresponding sums of products of Clebsch-Gordan coefficients. The results are given in Tables III to XI¹¹⁾, and the procedure used is described in Appendix II.

C - Identity of the external and internal baryons

We have up to now treated δf_{ijk} as if the two baryons belonged to two different octets. If this were true, all the $\delta f_{\beta_{\gamma}}^{N_{\Gamma}}$ defined by formula (7-a) would be independent. In particular, for $N = 8$, there would be 8 parameters ($\beta = 1$; $\beta_{\gamma} = 8_s$, $N_{\Gamma} = 8_s$ and 8_a ; $\beta_{\gamma} = 8_a$, $N_{\Gamma} = 8_s$ and 8_a ; $\beta = 10$; $\beta = \overline{10}$ and $\beta = 27$). Actually, it is well known^{12), 13)} that only 5 of them are independent.

The effective Hamiltonian for the parity-conserving vertex $B_j + P_k \rightarrow B_i$ will be, up to a factor :

$$H_{ijk} \propto \delta f_{ijk} \overline{B_i} \gamma_5 B_j P_k \quad (38)$$

Now, if one imposes CP invariance, the fact that :

$$(CP) [\overline{B_i} \gamma_5 B_j P_k] (CP)^{-1} = \overline{B_j} \gamma_5 B_i \overline{P_k} \quad (39)$$

will result in the following restrictions on the δf 's :

$$\delta f_{ijk} = (-1)^k \delta f_{ji-k} \quad (40)$$

This complicates the problem rather seriously. In fact, we have just seen in Section II-B that the "dynamical factors" were simple when the basis used for the δf 's was that defined in Equation (7-a). But equality (40) is hard to express in this basis, which can easily be understood, since the two baryonic indices play a completely different role in (7-a).

This is why we are forced to consider a second basis, that of formula (8). In terms of the $\delta \overline{f}$'s, the right-hand side of (40) reads [once more for $n = (0,0,0)$] :

$$(-1)^k \delta f_{ji-k} = (-1)^k \sum_{\alpha \gamma, N_r} \overline{(-1)^j \begin{pmatrix} 8 & 8 & \alpha_f \\ -j & i & k \end{pmatrix} \begin{pmatrix} \alpha & 8 & N_r \\ k & -k & 0,0,0 \end{pmatrix}} \delta \overline{f}_{\alpha \gamma}^{N_r} \quad (41)$$

which can be rewritten in the form :

$$\begin{aligned}
(-1)^k \delta_{j, i-k}^p = & (-1)^i \sum_{\alpha_\gamma; N_\Gamma} \xi_1(8 \otimes 8; \alpha_\gamma) \xi_3(8 \otimes 8; \alpha_\gamma) \xi_3(\alpha \otimes 8; N_\Gamma) \times \\
& \times \begin{pmatrix} 8 & 8 & \overline{\alpha}_\gamma \\ -i & j & -k \end{pmatrix} \begin{pmatrix} \overline{\alpha} & 8 & N_\Gamma \\ -k & k & 0,0,0 \end{pmatrix} \delta_{\alpha_\gamma}^{\overline{N}_\Gamma} \quad (42)
\end{aligned}$$

Comparing with formula (8), one immediately sees that equality (40) implies :

$$\delta_{\alpha_\gamma}^{\overline{N}_\Gamma} = \xi_1(8 \otimes 8; \overline{\alpha}_\gamma) \xi_3(8 \otimes 8; \overline{\alpha}_\gamma) \xi_3(\overline{\alpha} \otimes 8; N_\Gamma) \delta_{\overline{\alpha}_\gamma}^{\overline{N}_\Gamma} \quad (43)$$

This means :

- a) For $N = 1$, there is no restriction (2 independent parameters). The matrix given in Table III can be kept as it is.
- b) For $N = 8$:

$$\begin{cases} \delta_{8_a}^{\overline{8}_a} = 0 \\ \delta_{8_a}^{\overline{8}_a} = 0 \\ \delta_{10}^{\overline{8}} = \delta_{10}^{\overline{8}} \end{cases} \quad (44)$$

The number of independent parameters is then 5 instead of 8, and one has to use the V matrix of Table I.

One first combines the second and third (respectively 4th and 5th) rows of this matrix so that they correspond to $\delta_{8_\lambda}^{8_s}$ and $\delta_{8_\lambda}^{8_s}$ (respectively $\delta_{8_\lambda}^{8_a}$ and $\delta_{8_\lambda}^{8_a}$). Then one crosses out the

columns labelled by $(\alpha_\gamma, N_\Gamma) = (8_s, 8_a)$ and $(8_a, 8_a)$, and replaces the 6th and 7th columns by a single one corresponding to $(\alpha) = \sqrt{\frac{1}{2}} [(10) + (\overline{10})]$. This gives an 8×5 matrix U^8 .

Finally, the 12×12 matrix obtained by grouping Tables IV to VII is multiplied from the right by the 12×9 one :

$$U_c^8 = \begin{pmatrix} U^8 & 0 \\ 0 & 1 \end{pmatrix} \quad (45)$$

where 1 stands for the 4×4 unity, and from the left by $\tilde{U}_c^8$. The resulting 9×9 matrix is the one that was called B^8 in Section I.

c) For $N = 27$:

$$\begin{aligned} \delta \bar{f}_{27}^{27a} &= 0 \\ \delta \bar{f}_{10}^{27} &= - \delta \bar{f}_{\overline{10}}^{27} \end{aligned} \quad (46)$$

We are left with 4 parameters, and follow a procedure similar to that used for $N = 8$: replace the first two rows of Table II by their combinations (8_λ) and $(8_{\bar{\lambda}})$, drop the last column and keep only $\sqrt{\frac{1}{2}} [(10) - (\overline{10})]$ in place of the 3rd and 4th columns, to obtain U^{27} , which is then 6×4 ; form U_c^{27} by "completing" U^{27} with the 5×5 unit matrix ; B^{27} is found by multiplying the 11×11 matrix displayed in Tables VIII to XI, from the right by U_c^{27} and from the left by $\tilde{U}_c^{27}$. One ends up with a 9×9 matrix B^{27} .

III. RESULTS AND DISCUSSION

A - Eigenvalues of the B matrices

The eigenvalues of B^1 , B^8 and B^{27} were determined numerically on the CERN CDC computer, for values of the F/D ratio λ varying from $\frac{1}{3}$ to $\frac{2}{3}$ ¹⁴⁾. The results for B^8 and B^{27} are displayed in Tables XII and XIII.

B^1 is of course less interesting. For $\lambda = \frac{1}{3}$, its three eigenvalues are : 0.83, -0.09 and -0.65.

Tables XII and XIII show that the eigenvalues of B^8 and B^{27} vary smoothly when λ goes from $\frac{1}{3}$ to $\frac{2}{3}$.

The most important point is certainly that both B^8 and B^{27} have one eigenvalue near 1 (due to the approximations used, it is impossible to say that one is actually smaller than the other). For semi-strong interactions, a violation of SU_3 transforming partly like 8 and partly like 27 could be accepted. However, these results mean that the $\Delta I = \frac{1}{2}$ rule for parity-conserving weak interactions (more precisely, we are studying here the non-leptonic decays of the hyperons) does not come out as a consequence of a dynamical enhancement of $N = 8$ compared to $N = 27$, as was the case for the corresponding parity-violating amplitudes ¹⁵⁾. This is rather bad, since the $\Delta I = \frac{1}{2}$ rule seems experimentally to hold fairly well for the non-leptonic decays of the hyperons ¹⁶⁾.

B - Is octet enhancement nevertheless present ?

But there exists another mechanism through which octet dominance could come in.

Since the δM 's found in solving equation (2) of Section I behave essentially like octet members, Equation (6) actually writes :

$$\delta G^8 = B^8 \delta G^8 + X^8 \delta M^8 + R_G^8 \quad (47-a)$$

$$\delta G^{27} = B^{27} \delta G^{27} + R_G^{27} \quad (47-b)$$

As explained in Section I, the enhancement argument postulates that the inhomogeneous terms are of the same order of magnitude in all similar equations. In (47) this would be for instance the case if :

$$R_G^8 \approx R_G^{27} \gg X^8 \delta M^8 \quad (48)$$

But the term $X^8 \delta M^8$ is likely to be important, since it contains the influence of mass-differences of the external particles, which is generally dominant (see for instance Ref.²⁾). We are thus led to try a hypothesis contrary to (48) :

$$R_G^8 \approx R_G^{27} \ll X^8 \delta M^8 \quad (49)$$

Octet enhancement would then be due to the influence of mass-terms : δG^8 would be much larger than δG^{27} , not because of the eigenvalues of B^8 and B^{27} , but because the inhomogeneous terms in (47-a) would be much larger than in (47-b).

However, this argument cannot be applied to strangeness changing weak interactions, which, to first order, give no mass-shift. The situation is then rather annoying : as mentioned above, one could accept a 27 term for semi-strong couplings, but not for parity-conser-

ving weak interactions, if one wants to incorporate the $\Delta I = \frac{1}{2}$ rule ¹⁷⁾ : now the presence of mass-terms can only be invoked in the first case.

We will nevertheless try the hypothesis that $N = 8$ dominates both semi-strong ¹⁸⁾ and parity conserving weak couplings (perhaps the difference in the eigenvalues of B^8 and B^{27} is significant ?), to see whether it is able to fit experimental data.

We have two pieces of data to fit : the widths of the $\frac{3}{2}^+$ resonances and the P waves of non-leptonic hyperon-decays. For the first, we take the currently accepted values ¹⁹⁾, except for the Y_1^* , where a radical change has recently taken place ²⁰⁾ (coupling constants are of course extracted from experimental widths by dividing out phase space ²¹⁾). For the second, it seems natural to rely on Ref. ²²⁾, since the S wave amplitudes reported there already agree with the theory of Ref. ¹⁵⁾.

Even if octet enhancement is not due to the mere fact that B^8 has an eigenvalue around 0.9, the corresponding eigenvector, displayed in Table XIV, should give the ratios between the various $\delta f^{8'}$'s and $\delta g^{8'}$'s.

a) Widths of the $\frac{3}{2}^+$ resonances :

We try here a two-parameter fit (the unperturbed coupling constant g and the common factor α of the δg 's ²³⁾).

The four known coupling constants are given in terms of g and the $\delta g^{8'}$'s by :

$$g_{N^*N\pi} = -\frac{1}{\sqrt{2}}g - \frac{1}{2\sqrt{5}}\delta g^8_{10} - \frac{1}{2\sqrt{3}}\delta g^8_{27} \quad (50-a)$$

$$g_{Y_1^*\Lambda\pi} = -\frac{1}{2}g - \frac{1}{5}\delta g^8_{8_\Lambda} - \frac{1}{5}\sqrt{\frac{2}{3}}\delta g^8_{27} \quad (50-b)$$

$$g_{Y^* \Sigma \pi} = \frac{1}{\sqrt{6}} g - \sqrt{\frac{2}{15}} \delta g^8_{8a} \quad (50-c)$$

$$g_{\Xi^* \Xi \pi} = \frac{1}{2} g + \frac{3}{10} \delta g^8_{8_0} - \frac{1}{2\sqrt{5}} \delta g^8_{8a} - \frac{1}{2\sqrt{10}} \delta g^8_{10} + \frac{1}{20} \sqrt{\frac{2}{3}} \delta g^8_{27} \quad (50-d)$$

The procedure used consisted in calculating the two parameters from $g_{Y^* \Lambda \pi}$ and $g_{\Xi^* \Xi \pi}$ (the width of the Ξ^* being varied from 5 to 9 MeV²¹) and finding the corresponding values for $g_{N^* N \pi}$ and $g_{Y^* \Sigma \pi}$.

The results show roughly the same features throughout the range of λ and $g_{\Xi^* \Xi \pi}$. A typical example is the following :

$$\left\{ \begin{array}{l} \lambda = \frac{1}{3} \\ g_{Y^* \Lambda \pi} = -0.21 \\ g_{\Xi^* \Xi \pi} = 0.18 \end{array} \right\} \text{ values given}$$

$$\left\{ \begin{array}{l} g_{N^* N \pi} = -0.32 \\ g_{Y^* \Sigma \pi} = 0.14 \end{array} \right\} \text{ values found}$$

$$\begin{array}{l} g = 0.29 \\ \alpha = 0.32 \end{array}$$

(51)

This is not so bad as far as the widths themselves are concerned : one finds $\Gamma_{N^*\pi} \simeq 100 \text{ MeV}$ and $\Gamma_{Y_1^*\pi} \simeq 2.9 \text{ MeV}$. Furthermore, the relative signs of the coupling constants come out as wanted ²⁴⁾.

However, the δg 's are not sufficiently small compared to g . To make the comparison, one has to multiply the value of g by $\sqrt{10}$, since the definition (15) does not contain :

$$(-1)^i \begin{pmatrix} 10 & 10 & 1 \\ -i & i & 0,0,0 \end{pmatrix} = -\frac{1}{\sqrt{10}} \quad (52)$$

whereas similar factors do appear in the δg 's (see definition (7-b)). But, even with this factor, the unperturbed coupling constant is only about three times larger than the biggest δg , which casts serious doubts on the validity of first order perturbation theory. We find ourselves in a situation similar to that of Ref. ¹³⁾, even after the change in the experimental parameters of the Y_1^* ²⁰⁾. This can even affect the original calculation of the mass-shifts ²⁾ : if $\frac{\delta g}{g}$ is really so large, perhaps one should not neglect the off-diagonal terms giving the influence of δG on δM (see Equation (1)).

b) P waves of non-leptonic hyperon decays

There are seven observable non-leptonic decays of hyperons. However, as we try here a fit with only $N = 8$, the $\Delta I = \frac{1}{2}$ relations will be automatically satisfied. There remain four independent parity conserving coupling constants, which one can take as :

$$P(\Sigma_0^+) = -\frac{3}{10\sqrt{2}} \delta f_{8_s}^{8_s} + \frac{1}{2\sqrt{10}} \delta f_{8_a}^{8_s} + \frac{1}{2\sqrt{5}} \delta f_{\frac{10+10}{2}}^{8_s} - \frac{1}{5\sqrt{3}} \delta f_{27}^{8_s} \quad (53-a)$$

$$P(\Sigma^+) = \frac{1}{\sqrt{10}} \delta \int_{\frac{10+10}{\sqrt{2}}}^8 - \frac{1}{3} \sqrt{\frac{2}{3}} \delta \int_3^8 \quad (53-b)$$

$$P(\Lambda_-) = -\frac{1}{10} \sqrt{\frac{3}{2}} \delta \int_{8_s}^8 - \frac{1}{2} \sqrt{\frac{3}{10}} \delta \int_{8_a}^8 + \frac{1}{2\sqrt{15}} \delta \int_{\frac{10+10}{\sqrt{2}}}^8 - \frac{1}{15} \delta \int_{27}^8 \quad (53-c)$$

$$P(\Xi^-) = \frac{1}{10} \sqrt{\frac{3}{2}} \delta \int_{8_s}^8 - \frac{1}{2} \sqrt{\frac{3}{10}} \delta \int_{8_a}^8 + \frac{1}{2\sqrt{15}} \delta \int_{\frac{10+10}{\sqrt{2}}}^8 + \frac{1}{15} \delta \int_{27}^8 \quad (53-d)$$

The one-parameter fit tried with these formulae and Table XIV completely fails : neither the relative magnitudes nor the signs of the amplitudes agree with experiment.

IV. CONCLUSION

This attempt to continue the investigation of Dashen and Frautschi on the baryon octet and decuplet ²⁾ by studying the parity-conserving SU_3 violations in the coupling constants has then failed, at least in important aspects :

- a) Both $N = 8$ and $N = 27$ show dynamical enhancement, so that the $\Delta I = \frac{1}{2}$ rule does not come out of these arguments as it did for the parity-violating case ¹⁵⁾.
- b) The hypothesis that octet dominance is still present appears, when investigated within the framework of the method, to be incapable of fitting experimental results concerning the parity-conserving part of the non-leptonic hyperon decays.

The situation is less disastrous for the $\frac{3}{2}^+$ resonances (medium strong interactions) : the calculated widths come out in reasonable agreement with experiment. However, one is always stopped by the same difficulty ²⁶⁾ : the SU_3 violating constants needed are too large, compared to the SU_3 conserving one, for perturbation theory to be reliable.

Now, what can be the reasons of this failure ?

It does not seem likely that point a) would be radically changed if one could improve the approximations used (essentially : static model and truncated expressions for the unperturbed D functions).

On the other hand, point b) is on much less secure grounds : the contributions of the eight neglected eigenvectors to δG may not be negligible, especially that corresponding to the eigenvalue $\simeq 0.64$. Unfortunately, to evaluate these contributions seems very difficult : as remarked above, one would need a high precision on the largest eigenvalues to get reliable results.

It is then not impossible that, despite the negative conclusions reached in Section III-B in testing this hypothesis, octet terms dominate.

Anyway, the situation is far from satisfactory, when compared to that of mass-shifts ²⁾ and parity-violating couplings ¹⁵⁾.

ACKNOWLEDGEMENTS

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NOTE

After this work was performed, we received a preprint on the same subject by R.F. Dashen, Y. Dothan, S.C. Frautschi and D.H. Sharp (CALT 68-51) ; they find the same results as ours, but interpret them in a somewhat more optimistic way.

APPENDIX I

Formula (7-a) can be rewritten in the form :

$$\delta_{ijk}^{N_{\Gamma}, n} = \sum_{\beta_{\gamma}, N_{\Gamma}, n} e_{ijk}^{\beta_{\gamma}, N_{\Gamma}, n} \delta_{\beta_{\gamma}}^{N_{\Gamma}, n} \quad (\text{AI-1})$$

with

$$e_{ijk}^{\beta_{\gamma}, N_{\Gamma}, n} = \sum_y (-1)^i \begin{pmatrix} 8 & 8 & \beta_{\gamma} \\ j & k & y \end{pmatrix} \begin{pmatrix} 8 & \beta & N_{\Gamma} \\ -i & y & n \end{pmatrix} \quad (\text{AI-2})$$

The orthogonality relations among Clebsch-Gordan coefficients imply that :

$$\sum_{ijk} e_{ijk}^{\beta_{\gamma}, N_{\Gamma}, n} e_{ijk}^{\beta'_{\gamma}, N'_{\Gamma}, n'} = \delta_{\beta\beta'} \delta_{\gamma\gamma'} \delta_{NN'} \delta_{\Gamma\Gamma'} \delta_{nn'} \quad (\text{AI-3})$$

and

$$\sum_{\beta_{\gamma}, N_{\Gamma}, n} e_{ijk}^{\beta_{\gamma}, N_{\Gamma}, n} e_{i'j'k'}^{\beta_{\gamma}, N_{\Gamma}, n} = \delta_{ii'} \delta_{jj'} \delta_{kk'} \quad (\text{AI-4})$$

Formula (AI-1) can then be easily inverted :

$$\delta_{\beta_{\gamma}}^{N_{\Gamma}, n} = \sum_{ijk} e_{ijk}^{\beta_{\gamma}, N_{\Gamma}, n} \delta_{ijk}^{N_{\Gamma}, n} \quad (\text{AI-5})$$

When $\delta_{ijk}^{N_{\Gamma}, n}$ is replaced according to definition (8), one finds :

$$\delta_{\beta_{\gamma}}^{N_{\Gamma}, n} = \sum_{\alpha_{\gamma'}, N'_{\Gamma}, n'} V_{\beta_{\gamma} ; \alpha_{\gamma'}}^{N_{\Gamma}, n ; N'_{\Gamma}, n'} \delta_{\alpha_{\gamma'}}^{N'_{\Gamma}, n'} \quad (\text{AI-6})$$

where

$$V_{\beta\gamma; \alpha\gamma'}^{N_r, n; N_{r'}, n'} = \sum_{ijk y z} \begin{pmatrix} 8 & 8 & \beta\gamma \\ j & k & y \end{pmatrix} \begin{pmatrix} 8 & \beta & N_r \\ -i & y & n \end{pmatrix} \times \\ \times \begin{pmatrix} 8 & 8 & \alpha\gamma' \\ -i & j & z \end{pmatrix} \begin{pmatrix} \alpha & 8 & N_{r'} \\ z & k & n' \end{pmatrix} \quad (\text{AI-7})$$

Now (see Ref.⁸) :

$$\begin{pmatrix} 8 & 8 & \beta\gamma \\ j & k & y \end{pmatrix} = \xi_1(8 \otimes 8; \beta\gamma) \xi_2(8 \otimes 8; \beta\gamma) (-1)^k \sqrt{\frac{D_\beta}{8}} \begin{pmatrix} 8 & \bar{\beta} & 8\gamma \\ k & -y & -j \end{pmatrix} \\ \begin{pmatrix} 8 & 8 & \alpha\gamma' \\ -i & j & z \end{pmatrix} = \xi_2(8 \otimes 8; \alpha\gamma') (-1)^i \sqrt{\frac{D_\alpha}{8}} \begin{pmatrix} 8 & \bar{\alpha} & 8\gamma' \\ -i & -z & -j \end{pmatrix} \quad (\text{AI-8})$$

where D_β is the dimension of representation β .

(AI-7) then becomes :

$$V_{\beta\gamma; \alpha\gamma'}^{N_r, n; N_{r'}, n'} = \xi_1(8 \otimes 8; \beta\gamma) \xi_2(8 \otimes 8; \beta\gamma) \xi_2(8 \otimes 8; \alpha\gamma') \frac{\sqrt{D_\beta D_\alpha}}{8} \times \\ \times \sum_{ijk y z} \begin{pmatrix} 8 & \beta & N_r \\ -i & y & n \end{pmatrix} \begin{pmatrix} \alpha & 8 & N_{r'} \\ z & k & n' \end{pmatrix} (-1)^{i+k} \begin{pmatrix} 8 & \bar{\beta} & 8\gamma \\ k & -y & -j \end{pmatrix} \begin{pmatrix} 8 & \bar{\alpha} & 8\gamma' \\ -i & -z & -j \end{pmatrix} \quad (\text{AI-9})$$

Consider now a fictitious reaction :

the initial state being (N_r, n) and the final one $(N_{r'}, n')$.

Suppose that, in the crossed channel :

an octet of particles is exchanged (with a coupling through $8_{\gamma'}$ and 8_{γ} respectively, if α and β are octets). The corresponding crossing coefficient will be given by :

$$\sum_{abmm'l} \begin{pmatrix} 8 & \beta & N_{\rho'} \\ m & b & n \end{pmatrix} \begin{pmatrix} \alpha & 8 & N_{\rho'} \\ a & m' & n' \end{pmatrix} (-1)^{a+b} \times$$

$$\times \begin{pmatrix} 8 & \bar{\alpha} & 8_{\gamma'} \\ m & -a & l \end{pmatrix} \begin{pmatrix} 8 & \bar{\beta} & 8_{\gamma} \\ m' & -b & l \end{pmatrix} \quad (\text{AI-10})$$

and it is well known that such a crossing coefficient is proportional to $\delta_{NN'}$, $\delta_{nn'}$ and independent of n .

Expression (AI-10) is identical with the sum appearing in (AI-9) (change m to $-i$, b to y , a to z , m' to k and l to $-j$), since $(-1)^{i+k} = (-1)^{y+z}$, due to the fact that n and n' conserve electric charge : $(-1)^n = (-1)^{n'} = +1$

$$\sqrt{\begin{matrix} N_{\rho}, n & N_{\rho'}, n' \\ \beta_{\gamma} & \alpha_{\gamma'} \end{matrix}}$$

is then proportional to $\delta_{NN'}$, $\delta_{nn'}$ and independent of n .

APPENDIX II

We want here to sketch the method used to obtain the results of Tables III to XI. We will for that purpose take one example.

One of the terms of Equation (36) reads :

$$\delta g_{i(\beta_\gamma, i)}^{(1)} = \frac{1}{3} \sum_{lmjkv} (-1)^{m+k} \begin{pmatrix} 8 & 8 & 10 \\ l & m & i \end{pmatrix} \times \\ \times \begin{pmatrix} 8 & 8 & \beta_\gamma \\ j & k & i \end{pmatrix} \begin{pmatrix} 8 & 8 & 10 \\ l & -k & v \end{pmatrix} \delta g_{vj-m} \quad (\text{AII-1})$$

Since $\delta g_{i(10, i)}$ contains a similar term, but with $\frac{1}{6}$ replacing $\frac{1}{3}$, we will provisionally forget about this factor and concentrate on the quantity :

$$\delta g'_{i(\beta_\gamma, i)} = \sum_{lmjkv} (-1)^{m+k} \begin{pmatrix} 8 & 8 & 10 \\ l & m & i \end{pmatrix} \times \\ \times \begin{pmatrix} 8 & 8 & \beta_\gamma \\ j & k & i \end{pmatrix} \begin{pmatrix} 8 & 8 & 10 \\ l & -k & v \end{pmatrix} \delta g_{vj-m} \quad (\text{AII-2})$$

in which now β can also be 10.

A very quick and convenient way to find formula (AII-2) is to consider the diagram of Figure 3 : this diagram visualizes the coupling of particle i of the decuplet to the BP state (β_γ, i) ; to obtain the influence of δg on itself, the "structure" of the vertex has to be due to Δ exchange, and the circle, in Figure 3, points out the perturbed vertex (of course, if the non-invariant vertex is taken at the other end of the v line, one gets the second term of (36) containing δg).

We insert the expansion (7-b) for δg_{vj-m} (as we have taken the case of semi-strong SU_3 violations, n is the centre of the representation N), and the following expression for $\delta g'_i(\beta_\gamma, i)$:

$$\delta g'_i(\beta_\gamma, i) = (-1)^i \sum_N \begin{pmatrix} \overline{10} & \beta & N \\ -i & i & 0,0,0 \end{pmatrix} \delta g'_{\beta_\gamma}{}^N \quad (\text{AII-3})$$

This results in :

$$\delta g'_{\beta_\gamma}{}^N = \sum_{\beta'_\gamma} B'_{\beta_\gamma \beta'_\gamma}{}^N \delta g_{\beta'_\gamma}{}^N \quad (\text{AII-4})$$

where

$$B'_{\beta_\gamma \beta'_\gamma}{}^N = \sum_{ijklmv} (-1)^{i+m+k+v} \begin{pmatrix} \overline{10} & \beta & N \\ -i & i & 0,0,0 \end{pmatrix} \begin{pmatrix} 8 & 8 & 10 \\ l & m & i \end{pmatrix} \times \\ \times \begin{pmatrix} 8 & 8 & \beta_\gamma \\ j & k & i \end{pmatrix} \begin{pmatrix} 8 & 8 & 10 \\ l & -k & v \end{pmatrix} \begin{pmatrix} 8 & 8 & \beta'_\gamma \\ j & -m & v \end{pmatrix} \begin{pmatrix} \overline{10} & \beta' & N \\ -v & v & 0,0,0 \end{pmatrix} \quad (\text{AII-5})$$

Once B' is known, B is obtained by multiplying each row by the corresponding dynamical factor (here $\frac{1}{3}$ for $\beta \neq 10$ and $\frac{1}{6}$ for $\beta = 10$).

It is also possible to verify here, as in Appendix I, that expression (AII-5) is different from zero only if the same N and n appear in the first and last Clebsch-Gordan coefficient, and independent of n .

This property can be used to simplify the calculations. In fact, due to it, (AII-5) can be rewritten :

$$\begin{aligned}
 B_{\beta_Y; \beta_{Y'}}^{N'} \delta_{NN'} \delta_{nn'} &= \sum_{ijklm \nu y z} (-1)^{i+m+k+\nu} \times \\
 &\times \begin{pmatrix} \overline{10} & \beta & N' \\ -i & y & n' \end{pmatrix} \begin{pmatrix} 8 & 8 & 10 \\ l & m & i \end{pmatrix} \begin{pmatrix} 8 & 8 & \beta_Y \\ j & k & y \end{pmatrix} \times \\
 &\times \begin{pmatrix} 8 & 8 & 10 \\ l & -k & \nu \end{pmatrix} \begin{pmatrix} 8 & 8 & \beta_{Y'} \\ j & -m & z \end{pmatrix} \begin{pmatrix} \overline{10} & \beta' & N \\ -\nu & z & n \end{pmatrix}
 \end{aligned} \tag{AII-6}$$

and the following formula then gives $B_{\beta_Y; \beta_{Y'}}^{N'}$:

$$\begin{aligned}
 (-1)^i \begin{pmatrix} \overline{10} & \beta & N \\ -i & y & n \end{pmatrix} B_{\beta_Y; \beta_{Y'}}^{N'} &= \sum_{jklm \nu z} (-1)^{m+k+\nu} \times \\
 &\times \begin{pmatrix} 8 & 8 & 10 \\ l & m & i \end{pmatrix} \begin{pmatrix} 8 & 8 & \beta_Y \\ j & k & y \end{pmatrix} \begin{pmatrix} 8 & 8 & 10 \\ l & -k & \nu \end{pmatrix} \times \\
 &\times \begin{pmatrix} 8 & 8 & \beta_{Y'} \\ j & -m & z \end{pmatrix} \begin{pmatrix} \overline{10} & \beta' & N \\ -\nu & z & n \end{pmatrix}
 \end{aligned} \tag{AII-7}$$

With a minimum of cleverness in the choice of n , i and y , formula (AII-7) can be handled as it stands, using the Clebsch-Gordan tables of Ref.²⁷⁾ and the various $\sum$ phase factors defined and tabulated in Ref.⁸⁾ (actually, this was not quite sufficient, and we also needed Ref.²⁸⁾). The only difficulty is to keep track of signs and square roots.

Of course, one can use several sets of values for n , i and y and make sure that they give the same results. Another fortunate a posteriori check comes from the fact that, for a given N , the B' matrices are symmetric with respect to β and β' (this property can be easily verified on formula (AII-5)).

Another possibility is to use isoscalar factors ⁸⁾ instead of Clebsch-Gordan coefficients. This reduces the number of terms to be considered.

When separating the Clebsch-Gordan coefficients appearing in (AII-7) into isoscalar factors and SU_2 Clebsch-Gordan coefficients, the sum over the various T_3 's obtained reads (we have chosen $n = (0,0,0)$, so that $y = i$ and $z = v$) :

$$\sum_{jklmv} (-1)^{m+k+v} C \begin{pmatrix} T_e & T_m & T_i \\ T_{3e} & T_{3m} & T_{3i} \end{pmatrix} C \begin{pmatrix} T_j & T_k & T_i \\ T_{3j} & T_{3k} & T_{3i} \end{pmatrix} \times$$

$$\times C \begin{pmatrix} T_e & T_k & T_v \\ T_{3e} & -T_{3k} & T_{3v} \end{pmatrix} C \begin{pmatrix} T_j & T_m & T_v \\ T_{3j} & -T_{3m} & T_{3v} \end{pmatrix} C \begin{pmatrix} T_v & T_v & 0 \\ -T_{3v} & T_{3v} & 0 \end{pmatrix}$$

(AII-8)

The last coefficient has the value :

$$C \begin{pmatrix} T_v & T_v & 0 \\ -T_{3v} & T_{3v} & 0 \end{pmatrix} = \frac{(-1)^{T_v + T_{3v}}}{\sqrt{2T_v + 1}}$$

(AII-9)

The sign in (AII-9) combines with $(-1)^v$ to give :

$$(-1)^{T_v + 2T_{3v} + \frac{1}{2}Y_v} = (-1)^{\frac{1}{2}Y_v - T_v} \quad (\text{AII-10})$$

which is independent of T_{3v} and factorizes out of the sum (AII-8).

What remains :

$$\sum_{jklmv} (-1)^{k+m} C \begin{pmatrix} T_l & T_m & T_i \\ T_{3l} & T_{3m} & T_{3i} \end{pmatrix} C \begin{pmatrix} T_j & T_k & T_i \\ T_{3j} & T_{3k} & T_{3i} \end{pmatrix} \times$$

$$\times C \begin{pmatrix} T_l & T_k & T_v \\ T_{3l} & -T_{3k} & T_{3v} \end{pmatrix} C \begin{pmatrix} T_j & T_m & T_v \\ T_{3j} & -T_{3m} & T_{3v} \end{pmatrix}$$

(AII-11)

is nothing but an isospin crossing coefficient, as visualized in Figure 4. If this crossing coefficient is called $R[(lm)_i; (jk)_i; v]$, equation (AII-7) becomes :

$$(-1)^i \begin{pmatrix} \overline{10} & \beta & N \\ -i & i & 0,0,0 \end{pmatrix} B'_{\beta\gamma; \beta'\gamma'}^N = \sum_{jklmv} \frac{(-1)^{\frac{1}{2}Y_v - T_v}}{\sqrt{2T_v + 1}} \times$$

$$\times R[(lm)_i; (jk)_i; v] \times \begin{pmatrix} 8 & 8 & \overline{10} \\ l & m & i \end{pmatrix} \times$$

$$\times \begin{pmatrix} 8 & 8 & \beta\gamma \\ j & k & i \end{pmatrix} \begin{pmatrix} 8 & 8 & \overline{10} \\ l & -k & v \end{pmatrix} \begin{pmatrix} 8 & 8 & \beta'\gamma' \\ j & -m & v \end{pmatrix} \begin{pmatrix} \overline{10} & \beta' & N \\ -v & v & 0,0 \end{pmatrix}$$

(AII-12)

where the indices on the right-hand side represent now (Y, T) only.

The isospin crossing coefficients R are tabulated for instance in Ref.²⁹⁾.

More powerful methods can of course be found. In particular, one can use SU_3 crossing coefficients. The difficulty in doing so comes from the fact that not many of these are given in the literature. However, the matrices relating δf 's to themselves can be calculated this way.

REFERENCES AND FOOTNOTES

- 1) R.F. Dashen and S.C. Frautschi - Phys.Rev 137, B1318 (1965).
- 2) R.F. Dashen and S.C. Frautschi - Phys.Rev 137, B1331 (1965).
- 3) Notice that this argument is not invoked here for the unperturbed problem, as was the case in the "reciprocal bootstrap" model ⁴⁾. The unperturbed solution is supposed to be exact, and the nearby singularity considerations applied only to the perturbation formulae.
- 4) R.F. Dashen - Phys.Letters 11, 89 (1964);
Y. Hara - Phys.Rev. 135, B1079 (1964) .
- 5) Each representation N which δM and δG can transform like will give one such system, different representations being unmixed (see Refs. ¹⁾ and ²⁾).
- 6) If for instance we had calculated that α_1 lies between 0.85 and 0.95, δm_1 would be known only up to a factor of 3.
- 7) $\begin{pmatrix} \mu_1 & \mu_2 & \mu_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{pmatrix}$ is the Clebsch-Gordan coefficient of SU_3 combining the members γ_1 and γ_2 of the representations μ_1 and μ_2 to obtain γ_3 in the representation μ_3 . Notice that the first index i represents a particle which is created in the vertex, i.e., which transforms like $-i$, with a phase factor defined in Ref. ⁸⁾: $(-1)^i = (-1)^{\frac{1}{2}Y_i + T_3 i}$. The indices γ and Γ operate when a given representation is obtained several times in the product. More generally, the notations used in this paper will be those of Ref. ⁸⁾.

- 8) J.J. de Swart - Rev.Modern Phys. 35, 916 (1963).
- 9) This means that we take : $D_\beta(M) \simeq D_\beta(M^*)$ for $\beta \neq 10$ and $D_{10}(M) \simeq (M-M^*) D'_{10}(M^*)$. Actually, the term in $(M-M^*) \frac{D'_{10}(M^*)}{D'_{10}(M^*)}$ cancels out of expression (32-b).
- 10) This factor comes of course from the two appearing in the left-hand side of Equation (23-b). It originates from the fact that, when both the initial and final states of $B + P$ are in 10 , the violation of SU_3 can occur at one or the other vertex in the pole diagram.
- 11) To minimize the possibility of errors, each element of Tables III to XI was calculated at least twice, by different methods and authors.
- 12) M. Muraskin and S.L. Glashow - Phys.Rev. 132, 482 (1963).
- 13) B. Diu, H.R. Rubinstein and J.L. Basdevant - Nuovo Cimento 35, 460 (1965).
- 14) As in Ref.²⁾, the value of $\frac{g}{f}$ was taken from the "reciprocal bootstrap" model for B and Δ ⁴⁾, which gives $\frac{g}{f} = 4 \sqrt{\frac{1+3\lambda}{11(5+9\lambda^2)}}$. Notice, though, that the eigenvalues of the B matrices do not depend on $\frac{g}{f}$.
- 15) R.F. Dashen, S.C. Frautschi and D.H. Sharp - Phys.Rev.Letters 13, 777 (1964).
- 16) See for instance R.H. Dalitz - Varenna lectures (June 1964).

- 17) The representation 27 of course contains a $\Delta I = \frac{1}{2}$ term, but one has no reason, in this model, to postulate that this one is present and not the $\Delta I = \frac{3}{2}$ one.
- 18) It would be of no great use to try and evaluate $X^8 \delta M^8$, because of the remark already made in Section I : we are here limited by the very fact that B^8 has an eigenvalue near 1.
- 19) A.H. Rosenfeld et al. - Revs. Modern Phys. 36, 977 (1964).
- 20) R. Armenteros et al. - CERN preprint TC 65-16.
- 21) The formula used is :

$$g^2 = \frac{\Gamma}{2} \frac{M^{*2}}{q M^2 (M^* - M)^2 - \mu^2} \frac{1}{\mu^2}$$

(M^* , Γ = mass and width of the $\frac{3}{2}^+$ resonance ; M , μ = masses of the $\frac{1}{2}^+$ baryon and meson, respectively ; q = centre-of-mass momentum). The relative signs between the four coupling constants to be fit are taken from exact SU_3 , which is natural, since we have treated semi-strong interactions as a perturbation of the SU_3 invariant ones. The results are :

$$g_{N^* N \pi} \simeq -0.37 \mu^{-2} \text{ for } \Gamma = 125 \text{ MeV } (-0.33 \mu^{-2} \text{ for } \Gamma = 100 \text{ MeV})$$

$$g_{Y^* \Lambda \pi} \simeq -0.21 \mu^{-2} \text{ for } \Gamma = 30 \text{ MeV}$$

$$g_{Y^* \Sigma \pi} \simeq 0.19 \mu^{-2} \text{ for } \Gamma = 5 \text{ MeV}$$

$$g_{\Xi^* \Xi \pi} \simeq 0.18 \mu^{-2} \text{ for } \Gamma = 9 \text{ MeV } (0.14 \mu^{-2} \text{ for } \Gamma = 6 \text{ MeV})$$

- 22) M.L. Stevenson et al. - Phys. Letters 9, 349 (1964).
- 23) The unperturbed coupling constant g could be, as above ¹⁴⁾, taken from the "reciprocal bootstrap" model for B and Δ . However, it is perhaps not fair to impose the value of g a priori, especially since the matrix B^1 has also an eigenvalue near 1.
- 24) Notice that, since the violation of SU_3 considered transforms like 8, the following sum rule is satisfied ²⁵⁾:
- $$-2g_{N^*N\pi} + 3\sqrt{2}g_{Y_1^*N\pi} - \sqrt{3}g_{Y_1^*\Sigma\pi} + 2\sqrt{2}g_{\Xi^*\Xi\pi} = 0$$
- 25) M. Konuma and Y. Tomozawa - Phys. Letters 10, 347 (1964).
- 26) B. Diu and H.R. Rubinstein - Nuovo Cimento 37, 1673 (1965).
- 27) P. McNamee and F. Chilton - Revs. Modern Phys. 36, 1005 (1964).
- 28) L.K. Pandit and N. Mukunda - University of Rochester preprint UR 875 - 73.
- 29) B. Diu, H.R. Rubinstein and J.L. Basdevant - Nuovo Cimento 36, 322 (1965).

FIG.1

FIG.2

FIG. 3

FIG. 4

T A B L E I

$\beta_Y, N_\Gamma \backslash \begin{matrix} \alpha_{Y'} \\ N_{\Gamma'} \end{matrix}$	$\begin{matrix} 1 \\ 8 \end{matrix}$	$\begin{matrix} 8_s \\ 8_s \end{matrix}$	$\begin{matrix} 8_a \\ 8_s \end{matrix}$	$\begin{matrix} 8_s \\ 8_a \end{matrix}$	$\begin{matrix} 8_a \\ 8_a \end{matrix}$	$\begin{matrix} 10 \\ 8 \end{matrix}$	$\begin{matrix} \overline{10} \\ 8 \end{matrix}$	$\begin{matrix} 27 \\ 8 \end{matrix}$
$1, 8$	$\frac{1}{8}$	$-\frac{1}{2\sqrt{2}}$	0	0	$-\frac{1}{2\sqrt{2}}$	$\frac{1}{4}\sqrt{\frac{5}{2}}$	$-\frac{1}{4}\sqrt{\frac{5}{2}}$	$\frac{3\sqrt{3}}{8}$
$8_s, 8_s$	$-\frac{1}{2\sqrt{2}}$	$-\frac{3}{10}$	0	0	$\frac{1}{2}$	$\frac{1}{\sqrt{5}}$	$-\frac{1}{\sqrt{5}}$	$-\frac{9}{10\sqrt{6}}$
$8_a, 8_s$	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	0
$8_s, 8_a$	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0
$8_a, 8_a$	$-\frac{1}{2\sqrt{2}}$	$\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0	$\frac{\sqrt{6}}{4}$
$10, 8$	$-\frac{1}{4}\sqrt{\frac{5}{2}}$	$-\frac{1}{\sqrt{5}}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}\sqrt{\frac{3}{10}}$
$\overline{10}, 8$	$\frac{1}{4}\sqrt{\frac{5}{2}}$	$\frac{1}{\sqrt{5}}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}\sqrt{\frac{3}{10}}$
$27, 8$	$\frac{3\sqrt{3}}{8}$	$-\frac{9}{10\sqrt{6}}$	0	0	$\frac{\sqrt{6}}{4}$	$-\frac{1}{4}\sqrt{\frac{3}{10}}$	$\frac{1}{4}\sqrt{\frac{3}{10}}$	$\frac{7}{40}$

THE V MATRIX GIVING THE $\delta f_{\beta_Y}^{N_\Gamma, n}$ IN TERMS OF THE $\delta \bar{f}_{\alpha_{Y'}}^{N_{\Gamma'}, n}$ FOR $N = 8$

T A B L E II

$\beta_Y, N_\Gamma \backslash \begin{matrix} \alpha_{Y'} \\ N_{\Gamma'} \end{matrix}$	$\begin{matrix} 8_s \\ 27 \end{matrix}$	$\begin{matrix} 8_a \\ 27 \end{matrix}$	$\begin{matrix} 10 \\ 27 \end{matrix}$	$\begin{matrix} \overline{10} \\ 27 \end{matrix}$	$\begin{matrix} 27 \\ 27_s \end{matrix}$	$\begin{matrix} 27 \\ 27_a \end{matrix}$
$8_s, 27$	$\frac{1}{5}$	0	$-\frac{1}{\sqrt{5}}$	$-\frac{1}{\sqrt{5}}$	$\frac{\sqrt{14}}{5}$	0
$8_a, 27$	0	$-\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	0	$\sqrt{\frac{2}{3}}$
$10, 27$	$-\frac{1}{\sqrt{5}}$	$-\frac{1}{3}$	$-\frac{1}{6}$	$\frac{2}{3}$	$\frac{1}{2}\sqrt{\frac{7}{10}}$	$\frac{1}{2\sqrt{6}}$
$\overline{10}, 27$	$-\frac{1}{\sqrt{5}}$	$\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{6}$	$\frac{1}{2}\sqrt{\frac{7}{10}}$	$-\frac{1}{2\sqrt{6}}$
$27, 27_s$	$\frac{\sqrt{14}}{5}$	0	$\frac{1}{2}\sqrt{\frac{7}{10}}$	$\frac{1}{2}\sqrt{\frac{7}{10}}$	$\frac{3}{10}$	0
$27, 27_a$	0	$\sqrt{\frac{2}{3}}$	$-\frac{1}{2\sqrt{6}}$	$\frac{1}{2\sqrt{6}}$	0	$\frac{1}{2}$

THE V MATRIX GIVING THE $\delta f_{\beta_Y}^{N_\Gamma, n}$ IN TERMS OF THE $\delta \bar{f}_{\alpha_{Y'}}^{N_{\Gamma'}, n}$ FOR $N = 27$

T A B L E III

	$\delta f_{s\lambda}^1$	$\delta f_{s\bar{\lambda}}^1$	δg_{10}^1
$\delta f_{s\lambda}^1$	$\frac{1}{2} \frac{5 + 30\lambda^2 - 27\lambda^4}{(5 + 9\lambda^2)^2}$	$\frac{\lambda(1 - 9\lambda^2)}{(5 + 9\lambda^2)^2} \sqrt{5}$	$-\frac{4}{3} \frac{g}{f} \frac{1 + 3\lambda}{5 + 9\lambda^2} \sqrt{5}$
$\delta f_{s\bar{\lambda}}^1$	$\frac{2\lambda(1 - 9\lambda^2)}{(5 + 9\lambda^2)^2} \sqrt{5}$	$-\frac{36\lambda^2}{(5 + 9\lambda^2)^2}$	$-\frac{4}{3} \frac{g}{f} \frac{5 - 6\lambda - 9\lambda^2}{5 + 9\lambda^2}$
δg_{10}^1	$-\frac{2}{3} \frac{f}{g} \frac{1 + 3\lambda}{5 + 9\lambda^2} \sqrt{5}$	$-\frac{1}{3} \frac{f}{g} \frac{5 - 6\lambda - 9\lambda^2}{5 + 9\lambda^2}$	$\frac{1}{12}$

THE MATRIX RELATING CROSSED COUPLING-SHIFTS TO DIRECT ONES FOR $N = 1$

TABLE IV

$\beta_{\gamma}, N_{\Gamma}$	$\beta_{\gamma'}, N_{\Gamma'}$	1	8_{λ}	8_{λ}	8_{λ}	8_{λ}	10	10	27
		8	8_s	8_a	8_{λ}	8_{λ}	8_a	8	8
1, 8		$-\frac{5}{12} \frac{1}{5+9\lambda^2}$	$-\frac{1}{2} \frac{\sqrt{5}}{2} \frac{1+3\lambda^2}{(5+9\lambda^2)^{3/2}}$	$-\frac{\lambda}{\sqrt{2}} \frac{1}{(5+9\lambda^2)^{3/2}}$	$-\frac{\lambda}{2\sqrt{2}} \frac{5-9\lambda^2}{(5+9\lambda^2)^{3/2}}$	$-\frac{3\lambda^2 \sqrt{10}}{2(5+9\lambda^2)^{3/2}}$	$-\frac{1}{2} \frac{\sqrt{5}}{2} \frac{\lambda}{(5+9\lambda^2)^{3/2}}$	$-\frac{1}{2} \frac{\sqrt{5}}{2} \frac{\lambda}{(5+9\lambda^2)^{3/2}}$	$-\frac{\sqrt{3}}{4} \frac{1}{5+9\lambda^2}$
$u_{\lambda}, 8_s$		$-\frac{1}{4} \frac{\sqrt{5}}{2} \frac{1+3\lambda^2}{(5+9\lambda^2)^{3/2}}$	$-\frac{4}{3} \frac{1+9\lambda^2}{(5+9\lambda^2)^{3/2}}$	$-\frac{6}{\sqrt{5}} \frac{\lambda(1-3\lambda^2)}{(5+9\lambda^2)^{3/2}}$	$-\frac{4\lambda\sqrt{5}}{(5+9\lambda^2)^{3/2}}$	$\frac{12\lambda^2}{(5+9\lambda^2)^{3/2}}$	$\frac{\lambda(1-12\lambda-9\lambda^2)}{4(5+9\lambda^2)^{3/2}}$	$\frac{\lambda(1+12\lambda-9\lambda^2)}{4(5+9\lambda^2)^{3/2}}$	$\frac{1}{4} \frac{\sqrt{3}}{2} \frac{13-9\lambda^2}{(5+9\lambda^2)^{3/2}}$
$u_{\lambda}, 8_s$		$-\frac{\lambda}{\sqrt{2}} \frac{1}{(5+9\lambda^2)^{3/2}}$	$-\frac{12}{\sqrt{5}} \frac{\lambda(1-3\lambda^2)}{(5+9\lambda^2)^{3/2}}$	$-\frac{4}{3} \frac{5-9\lambda^2}{(5+9\lambda^2)^{3/2}}$	$-\frac{4}{3} \frac{5-9\lambda^2}{(5+9\lambda^2)^{3/2}}$	$\frac{4}{\sqrt{5}} \frac{\lambda(5-9\lambda^2)}{(5+9\lambda^2)^{3/2}}$	$-\frac{1}{\sqrt{5}} \frac{\lambda(5+9\lambda-9\lambda^2)}{(5+9\lambda^2)^{3/2}}$	$\frac{1}{\sqrt{5}} \frac{\lambda(5-9\lambda-9\lambda^2)}{(5+9\lambda^2)^{3/2}}$	$-\frac{27}{5} \frac{\sqrt{3}}{2} \frac{\lambda}{(5+9\lambda^2)^{3/2}}$
$u_{\lambda}, 8_a$		$-\frac{\lambda}{4\sqrt{2}} \frac{5-9\lambda^2}{(5+9\lambda^2)^{3/2}}$	$-\frac{4\lambda\sqrt{5}}{(5+9\lambda^2)^{3/2}}$	$-\frac{2}{3} \frac{5-9\lambda^2}{(5+9\lambda^2)^{3/2}}$	0	0	$-\frac{\sqrt{5}}{12} \frac{1+12\lambda-9\lambda^2}{(5+9\lambda^2)^{3/2}}$	$-\frac{\sqrt{5}}{12} \frac{1-12\lambda-9\lambda^2}{(5+9\lambda^2)^{3/2}}$	$-\frac{3}{4} \frac{\sqrt{3}}{2} \frac{\lambda(1-5\lambda^2)}{(5+9\lambda^2)^{3/2}}$
$u_{\lambda}, 8_a$		$-\frac{\lambda^2 \sqrt{10}}{2(5+9\lambda^2)^{3/2}}$	$\frac{24\lambda^2}{(5+9\lambda^2)^{3/2}}$	$\frac{4}{\sqrt{5}} \frac{\lambda(5-9\lambda^2)}{(5+9\lambda^2)^{3/2}}$	0	0	$-\frac{1}{3} \frac{5-9\lambda-9\lambda^2}{(5+9\lambda^2)^{3/2}}$	$\frac{1}{3} \frac{5+9\lambda-9\lambda^2}{(5+9\lambda^2)^{3/2}}$	$17 \frac{\sqrt{3}}{10} \frac{\lambda^2}{(5+9\lambda^2)^{3/2}}$
10, 8		$-\frac{1}{2} \frac{\sqrt{5}}{2} \frac{\lambda}{5+9\lambda^2}$	$\frac{\lambda(1-12\lambda-9\lambda^2)}{2(5+9\lambda^2)^{3/2}}$	$-\frac{1}{\sqrt{5}} \frac{\lambda(5+9\lambda-9\lambda^2)}{(5+9\lambda^2)^{3/2}}$	$-\frac{\sqrt{5}}{6} \frac{1+12\lambda-9\lambda^2}{(5+9\lambda^2)^{3/2}}$	$-\frac{1}{3} \frac{5-9\lambda-9\lambda^2}{(5+9\lambda^2)^{3/2}}$	$-\frac{1}{2} \frac{1+2\lambda}{5+9\lambda^2}$	$\frac{1}{2} \frac{1}{5+9\lambda^2}$	$\frac{1}{2} \frac{\sqrt{3}}{10} \frac{(1+4\lambda)\lambda}{5+9\lambda^2}$
10, 8		$-\frac{1}{2} \frac{\sqrt{5}}{2} \frac{\lambda}{5+9\lambda^2}$	$\frac{\lambda(1+12\lambda-9\lambda^2)}{2(5+9\lambda^2)^{3/2}}$	$\frac{1}{\sqrt{5}} \frac{\lambda(5-9\lambda-9\lambda^2)}{(5+9\lambda^2)^{3/2}}$	$-\frac{\sqrt{5}}{6} \frac{1-12\lambda-9\lambda^2}{(5+9\lambda^2)^{3/2}}$	$\frac{1}{3} \frac{5+9\lambda-9\lambda^2}{(5+9\lambda^2)^{3/2}}$	$\frac{1}{2} \frac{1}{5+9\lambda^2}$	$\frac{1}{2} \frac{1}{5+9\lambda^2}$	$\frac{1}{2} \frac{\sqrt{3}}{10} \frac{(1-4\lambda)\lambda}{5+9\lambda^2}$
27, 8		$-\frac{\sqrt{3}}{4} \frac{1}{5+9\lambda^2}$	$\frac{1}{2} \frac{\sqrt{3}}{10} \frac{13-9\lambda^2}{(5+9\lambda^2)^{3/2}}$	$-\frac{27}{5} \frac{\sqrt{3}}{2} \frac{\lambda}{(5+9\lambda^2)^{3/2}}$	$-\frac{3}{2} \frac{\sqrt{3}}{2} \frac{\lambda(1-5\lambda^2)}{(5+9\lambda^2)^{3/2}}$	$17 \frac{\sqrt{3}}{10} \frac{\lambda^2}{(5+9\lambda^2)^{3/2}}$	$\frac{1}{2} \frac{\sqrt{3}}{10} \frac{\lambda(1+4\lambda)}{5+9\lambda^2}$	$\frac{1}{2} \frac{\sqrt{3}}{10} \frac{\lambda(1-4\lambda)}{5+9\lambda^2}$	$-\frac{43}{50} \frac{1}{5+9\lambda^2}$

THE MATRIX FOR $N = 8$: INFLUENCE OF $\delta f_{\gamma'}$ ON δf_{γ}

TABLE V

$\beta_Y, N_T \backslash \beta_Y'$	8_s	8_a	10	27
1, 8	0	$\frac{2}{3} \sqrt{\frac{5}{2}} \frac{g}{f} \frac{1}{\sqrt{5+9\lambda^2}}$	$-\frac{\sqrt{5}}{3} \frac{g}{f} \frac{1-3\lambda}{\sqrt{5+9\lambda^2}}$	0
$8_\lambda, 8_s$	0	$-\frac{1}{3} \frac{g}{f} \frac{1-12\lambda-9\lambda^2}{5+9\lambda^2}$	$\frac{\sqrt{2}}{3} \frac{g}{f} \frac{5+6\lambda+9\lambda^2}{5+9\lambda^2}$	0
$8_{\bar{\lambda}}, 8_s$	0	$\frac{4}{3\sqrt{5}} \frac{g}{f} \frac{5+9\lambda-9\lambda^2}{5+9\lambda^2}$	$\frac{4}{3\sqrt{10}} \frac{g}{f} \frac{5-9\lambda^2}{5+9\lambda^2}$	0
$8_\lambda, 8_a$	$\frac{1}{3} \frac{g}{f} \frac{1+12\lambda-9\lambda^2}{5+9\lambda^2}$	0	0	$-\sqrt{6} \frac{g}{f} \frac{(1+\lambda)^2}{5+9\lambda^2}$
$8_{\bar{\lambda}}, 8_a$	$\frac{4}{3\sqrt{5}} \frac{g}{f} \frac{5-9\lambda-9\lambda^2}{5+9\lambda^2}$	0	0	$-2 \sqrt{\frac{2}{15}} \frac{g}{f} \frac{(1+\lambda)(5-9\lambda)}{5+9\lambda^2}$
10, 8	$\frac{2}{\sqrt{5}} \frac{g}{f} \frac{1+\lambda}{\sqrt{5+9\lambda^2}}$	$\frac{2}{3} \frac{g}{f} \frac{1}{\sqrt{5+9\lambda^2}}$	$-\sqrt{2} \frac{g}{f} \frac{1+\lambda}{\sqrt{5+9\lambda^2}}$	$\frac{4}{\sqrt{30}} \frac{g}{f} \frac{1+\lambda}{\sqrt{5+9\lambda^2}}$
$\bar{10}, 8$	$-\frac{2}{\sqrt{5}} \frac{g}{f} \frac{1+\lambda}{\sqrt{5+9\lambda^2}}$	$\frac{2}{3} \frac{g}{f} \frac{1}{\sqrt{5+9\lambda^2}}$	$-\sqrt{2} \frac{g}{f} \frac{1+\lambda}{\sqrt{5+9\lambda^2}}$	$-\frac{4}{\sqrt{30}} \frac{g}{f} \frac{1+\lambda}{\sqrt{5+9\lambda^2}}$
27, 8	0	$-\frac{2}{\sqrt{30}} \frac{g}{f} \frac{1+4\lambda}{\sqrt{5+9\lambda^2}}$	$\frac{1}{\sqrt{15}} \frac{g}{f} \frac{5+\lambda}{\sqrt{5+9\lambda^2}}$	0

THE MATRIX FOR $N = 8$: INFLUENCE OF $\delta_{\beta_Y'}^a$ ON $\delta_{\beta_Y}^a$

TABLE VI

β_Y	β_Y'	1	8_λ	8_λ	8_λ	8_λ	10	10	27
$N_{T'}$	$N_{T'}$	8	8_s	8_s	8_a	8_a	8	8	8
8_s		0	0	0	$\frac{1}{3} \frac{f}{g} \frac{1+12\lambda-9\lambda^2}{5+9\lambda^2}$	$\frac{2}{3\sqrt{5}} \frac{f}{g} \frac{5-9\lambda-9\lambda^2}{5+9\lambda^2}$	$-\frac{1}{\sqrt{5}} \frac{f}{g} \frac{1+\lambda}{\sqrt{5+9\lambda^2}}$	$-\frac{1}{\sqrt{5}} \frac{f}{g} \frac{1+\lambda}{\sqrt{5+9\lambda^2}}$	0
8_a		$\frac{1}{3} \sqrt{\frac{5}{2}} \frac{f}{g} \frac{1}{\sqrt{5+9\lambda^2}}$	$-\frac{1}{3} \frac{f}{g} \frac{1-12\lambda-9\lambda^2}{5+9\lambda^2}$	$\frac{2}{3\sqrt{5}} \frac{f}{g} \frac{5+9\lambda-9\lambda^2}{5+9\lambda^2}$	0	0	$\frac{1}{3} \frac{f}{g} \frac{1}{\sqrt{5+9\lambda^2}}$	$\frac{1}{3} \frac{f}{g} \frac{1}{\sqrt{5+9\lambda^2}}$	$-\frac{1}{\sqrt{30}} \frac{f}{g} \frac{1+4\lambda}{\sqrt{5+9\lambda^2}}$
10		$-\frac{\sqrt{5}}{12} \frac{f}{g} \frac{1-3\lambda}{\sqrt{5+9\lambda^2}}$	$\frac{1}{3\sqrt{2}} \frac{f}{g} \frac{5+6\lambda+9\lambda^2}{5+9\lambda^2}$	$\frac{1}{3\sqrt{10}} \frac{f}{g} \frac{5-9\lambda^2}{5+9\lambda^2}$	0	0	$-\frac{1}{2\sqrt{2}} \frac{f}{g} \frac{1+\lambda}{\sqrt{5+9\lambda^2}}$	$-\frac{1}{2\sqrt{2}} \frac{f}{g} \frac{1+\lambda}{\sqrt{5+9\lambda^2}}$	$\frac{1}{4\sqrt{15}} \frac{f}{g} \frac{5+\lambda}{\sqrt{5+9\lambda^2}}$
27		0	0	0	$-\sqrt{6} \frac{f}{g} \frac{(1+\lambda)^2}{5+9\lambda^2}$	$-\frac{f}{g} \sqrt{\frac{2}{15}} \frac{(1+\lambda)(5-9\lambda)}{5+9\lambda^2}$	$-\frac{2}{\sqrt{30}} \frac{f}{g} \frac{1+\lambda}{\sqrt{5+9\lambda^2}}$	$-\frac{2}{\sqrt{30}} \frac{f}{g} \frac{1+\lambda}{\sqrt{5+9\lambda^2}}$	0

THE MATRIX FOR $N = 8$: INFLUENCE OF $\delta f_{\beta_Y}'$ ON $\delta g_{\beta_Y}''$

T A B L E VII

$\beta_Y \backslash \beta_{Y'}$	8_s	8_a	10	27
8_s	$\frac{1}{10}$	$-\frac{1}{6\sqrt{5}}$	$\frac{1}{2\sqrt{10}}$	$\frac{1}{5\sqrt{6}}$
8_a	$-\frac{1}{6\sqrt{5}}$	0	$-\frac{1}{6\sqrt{2}}$	$-\frac{1}{3\sqrt{30}}$
10	$\frac{1}{4\sqrt{10}}$	$-\frac{1}{12\sqrt{2}}$	0	$-\frac{1}{6\sqrt{15}}$
27	$\frac{1}{5\sqrt{6}}$	$-\frac{1}{3\sqrt{30}}$	$-\frac{1}{3\sqrt{15}}$	$\frac{1}{15}$

THE MATRIX FOR $N = 8$: INFLUENCE OF $\delta g_{\beta_{Y'}}^8$ ON $\delta g_{\beta_Y}^8$

T A B L E V I I I

β_{γ}, N_T	$\beta_{\gamma'}, N_{T'}$	8_{λ}	8_{λ}	10	$\overline{10}$	27	27	27
		27	27	27	27	27 _s	27 _a	27
$8_{\lambda}, 27$	$-\frac{1}{6} \frac{13-18\lambda^2+45\lambda^4}{(5+9\lambda^2)^2}$	$-\frac{1}{6} \frac{13-18\lambda^2+45\lambda^4}{(5+9\lambda^2)^2}$	$\frac{\lambda}{\sqrt{5}} \frac{9-17\lambda^2}{(5+9\lambda^2)^2}$	$-\frac{\lambda(1-\lambda)^2}{(5+9\lambda^2)^{3/2}}$	$\frac{\lambda(1+\lambda)^2}{(5+9\lambda^2)^{3/2}}$	$\frac{\sqrt{70}}{15} \frac{1-3\lambda^2}{(5+9\lambda^2)^{3/2}}$	$2\sqrt{\frac{2}{3}} \frac{\lambda}{(5+9\lambda^2)^{3/2}}$	
$8_{\lambda}, 27$	$\frac{2\lambda}{\sqrt{5}} \frac{9-17\lambda^2}{(5+9\lambda^2)^2}$	$-\frac{332}{15} \frac{\lambda^2}{(5+9\lambda^2)^2}$	$\frac{2}{3\sqrt{5}} \frac{\lambda(1-\lambda)(5+9\lambda)}{(5+9\lambda^2)^{3/2}}$	$\frac{2}{3\sqrt{5}} \frac{\lambda(1-\lambda)(5+9\lambda)}{(5+9\lambda^2)^{3/2}}$	$\frac{2}{3\sqrt{5}} \frac{\lambda(1+\lambda)(5-9\lambda)}{(5+9\lambda^2)^{3/2}}$	$-16 \frac{\sqrt{14}}{15} \frac{\lambda}{(5+9\lambda^2)^{3/2}}$	$\frac{2}{3} \sqrt{\frac{2}{15}} \frac{5-9\lambda^2}{(5+9\lambda^2)^{3/2}}$	
10, 27	$-\frac{2\lambda(1-\lambda)^2}{(5+9\lambda^2)^{3/2}}$	$\frac{2}{3\sqrt{5}} \frac{\lambda(1-\lambda)(5+9\lambda)}{(5+9\lambda^2)^{3/2}}$	$-\frac{1}{12} \frac{1+2\lambda+5\lambda^2}{5+9\lambda^2}$	$-\frac{1}{12} \frac{1-5\lambda^2}{5+9\lambda^2}$	$-\frac{1}{12} \frac{1-5\lambda^2}{5+9\lambda^2}$	$\frac{1}{6} \sqrt{\frac{7}{10}} \frac{1+9\lambda}{5+9\lambda^2}$	$\frac{1}{6\sqrt{6}} \frac{11-9\lambda}{5+9\lambda^2}$	
$\overline{10}, 27$	$\frac{2\lambda(1+\lambda)^2}{(5+9\lambda^2)^{3/2}}$	$\frac{2}{3\sqrt{5}} \frac{\lambda(1+\lambda)(5-9\lambda)}{(5+9\lambda^2)^{3/2}}$	$-\frac{1}{12} \frac{1-5\lambda^2}{5+9\lambda^2}$	$-\frac{1}{12} \frac{1-2\lambda+5\lambda^2}{5+9\lambda^2}$	$-\frac{1}{12} \frac{1-2\lambda+5\lambda^2}{5+9\lambda^2}$	$-\frac{1}{6} \sqrt{\frac{7}{10}} \frac{1-9\lambda}{5+9\lambda^2}$	$-\frac{1}{6\sqrt{6}} \frac{11+9\lambda}{5+9\lambda^2}$	
27, 27 _s	$\frac{2\sqrt{70}}{15} \frac{1-3\lambda^2}{(5+9\lambda^2)^{3/2}}$	$-\frac{16\sqrt{14}}{15} \frac{\lambda}{(5+9\lambda^2)^{3/2}}$	$\frac{1}{6} \sqrt{\frac{7}{10}} \frac{1+9\lambda}{5+9\lambda^2}$	$-\frac{1}{6} \sqrt{\frac{7}{10}} \frac{1-9\lambda}{5+9\lambda^2}$	$-\frac{1}{6} \sqrt{\frac{7}{10}} \frac{1-9\lambda}{5+9\lambda^2}$	$\frac{1}{30} \frac{1}{5+9\lambda^2}$	$\frac{\sqrt{105}}{10} \frac{\lambda}{5+9\lambda^2}$	
27, 27 _a	$4\sqrt{\frac{2}{3}} \frac{\lambda}{(5+9\lambda^2)^{3/2}}$	$\frac{2}{3} \sqrt{\frac{2}{15}} \frac{5-9\lambda^2}{(5+9\lambda^2)^{3/2}}$	$\frac{1}{6\sqrt{6}} \frac{11-9\lambda}{5+9\lambda^2}$	$-\frac{1}{6\sqrt{6}} \frac{11+9\lambda}{5+9\lambda^2}$	$-\frac{1}{6\sqrt{6}} \frac{11+9\lambda}{5+9\lambda^2}$	$\frac{\sqrt{105}}{10} \frac{\lambda}{5+9\lambda^2}$	$-\frac{5}{2} \frac{\lambda^2}{5+9\lambda^2}$	

THE MATRIX FOR N = 27: INFLUENCE OF $\delta f_{\beta_{\gamma'}}$ ON $\delta f_{\beta_{\gamma}}$

TABLE IX

$\beta_{Y'}^{N_T}$	8_s	8_a	10	$\overline{10}$	27
$8_\lambda, 27$	0	$\frac{4}{3} \frac{g}{f} \frac{(1-\lambda)^2}{5+9\lambda^2}$	$\frac{4\sqrt{7}}{3} \frac{g}{f} \frac{\lambda(1-\lambda)}{5+9\lambda^2}$	$\frac{2\sqrt{10}}{3} \frac{g}{f} \frac{1-\lambda^2}{5+9\lambda^2}$	0
$8_{\overline{\lambda}}, 27$	0	$-\frac{8}{9\sqrt{5}} \frac{g}{f} \frac{(1-\lambda)(5+9\lambda)}{5+9\lambda^2}$	$\frac{4}{9} \sqrt{\frac{7}{5}} \frac{g}{f} \frac{5-10\lambda-9\lambda^2}{5+9\lambda^2}$	$-\frac{56\sqrt{2}}{9} \frac{g}{f} \frac{\lambda}{5+9\lambda^2}$	0
10, 27	$\frac{1}{3\sqrt{5}} \frac{g}{f} \frac{1+\lambda}{\sqrt{5+9\lambda^2}}$	$\frac{1}{9} \frac{g}{f} \frac{1+5\lambda}{\sqrt{5+9\lambda^2}}$	$\frac{4\sqrt{7}}{9} \frac{g}{f} \frac{1+\lambda}{\sqrt{5+9\lambda^2}}$	$\frac{2\sqrt{10}}{9} \frac{g}{f} \frac{\lambda}{\sqrt{5+9\lambda^2}}$	$2\sqrt{\frac{7}{15}} \frac{g}{f} \frac{1+\lambda}{\sqrt{5+9\lambda^2}}$
$\overline{10}, 27$	$\frac{1}{3\sqrt{5}} \frac{g}{f} \frac{1+\lambda}{\sqrt{5+9\lambda^2}}$	$-\frac{1}{9} \frac{g}{f} \frac{1+5\lambda}{\sqrt{5+9\lambda^2}}$	$-\frac{4\sqrt{7}}{9} \frac{g}{f} \frac{1+\lambda}{\sqrt{5+9\lambda^2}}$	$-\frac{2\sqrt{10}}{9} \frac{g}{f} \frac{\lambda}{\sqrt{5+9\lambda^2}}$	$2\sqrt{\frac{7}{15}} \frac{g}{f} \frac{1+\lambda}{\sqrt{5+9\lambda^2}}$
27, 27_s	0	$-\frac{\sqrt{70}}{45} \frac{g}{f} \frac{1+9\lambda}{\sqrt{5+9\lambda^2}}$	$-\frac{4\sqrt{10}}{45} \frac{g}{f} \frac{5+9\lambda}{\sqrt{5+9\lambda^2}}$	$\frac{4\sqrt{7}}{9} \frac{g}{f} \frac{1}{\sqrt{5+9\lambda^2}}$	0
27, 27_a	$-\frac{2}{3\sqrt{30}} \frac{g}{f} \frac{11-9\lambda}{\sqrt{5+9\lambda^2}}$	0	0	0	$-\frac{4}{3} \sqrt{\frac{7}{10}} \frac{g}{f} \frac{1+\lambda}{\sqrt{5+9\lambda^2}}$

THE MATRIX FOR N = 27: INFLUENCE OF $\delta g_{\beta_{Y'}}^{27}$ ON $\delta f_{\beta_Y}^{27\Gamma}$

TABLE X

$\beta_Y \backslash N_{T'}$	8_λ	$8_{-\lambda}$	10	$\overline{10}$	27	27
	27	27	27	27	27_s	27_a
8_s	0	0	$\frac{1}{6\sqrt{5}} \frac{f}{g} \frac{1+\lambda}{\sqrt{5+9\lambda^2}}$	$\frac{1}{6\sqrt{5}} \frac{f}{g} \frac{1+\lambda}{\sqrt{5+9\lambda^2}}$	0	$-\frac{1}{3\sqrt{30}} \frac{f}{g} \frac{11-9\lambda}{\sqrt{5+9\lambda^2}}$
8_a	$\frac{4}{3} \frac{f}{g} \frac{(1-\lambda)^2}{5+9\lambda^2}$	$-\frac{4}{9\sqrt{5}} \frac{f}{g} \frac{(1-\lambda)(5+9\lambda)}{5+9\lambda^2}$	$\frac{1}{18} \frac{f}{g} \frac{1+5\lambda}{\sqrt{5+9\lambda^2}}$	$-\frac{1}{18} \frac{f}{g} \frac{1+5\lambda}{\sqrt{5+9\lambda^2}}$	$-\frac{\sqrt{70}}{90} \frac{f}{g} \frac{1+9\lambda}{\sqrt{5+9\lambda^2}}$	0
10	$\frac{2\sqrt{7}}{3} \frac{f}{g} \frac{\lambda(1-\lambda)}{5+9\lambda^2}$	$\frac{1}{9} \sqrt{\frac{7}{5}} \frac{f}{g} \frac{5-10\lambda-9\lambda^2}{5+9\lambda^2}$	$\frac{\sqrt{7}}{9} \frac{f}{g} \frac{1+\lambda}{\sqrt{5+9\lambda^2}}$	$-\frac{\sqrt{7}}{9} \frac{f}{g} \frac{1+\lambda}{\sqrt{5+9\lambda^2}}$	$-\frac{\sqrt{10}}{45} \frac{f}{g} \frac{5+9\lambda}{\sqrt{5+9\lambda^2}}$	0
$\overline{10}$	$\frac{2\sqrt{10}}{3} \frac{f}{g} \frac{1-\lambda^2}{5+9\lambda^2}$	$-\frac{28\sqrt{2}}{9} \frac{f}{g} \frac{\lambda}{5+9\lambda^2}$	$\frac{\sqrt{10}}{9} \frac{f}{g} \frac{\lambda}{\sqrt{5+9\lambda^2}}$	$-\frac{\sqrt{10}}{9} \frac{f}{g} \frac{\lambda}{\sqrt{5+9\lambda^2}}$	$\frac{2\sqrt{7}}{9} \frac{f}{g} \frac{1}{\sqrt{5+9\lambda^2}}$	0
27	0	0	$\sqrt{\frac{7}{15}} \frac{f}{g} \frac{1+\lambda}{\sqrt{5+9\lambda^2}}$	$\sqrt{\frac{7}{15}} \frac{f}{g} \frac{1+\lambda}{\sqrt{5+9\lambda^2}}$	0	$-\frac{2}{3} \sqrt{\frac{7}{10}} \frac{f}{g} \frac{1+\lambda}{\sqrt{5+9\lambda^2}}$

THE MATRIX FOR N = 27: INFLUENCE OF $\delta f_{\beta_Y}^{27T'}$ ON $\delta g_{\beta_Y}^{27}$

TABLE XI

$\beta_Y \backslash \beta_{Y'}$	8_s	8_a	10	$\overline{10}$	27
8_s	$\frac{1}{60}$	$-\frac{1}{36\sqrt{5}}$	$-\frac{1}{9}\sqrt{\frac{7}{5}}$	0	$\frac{1}{10}\sqrt{\frac{7}{3}}$
8_a	$-\frac{1}{36\sqrt{5}}$	$\frac{5}{108}$	$\frac{\sqrt{7}}{27}$	$\frac{1}{27}\sqrt{\frac{5}{2}}$	$-\frac{1}{6}\sqrt{\frac{7}{15}}$
10	$-\frac{1}{18}\sqrt{\frac{7}{5}}$	$\frac{\sqrt{7}}{54}$	$-\frac{5}{108}$	$-\frac{\sqrt{70}}{108}$	$\frac{1}{6\sqrt{15}}$
$\overline{10}$	0	$\frac{1}{27}\sqrt{\frac{5}{2}}$	$-\frac{\sqrt{70}}{54}$	$\frac{2}{27}$	0
27	$\frac{1}{10}\sqrt{\frac{7}{3}}$	$-\frac{1}{6}\sqrt{\frac{7}{15}}$	$\frac{1}{3\sqrt{15}}$	0	$\frac{1}{90}$

THE MATRIX FOR $N = 27$: INFLUENCE OF $\delta g_{\beta_{Y'}}^{27}$ ON $\delta g_{\beta_Y}^{27}$

T A B L E X I I

$\lambda = \frac{1}{3}$	$\lambda = 0.44$	$\lambda = 0.55$	$\lambda = \frac{2}{3}$
0.92	0.93	0.93	0.93
0.65	0.65	0.64	0.62
0.42	0.42	0.42	0.42
0.092	0.11	0.13	0.15
- 0.047	- 0.058	- 0.054	- 0.048
- 0.21	- 0.24	- 0.29	- 0.32
- 0.45	- 0.43	- 0.41	- 0.39
- 0.78	- 0.78	- 0.77	- 0.77
- 0.80	- 0.81	- 0.82	- 0.82

EIGENVALUES OF THE B MATRIX FOR N = 8

T A B L E XIII

$\lambda = \frac{1}{3}$	$\lambda = 0.44$	$\lambda = 0.55$	$\lambda = \frac{2}{3}$
0.86	0.87	0.86	0.85
0.68	0.66	0.65	0.64
0.48	0.49	0.49	0.48
0.039	0.024	0.027	0.036
8.7×10^{-4}	5.9×10^{-3}	$- 3.5 \times 10^{-3}$	- 0.015
- 0.32	- 0.31	- 0.29	- 0.26
- 0.50 + 0.027 i	- 0.50 + 0.029 i	- 0.50 + 8.6×10^{-3} i	- 0.47
- 0.50 - 0.027 i	- 0.50 - 0.029 i	- 0.50 - 8.6×10^{-3} i	- 0.54
- 0.84	- 0.86	- 0.87	- 0.87

EIGENVALUES OF THE B MATRIX FOR N = 27

T A B L E X I V

	$\lambda = \frac{1}{3}$	$\lambda = 0.44$	$\lambda = 0.55$	$\lambda = \frac{2}{3}$
$\delta \bar{f}_1^s$	- 0.052	- 0.091	- 0.12	- 0.13
$\delta \bar{f}_{s_s}^s$	- 0.71	- 0.71	- 0.70	- 0.67
$\delta \bar{f}_{s_a}^s$	- 0.43	- 0.45	- 0.46	- 0.46
$\delta \bar{f}_{1_0+1_0}^s$	0.30	0.32	0.35	0.38
$\frac{\sqrt{2}}{\delta \bar{f}_{27}^s}$	- 0.25	- 0.19	- 0.13	- 0.084
$\delta g_{s_s}^s$	0.18	0.22	0.24	0.24
$\delta g_{s_a}^s$	- 0.20	- 0.17	- 0.14	- 0.11
$\delta g_{1_0}^s$	0.28	0.28	0.28	0.29
δg_{27}^s	1	1	1	1

THE EIGENVECTOR OF B^s CORRESPONDING TO ITS LARGEST EIGENVALUE (≈ 0.93)