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NUCLEAR ENHANCEMENT OF T VIOLATION EFFECTS

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A B S T R A C T

The enhancement factor in non-resonant statistical reactions is shown to be roughly $(W/\Gamma)^{\frac{1}{2}}$. Present experiments give then the limit $(1-2) \times 10^{-3}$ for T violation effects in nuclei with a 50% confidence limit.

Detailed balance provides an important test of T violation. It has been pointed out long ago that nuclear reactions of strong dynamical complexity are most likely to be sensitive to T violation effects ¹⁾. This condition is particularly well fulfilled by statistical reactions for which the compound nuclear widths Γ are much larger than level spacing D ("fluctuation reactions"). Consequently, precision studies of detailed balance in such reactions are presently being done by several groups.

The present paper will :

- A. discuss the most favourable observational conditions for T violation effects;
- B. derive a semi-quantitative expression for the intrinsic nuclear enhancement of T violation in statistical reactions and
- C. analyze the limit on T violation implied by a recent experiment.

A. MOST FAVOURABLE EXPERIMENTAL CONDITIONS

We remind in this Section of some simple properties of cross-sections and amplitudes which are of great importance for a choice of significant tests.

- 1) The reaction should be chosen so that a single amplitude dominates the reaction. If $\beta = (m_1, m_2; m_3, m_4)$ denotes the helicities of the colliding and outgoing particle spins, the differential cross-section $\sigma(\theta)$ can be expressed ²⁾ as a sum of partial cross-sections $\sigma_{\beta}(\theta)$

$$\sigma(\theta) = \sum_{\beta} \sigma_{\beta}(\theta) \quad (1)$$

These partial cross-sections $\sigma_{\beta}(\theta)$ are in general not independent. In particular, parity implies that $\sigma_{\beta} = \sigma_{-\beta}$. In terms of the T conserving amplitude $f_{\beta}(\theta)$ and the T violating one $f'_{\beta}(\theta)$

$$\sigma(\theta) = \sum_{\beta} |f_{\beta}(\theta) + f'_{\beta}(\theta)|^2 \quad (2)$$

For dynamically complex reactions, it is common that the various channels contribute similar amounts to $\sigma(\theta)$ and that the phases of the amplitudes are random or nearly random. There is therefore usually a relative reduction of effects if more than one amplitude contributes to the reaction.

Two-body reactions between spinless particles are described by a single amplitude, so that T violation effects are maximal in this case. Even for unpolarized particles, the condition that the orbital angular momentum is perpendicular to the direction of motion restricts the amplitudes to a single one for 0° or 180° scattering when two of the reacting particles have zero spin and one other has spin $\frac{1}{2}$ or zero.

- 2) Assume a one-channel reaction with a small T violating amplitude. Provided the T violating amplitude is independent (or approximately independent) of the T conserving amplitude, it is clearly advantageous to search for T violating effects where $f(\theta)$ and $f'(\theta)$ are of comparable magnitude, since the relative effects are largest there. Since T violation effects are expected to be small, this means in practice a search for effects near minima in $\sigma(\theta)$. A decoupling of $f(\theta)$ and $f'(\theta)$ is expected in non-resonant statistical reactions (though not for statistical reactions with isolated sharp resonances), which further have deep minima as a function of energy. If we denote the typical average T violation in the amplitude by ξ with

$$\xi^2 = \overline{|f'|^2} / \overline{|f|^2} \quad , \quad (3)$$

a minimum which is ν times smaller than the average cross-section enhances the apparent violation from ξ to ξ' :

$$\xi'^2 = \overline{|f|^2} / |f_{\min}|^2 \approx \nu \xi^2. \quad (4)$$

- 3) A corollary of point 2) above is that maxima of the cross-sections, especially large ones, can be used to normalize experimental cross-sections³⁾ and energies for reversed reactions, which is of great interest since calibration is notoriously difficult to high precision. The maxima depend only weakly on T violation, so that the violation effects can efficiently be searched for in the intermediate minima.

B. INTRINSIC ENHANCEMENT OF T VIOLATION

We will now give a rough estimate of T violation effects assuming a complicated, statistical nature of the nuclear motion. In a statistical reaction of fluctuation type, the effect is then enhanced roughly by $(W/\Gamma)^{\frac{1}{2}}$ where Γ is the typical decay width of the compound system and W is the energy range over which matrix elements get thoroughly mixed due to the complicated statistical interaction. This latter quantity will later be identified with the energy region over which a single particle strength is distributed, i.e., ordinarily an energy interval of the order of a few MeV.

The estimate proceeds as follows: consider first $N \gg 1$ degenerate states i which are coupled by a T conserving (hence real and symmetric) Hamiltonian H_0 of statistical nature. By the latter statement, we mean that the matrix elements $\langle i | H_0 | j \rangle$ are assumed to be

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normally distributed with $\overline{\langle i|H_0|i\rangle^2} = 2 \overline{\langle i|H_0|j\rangle^2} = \text{const.}$ (We will only exploit this property very weakly.) Due to the interaction, the degeneracy is lifted and the levels spread out over an energy interval $W \approx ND$ where D is the typical level spacing. The average value $\overline{\langle i|H_0|j\rangle^2}$ is readily obtained, since :

$$T_E \{H_0^2\} = \sum_{n=1}^{\infty} E_n^2 \approx D^2 \sum_1^{\infty} n^2 \approx \frac{N^3}{3} D^2 \approx N^2 \overline{\langle i|H_0|j\rangle^2} \quad (5)$$

or

$$\overline{\langle i|H_0|j\rangle^2} \approx \frac{N}{3} D^2 \approx \frac{WD}{3} \quad (5')$$

We now introduce a T violating perturbation H' which we take to be imaginary $H'^* = H'^T = -H'$. Its matrix elements are assumed to be as statistical as those of H_0 and uncorrelated with H_0 . The typical strength of the H' matrix elements is taken to be x times that of H_0 which can be formally expressed by

$$T_E \{H'^2\} = x^2 T_E \{H_0^2\} \quad (6)$$

or, equivalently

$$\overline{|\langle i|H'|j\rangle|^2} \approx x^2 \overline{\langle i|H_0|j\rangle^2} \approx x^2 \frac{WD}{3} \quad (7)$$

Consider now a statistical nuclear reaction $a \rightarrow b$ characterized by $D \ll \Gamma \ll W$, so that many resonances contribute coherently to the amplitude. If T conservation holds $S_{ab} = S_{ba}$, where S is the scattering matrix. This quantity is essentially proportional to

$$S \propto \gamma \frac{1}{E - H_0 + i\frac{\Gamma}{2}} \gamma \quad (8)$$

where γ is the reduced width matrix. The denominator in (8) implies that a certain subset of all intermediate states i contributes coherently. The typical magnitude of S_{ab} is obtained by the usual statistical assumption of random sign for γ_{ai} and γ_{ib} :

$$|S_{ab}|^2 \propto \overline{\gamma_{ai}^2} \overline{\gamma_{ib}^2} \sum_i \frac{1}{(E-E_i)^2 + (\Gamma/2)^2} \approx \overline{\gamma_{ai}^2} \overline{\gamma_{ib}^2} \frac{2\pi}{\Gamma D} \quad (9)$$

The weak T violating interaction is now turned on; its typical matrix elements are still given by (7). We now enquire into the change of S_{ab} due to H' . For a rough estimate, we will neglect the effects in γ_{ai} and γ_{ib} as well as in the total width Γ . We will, however, include the effects of H' on the intermediate states within this crude approximation:

$$S'_{ab} \propto \gamma_a \frac{1}{E-H_0-H'+i\Gamma/2} \gamma_b, \quad (10)$$

so that the change in S_{ab} becomes to first order in H'

$$\delta S_{ab} \propto \gamma_a \frac{1}{(E-H_0+i\Gamma/2)} H' \frac{1}{(E-H_0+i\Gamma/2)} \gamma_b \quad (11)$$

Since the transpose of H' is $-H'$, we immediately see that this implies $\delta S_{ab} = -\delta S_{ba}$. Within the statistical assumption the typical value of δS_{ab} is straightforwardly calculated:

$$\begin{aligned} |\delta S_{ab}|^2 &\propto \overline{\gamma_{ai}^2} \overline{\gamma_{jb}^2} \sum_{ij} \left\{ \frac{|K(iH'j)|^2}{[(E-E_i)^2 + (\Gamma/2)^2] [(E-E_j)^2 + (\Gamma/2)^2]} \right\} \\ &\approx |S_{ab}|^2 \frac{2\pi}{\Gamma D} \overline{|K(iH'j)|^2} \approx X^2 \frac{2\pi}{3} \frac{W}{E} |S_{ab}|^2. \end{aligned} \quad (12)$$

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We thus conclude that there is a typical enhancement factor of x by $(aW/\Gamma)^{\frac{1}{2}}$ with a model dependent factor a of order unity. The factor $(W/\Gamma)^{\frac{1}{2}}$ is in heavier element commonly between one and two magnitudes and thus quite sizable.

To summarize Sections A and B : the observed T violation effect in the amplitude ξ ' can be boosted using minima in cross-sections and statistical enhancement with respect to the original violation x . The relation is

$$\xi'^2 \approx 2 a W/\Gamma x^2. \quad (13)$$

C. APPLICATION

The detailed balance of the $^{24}\text{Mg}(\alpha, p)^{27}\text{Al}$ reaction and its inverse has recently been measured as a function of energy for $E_\alpha \approx 13.5$ MeV and $\theta = 170^\circ$ in a remarkable experiment by v.Witsch et al. The cross-section is an extremely strong function of energy, and shows exceedingly good agreement for the two reactions. In particular, there is a very deep minimum in the cross-section with a cross-section determined to $\pm 6\%$ which is over a magnitude below the average cross-section. Hence $\xi' < 3 \times 10^{-2}$. From the discussion in Section A.1 we conclude that only one amplitude contributes at these large angles, though it would have been preferable to have the experiment performed even closer to 180° ; the effect is thus maximized. Further, the minimum enhances the limit by a factor ≈ 3 since $\nu \approx 11$. Hence $\xi < 1 \times 10^{-2}$. Since the typical width $\Gamma \approx 50$ keV ³⁾ and the typical mixing energy W is of the order of a few MeV, we have $W/\Gamma \approx 40$; hence the intrinsic enhancement is another factor ≈ 6 . We thus finally conclude that this experiment gives

$$\chi \lesssim (1-2) \times 10^{-3}$$

with a confidence limit of about 50%. The estimates are somewhat on the conservative side.

As known to us, this is at present by far the best limit on T violation in nuclear interactions. It would be of great interest to exploit the same method under conditions which give even higher precision by maximally exploiting enhancement as well as by investigating several minima. Tests of a similar kind in electromagnetic processes (photo-nuclear reactions) would be of extreme interest in view of current theoretical speculations.

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R E F E R E N C E S

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