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CURRENT ALGEBRA AND MASS FORMULAE

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A B S T R A C T

The Fubini-Furlan dispersion relation for current commutator is obtained in an unambiguous way and on its basis the general form of the mass formula is derived. It is then applied to get various relations among the electromagnetic mass differences. Two new formulae for the baryon decuplet are obtained and comparison with experiment is made.

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1. INTRODUCTION

Equal time commutation relations between different current-like quantities proved to be very useful in exploiting symmetry properties and deriving various sum rules. The relativistically invariant formulation of this method, using local commutativity, presents these sum rules in the form of dispersion relations ^{1),2)}. However, in the course of deriving these relations there were some uncertainties connected with the asymptotic behaviour of "charges" at $t \rightarrow \pm\infty$. We want to clarify this point here using the procedure proposed in Ref. ³⁾. Then we apply this method to obtain different mass formulae in broken $SU(3)$ symmetry. In doing so, main attention is paid to those including electromagnetic mass splittings. We are able to get the well-known Coleman-Glashow formula for the baryon octet and two new formulae for the baryon decuplet, taking into account the particular form of the interference between electromagnetic and medium-strong interactions breaking $SU(3)$ symmetry. We argue that these last ones are more accurate than those obtained previously ^{4),5)}.

In Section 2 the unambiguous relativistically invariant formulation of the commutation relations between "charges" and current-like densities is given.

In Section 3 the general sum rule for the mass splittings is obtained.

Some particular mass formulae, including electromagnetic ones, are considered in Section 4.

Section 5 is devoted to the discussion of the results.

2. RELATIVISTICALLY INVARIANT FORMULATION OF EQUAL TIME COMMUTATION RELATIONS

Suppose that we have a commutation relation of the type :

$$[Q^A, b(\vec{x}, 0)] = c(\vec{x}, 0), \quad (2.1)$$

2.

where the "charge"

$$Q^A = \int A_0(\vec{x}, 0) d^3x \quad (2.2)$$

and $A_0(x)$ is the time component of some current operator $A_\mu(x)$. The matrix element of (2.1), using the translation invariance and the complete set of states, takes the form:

$$\begin{aligned} \sum_n \langle f | Q^A | n \rangle \langle n | \beta(0) | i \rangle - \sum_{n'} \langle f | \beta(0) | n' \rangle \langle n' | Q^A | i \rangle \\ = \langle f | c(0) | i \rangle. \end{aligned} \quad (2.3)$$

Let us introduce the auxiliary quantity ³⁾

$$Q^A(k) = \int e^{-i\vec{k} \cdot \vec{x}} A_0(\vec{x}, 0) d^3x \quad (2.4)$$

so that

$$\langle m | Q^A | n \rangle = \lim_{\vec{k} \rightarrow 0} \langle m | Q^A(k) | n \rangle \quad (2.5)$$

Now the expression (2.4) for $Q^A(k)$ can be put in the form

$$\begin{aligned} Q^A(k) &= \int \theta(-x_0) \partial^\mu \{ A_\mu(x) e^{ikx} \} d^4x \\ &= \int \theta(-x_0) D^A(x) e^{ikx} d^4x + ik^\mu \int \theta(-x_0) A_\mu(x) e^{ikx} d^4x \end{aligned} \quad (2.6)$$

where

$$D^A(x) = \partial^\mu A_\mu(x), \quad kx = k^0 x^0 - \vec{k} \cdot \vec{x}, \quad \text{Im } k_0 < 0$$

Utilizing the translation invariance in the time variable, we can perform the matrix element of the second term in (2.6) as follows

$$i k^M \int \theta(-x_0) \langle m | A_M(x) | n \rangle e^{i k x} d^4 x = \\ = \frac{k^M \int \langle m | A_M(\vec{x}, 0) | n \rangle e^{-i \vec{k} \cdot \vec{x}} d^3 x}{E_m - E_n + k^0}$$

which, in the limit $\vec{k} \rightarrow 0$, reduces to

$$\frac{k^0 \int \langle m | A_0(\vec{x}, 0) | n \rangle d^3 x}{E_m - E_n + k^0} = \frac{k^0 \langle m | Q^A | n \rangle}{E_m - E_n + k^0} \quad (2.7)$$

It follows from Eq. (2.7) that, in the limit $k^0 \rightarrow 0$, this expression vanishes if $E_n \neq E_m$. In the opposite case, when $E_n = E_m$, we get simply $\langle m | Q^A | n \rangle$. From this one can conclude that if we are dealing only with non-diagonal (in energy) matrix elements of "charge" $Q^A(k)$ then in the limit $k^M \rightarrow 0$ it is sufficient to consider only the first term in (2.6). Thus, we get from (2.5) and (2.6) the representation

$$\langle m | Q^A | n \rangle = \lim_{k^M \rightarrow 0} \int \theta(-x_0) \langle m | D^A(x) | n \rangle e^{i k x} d^4 x, \quad (2.8)$$

$$\text{Im } k^0 < 0, \quad E_m \neq E_n$$

Now, if in Eq. (2.3) only non-diagonal (in energy) states participate, we can use representation (2.8), obtaining the result

$$\lim_{k^M \rightarrow 0} \int \theta(-x_0) \langle f | [D^A(x), \mathcal{B}(0)] | i \rangle e^{i k x} d^4 x = \quad (2.9) \\ = \langle f | \mathcal{C}(0) | i \rangle.$$

4.

Thus we have got a relativistically invariant function (as a result of presumed local commutativity)

$$F_{if}(s, t, u, k^2) = \int \theta(-x_0) \langle f | [D^A(x), \beta(0)] | i \rangle e^{i k x} d^4 x \quad (2.10)$$

where

$$s = (k + p_f)^2, \quad t = (p_f - p_i)^2, \quad u = (p_i + k)^2, \\ p_f^2 = m_f^2, \quad p_i^2 = m_i^2$$

Now we can make use of the arbitrariness of the vector k and put

$$k^2 = 0, \quad s - m_f^2 = u - m_i^2 = \omega$$

(the last relation implies the condition $kp_i = kp_f$).

The limit $k^\mu \rightarrow 0$ corresponds to $\omega \rightarrow 0$. Then, Eq. (2.9) can be rewritten in the form

$$\lim_{\omega \rightarrow 0} F_{if}(\omega, t) = \langle f | \beta(0) | i \rangle. \quad (2.11)$$

Proceeding now along the lines of Ref. ¹⁾, we write down the dispersion relation in ω at fixed t :

$$F_{if}(\omega, t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\Phi_{if}(\omega', t)}{\omega' - \omega} d\omega' \quad (2.12)$$

where the imaginary part

$$\Phi_{if}(\omega, t) = \frac{1}{2} (2\pi)^4 \left\{ \sum_n \delta^{(4)}(p_f + k - p_n) \langle f | D^A(0) | n \rangle \langle n | \beta(0) | i \rangle - \sum_{n'} \delta^{(4)}(p_i - k - p_{n'}) \langle f | \beta(0) | n' \rangle \langle n' | D^A(0) | i \rangle \right\}. \quad (2.13)$$

3. GENERAL FORM OF MASS RELATIONS

As it was shown in Refs. 1), 6), the covariant way of obtaining mass formulae is to postulate the commutation relation of the type

$$[Q^A, D^B(\vec{x}, 0)] = 0 \quad (3.1)$$

where

$$D^B = \partial^\mu B_\mu, \quad Q^A = \int A_0(\vec{x}, 0) d^3x$$

and A_μ, B_μ are vector currents of I, U or V spin. It means that in the general commutation relation (2.1) we are to specify

$$b(x) = D^B(x), \quad c(x) = 0 \quad (3.2)$$

Thus, Eq. (2.11) in this case reads

$$\lim_{\omega \rightarrow 0} F_{if}^{AB}(\omega, t) = 0 \quad (3.3)$$

where F_{if}^{AB} is defined by (2.12), (2.13) and (3.2).

Now we separate the one-particle intermediate state contribution in (2.13) and make use of the following parametrization of the matrix element of $D^A(0)$ between one-particle states [see Ref. 1] :

$$\langle p_1 | D^A(0) | p_2 \rangle = \begin{cases} i(m_1^2 - m_2^2) C_{12}^A G_{12}^A(t) & \text{for bosons} \\ i(m_1 - m_2) C_{12}^A G_{12}^A(t) \bar{u}_1 u_2 & \text{for fermions} \end{cases} \quad (3.4)$$

where $t = (p_1 - p_2)^2$, $p_1^2 = m_1^2$, $p_2^2 = m_2^2$, C_{12}^A is the appropriate Clebsch-Gordan coefficient, $G_{12}^A(t)$ is the corresponding form factor. Substituting (3.4) into (2.13) we get from (2.12) :

6.

$$\begin{aligned}
 F_{if}^{AB}(\omega, t) = & \sum_n \frac{(m_f^2 - m_n^2)(m_i^\alpha - m_n^\alpha)}{m_n^2 - m_f^2 - \omega} C_{fn}^A C_{ni}^B G_{fn}^A(0) G_{ni}^B(t) \\
 & + \sum_{n'} \frac{(m_f^\alpha - m_{n'}^\alpha)(m_i^2 - m_{n'}^2)}{m_{n'}^2 - m_i^2 + \omega} C_{fn'}^B C_{n'i}^A G_{fn'}^B(t) G_{n'i}^A(0) \\
 & + \frac{1}{\pi} \int \frac{\Phi_{if}^{AB}(\omega', t)}{\omega' - \omega} d\omega'
 \end{aligned} \quad (3.5)$$

where

$$t = (p_i - p_f)^2, \quad \alpha = \begin{cases} 1 & \text{for fermions} \\ 2 & \text{for bosons} \end{cases}$$

and

$$\begin{aligned}
 \Phi_{if}^{AB}(\omega, t) = & \frac{1}{2}(2\pi)^4 \left\{ \sum_n \delta^{(4)}(p_i + k - p_n) \langle f | D^A(0) | n \rangle \langle n | D^B(0) | i \rangle \right. \\
 & \left. - \sum_{n'} \delta^{(4)}(p_i - k - p_{n'}) \langle f | D^B(0) | n' \rangle \langle n' | D^A(0) | i \rangle \right\}.
 \end{aligned} \quad (3.6)$$

Now we can use Eq. (3.3) to obtain the required sum rule. In order to eliminate the dependence of this sum rule on the t variable, we shall put $t = 0$. In this way we get from (3.6) and (3.3) the general formula :

$$\begin{aligned}
 & \sum_n (m_i^\alpha - m_n^\alpha) C_{fn}^A C_{ni}^B \sum_{fn}^A \sum_{ni}^B + \sum_{n'} (m_f^\alpha - m_{n'}^\alpha) C_{fn'}^B C_{n'i}^A \sum_{fn'}^B \sum_{n'i}^A \\
 & = \frac{1}{\pi} \int \Phi_{if}^{AB}(\omega', 0) \frac{d\omega'}{\omega'},
 \end{aligned} \quad (3.7)$$

where again

$$\alpha = \begin{cases} 1 & \text{for fermions} \\ 2 & \text{for bosons,} \end{cases}$$

ϕ_{if}^{AB} is given by (3.6) and we have introduced the notation

$$\sum_{fn}^A = G_{fn}^A(0) \quad (3.8)$$

Equation (3.7) forms the basis of the following consideration.

4. MASS FORMULAE OF THE BROKEN SU(3) SYMMETRY

The simplest way to obtain particular SU(3) relations from (3.7) is to utilize the formalism of I, V and U spin ⁷⁾. The corresponding currents are usually denoted as $\vec{I}$, $\vec{K}$ and $\vec{L}$ ⁸⁾. The decomposition of baryons from SU(3) octet and decuplet into multiplets of U and V spin is shown in the Table. For bosons, the corresponding decomposition can be easily obtained by substituting

$$\begin{aligned} (\pi, \rho) &\rightarrow \Sigma; \quad (\eta, \phi) \rightarrow \Lambda; \quad (K, K^*) \rightarrow (P, N); \\ (\bar{K}, \bar{K}^*) &\rightarrow \Xi \end{aligned} \quad (4.1)$$

where ϕ' is a mixture of the physical ϕ and ω states, defined by SU(6) symmetry to be

$$|\phi'\rangle = -\sqrt{\frac{2}{3}} |\phi\rangle + \frac{1}{\sqrt{3}} |\omega\rangle \quad (4.2)$$

Now the only thing we need is the well-known matrix elements of SU(2) algebra :

$$\langle I, I_z \pm 1 | I_{\pm} | I, I_z \rangle = \left[(I \pm I_z)(I \pm I_z + 1) \right]^{1/2} \quad (4.3)$$

and the same for V spin changing I into K. For U spin, it is necessary to be careful with the phases of the matrix elements, which can be defined from commutation relations ⁸⁾ :

$$[I_{\pm}, K_{\pm}] = \pm L_{\pm} \quad (4.4)$$

Assume now that the mass splittings in Eq. (3.7) are of first order in the λ parameter ($\sim 1/10$) characterizing the medium-strong interaction, breaking SU(3) symmetry (this statement is equivalent to the hypothesis of the octet dominance). The integral term on the right-hand side of Eq. (3.7), as can be easily seen from (3.6), is of second order in λ , since the matrix elements of current divergences there have an order of magnitude of the interaction breaking SU(3) symmetry, [see Refs. 1), 6)]. As it follows from the Ademollo-Gatto theorem, the deviation of r_{fn}^A from unity is also of second order in λ . Thus, neglecting in Eq. (3.5) the integral term and putting $r = 1$, we get as first approximation various mass formulae of the broken SU(3) symmetry. For the sake of completeness, we shall write them down once again.

Thus, let $A = B = L_+$ (the generator of U spin) in Eq. (3.1). Then, from Eq. (3.7), using the Table, and Eqs. (4.1) - (4.4), we obtain immediately :

a) for the baryon octet :

$$i = \Xi^0; n = \Sigma^0, \Lambda; f = N$$

$$2m(\Xi^0) + 2m(N) - 3m(\Lambda) - m(\Sigma^0) = O(\lambda^2)$$

b) for the pseudoscalar meson octet

$$i = \bar{K}^0; n = \pi^0, \eta; f = K^0$$

$$4m^2(K^0) - m^2(\pi^0) - 3m^2(\eta) = O(\lambda^2) \quad (4.5)$$

c) for the vector meson nonet

$$i = \bar{K}^{*0}; n = \rho^0, \phi, \omega; f = K^{*0}$$

$$4m^2(K^{*0}) - m^2(\rho^0) - 2m^2(\phi) - m^2(\omega) = O(\lambda^2)$$

Note that all these relations include particles of equal electric charges (as a consequence of using U spin, where electric charge is an invariant) and therefore, generally speaking, take into account also electromagnetic corrections [see Ref. 5].

In the case of SU(3) multiplets, where all weights have multiplicity one, Eq. (3.7) takes on a simpler form for $A=B$:

$$\sum_{fn}^A \sum_{ni}^A C_{fn}^A C_{ni}^A [m_i^\alpha - m_n^\alpha + m_f^\alpha - m_n^\alpha] = \frac{1}{\pi} \int \Phi_{if}^{AA}(\omega, 0) \frac{d\omega'}{\omega'} \quad (4.6)$$

The interesting feature of Eq. (4.6) is the factorization of quantities r_{fn}^A . So that in this case the corresponding mass formulae do not depend on the particular value of r_{fn}^A . It is sufficient for r_{fn}^A to be of order of unity. This may be the reason why the mass formulae of the baryon decuplet are so amazingly accurate. To derive these formulae, we again put $A=L_+$ in (4.6) and then, using the Table and neglecting the integral term, get the so-called equal spacing rule

$$\begin{aligned} m(\Omega^-) - m(\Xi_\delta^-) &= m(\Xi_\delta^-) - m(\Sigma_\delta^-) \\ &= m(\Sigma_\delta^-) - m(\Delta_\delta^-) \end{aligned} \quad (4.7a)$$

and

$$m(\Xi_\delta^0) - m(\Sigma_\delta^0) = m(\Sigma_\delta^0) - m(\Delta_\delta^0) \quad (4.7b)$$

We now pass to the sum rules involving electromagnetic mass splittings. Obviously in this case we are to use the isotopic spin current $\vec{I}$. So let $A=B=I_+$ in Eq. (3.1). Then we get from (3.7) for the π multiplet:

$$2 \left[m^2(\pi^-) - m^2(\pi^0) + m^2(\pi^+) - m^2(\pi^0) \right] = \frac{1}{\pi} \int \phi_{\pi\pi}^{II}(\omega', 0) \frac{d\omega'}{\omega'}$$

Now as $m(\pi^-) = m(\pi^+)$ (charge invariance), this equality can be written as

$$m^2(\pi^+) - m^2(\pi^0) = \frac{1}{4\pi} \int \phi_{\pi\pi}^{II}(\omega', 0) \frac{d\omega'}{\omega'} > 0 \quad (4.8)$$

where $\phi_{\pi\pi}^{II}$ is given by (3.6) with $A=B=I_+$. Here, contrary to the previous cases, we do not get any mass formulae since the $\pi^+\pi^0$ mass difference is of second order in the electric charge e and the right-hand side in Eq. (4.8) is of the same order of magnitude. We obtain a similar equality for the Σ multiplet :

$$m(\Sigma^+) + m(\Sigma^-) - 2m(\Sigma^0) = \frac{1}{2\pi} \int \phi_{\Sigma\Sigma}^{II}(\omega', 0) \frac{d\omega'}{\omega'} > 0 \quad (4.9)$$

Within the Δ_8 multiplet we have from Eq. (3.7) two relations :

$$a) \quad i = \Delta_8^- ; n = \Delta_8^0 ; f = \Delta_8^+$$

$$2\sqrt{3} \left[m(\Delta_8^-) - m(\Delta_8^0) + m(\Delta_8^+) - m(\Delta_8^0) \right] = \frac{1}{\pi} \int \phi_{\Delta_8^- \Delta_8^+}^{II}(\omega', 0) \frac{d\omega'}{\omega'}$$

$$b) \quad i = \Delta_8^0 ; n = \Delta_8^+ ; f = \Delta_8^{++}$$

$$2\sqrt{3} \left[m(\Delta_8^0) - m(\Delta_8^+) + m(\Delta_8^{++}) - m(\Delta_8^+) \right] = \frac{1}{\pi} \int \phi_{\Delta_8^0 \Delta_8^{++}}^{II}(\omega', 0) \frac{d\omega'}{\omega'}$$

Here again both sides of these equations are of the order of e^2 . Let us subtract now the second relation from the first one

$$\begin{aligned} & [m(\Delta_{\delta}^{-}) + m(\Delta_{\delta}^{+}) - 2m(\Delta_{\delta}^0)] - [m(\Delta_{\delta}^0) + m(\Delta_{\delta}^{++}) - 2m(\Delta_{\delta}^{+})] \\ &= \frac{1}{2\pi\sqrt{3}} \int \left[\phi_{\Delta_{\delta}^{-}\Delta_{\delta}^{+}}^{II}(\omega, 0) - \phi_{\Delta_{\delta}^0\Delta_{\delta}^{++}}^{II}(\omega, 0) \right] \frac{d\omega'}{\omega'} = O(e^4) \end{aligned} \quad (4.10)$$

The mass differences on the left-hand side of (4.10) are of the order of e^2 , while the difference of the two imaginary parts on the right-hand side is necessarily of a higher order in e (at least $\sim e^4$). In fact, both imaginary parts are the matrix elements between the states which, in the absence of electromagnetic interaction, are degenerate since they belong to the same isotopic multiplet.

Now, writing down the explicit expression (3.6) for quantity $\phi_{\Delta\Delta}^{II}$ and making use of the Wigner-Eckart theorem and symmetry properties of Clebsch-Gordan coefficients, we obtain the above-mentioned result.

Thus neglecting the integral term in (4.10) we obtain the so-called second difference mass formula ⁴⁾:

$$m(\Delta_{\delta}^{-}) - m(\Delta_{\delta}^{++}) = 3[m(\Delta_{\delta}^0) - m(\Delta_{\delta}^{+})] \quad (4.11)$$

Consider now the case when $A = L_-$ and $B = I_+$ in Eq. (3.1). For pseudoscalar (and also for vector) mesons, as a result of invariance under charge conjugation, we get from (3.7) only one relation:

$$\begin{aligned} & i = \pi^{-}; n = \pi^0; n' = K^{-}; f = \bar{K}^0 \\ & m^2(\pi^{-}) - m^2(\pi^0) + m^2(K^{-}) - m^2(\bar{K}^0) = \frac{1}{\pi} \int \phi_{\pi K}^{LI}(\omega, 0) \frac{d\omega'}{\omega'} \end{aligned} \quad (4.12)$$

(neglecting the deviation of r from unity). However, within the baryon octet, we have from (3.7) two relations :

$$\begin{aligned} \text{a) } i = \Sigma^-; n = \Sigma^0; n' = \Xi^-; f = \Xi^0 \\ m(\Sigma^-) - m(\Sigma^0) + m(\Xi^0) - m(\Xi^-) = \frac{1}{\pi} \int_{\Sigma^-\Xi^0} \phi^{\text{LI}}(\omega', 0) \frac{d\omega'}{\omega'} \quad (4.13) \end{aligned}$$

$$\begin{aligned} \text{b) } i = N; n = P; n' = \Sigma^0; f = \Sigma^+ \\ m(N) - m(P) + m(\Sigma^+) - m(\Sigma^0) = \frac{1}{\pi} \int_{N\Sigma^+} \phi^{\text{LI}}(\omega', 0) \frac{d\omega'}{\omega'} \end{aligned}$$

A simple consideration shows that both sides of Eqs. (4.12) and (4.13) are of the order of (λe^2) . For the left-hand side this follows from the fact that within the I spin multiplets the mass differences are of the order of e^2 and within U spin (V spin) multiplets, these are of the order of λ . So, to get the mass formula, we subtract, in analogy with the case of the Δ multiplet, Eq. (4.13a) from Eq. (4.13b)

$$\begin{aligned} m(N) - m(P) + m(\Xi^-) - m(\Xi^0) + m(\Sigma^+) - m(\Sigma^-) \\ = \frac{1}{\pi} \int \left[\phi_{N\Sigma^+}^{\text{LI}}(\omega', 0) - \phi_{\Sigma^-\Xi^0}^{\text{LI}}(\omega', 0) \right] \frac{d\omega'}{\omega'} \quad (4.14) \end{aligned}$$

Now, under the integral in (4.14), we have the difference of matrix elements between states belonging to the same V spin multiplets $(\Sigma^+\Xi^0)$, $(N\Sigma^-)$ and thus degenerate in the absence of $SU(3)$ breaking interactions.

From the explicit expression (3.6) for the quantities ϕ^{LI} one can see, applying the Wigner-Eckart theorem, that if we take into account only the contribution of the intermediate states with $I=L$, then the difference of the two terms under the integral in (4.14) vanishes with

the accuracy of $(\lambda^2 e^2)$. In particular, the above-mentioned limitation on the set of intermediate states leads to omitting states belonging to the decuplet. It is possible that their contribution is damped because of the higher masses. Thus, neglecting the integral term in Eq. (4.14), we obtain the well-known Coleman-Glashow formula⁹⁾:

$$m(N) - m(P) + m(\Xi^-) - m(\Xi^0) + m(\Sigma^+) - m(\Sigma^-) = 0 \quad (4.15).$$

The important point to be underlined here is that relation (4.15) essentially takes into account the interference between medium-strong and electromagnetic interactions [cf., also Ref. 4].

Let us pass now to the baryon decuplet. We have here, from (3.7), three relations:

$$\begin{aligned} \text{a) } i = \Sigma_8^-; n = \Sigma_8^0; n' = \Xi_8^-; f = \Xi_8^0 \\ m(\Sigma_8^-) - m(\Sigma_8^0) + m(\Xi_8^0) - m(\Xi_8^-) = \frac{1}{2\pi} \int \phi_{\Sigma_8^- \Xi_8^0}^{\langle I \rangle}(\omega, 0) \frac{d\omega'}{\omega'} \end{aligned} \quad (4.16)$$

$$\begin{aligned} \text{b) } i = \Delta_8^-; n = \Delta_8^0; n' = \Sigma_8^-; f = \Sigma_8^0 \\ m(\Delta_8^-) - m(\Delta_8^0) + m(\Sigma_8^0) - m(\Sigma_8^-) = \frac{1}{\pi\sqrt{6}} \int \phi_{\Delta_8^- \Sigma_8^0}^{\langle I \rangle}(\omega, 0) \frac{d\omega'}{\omega'} \end{aligned}$$

$$\text{c) } i = \Delta_8^0; n = \Delta_8^+; n' = \Sigma_8^0; f = \Sigma_8^+$$

$$\begin{aligned} m(\Delta_8^0) - m(\Delta_8^+) + m(\Sigma_8^+) - m(\Sigma_8^0) = \\ = \frac{1}{2\pi} \int \phi_{\Delta_8^0 \Sigma_8^+}^{\langle I \rangle}(\omega, 0) \frac{d\omega'}{\omega'} \end{aligned}$$

Both sides of all the relations of Eq. (4.16) are again of the order of (λe^2) . Proceeding now in close analogy with the derivation of the Coleman-Glashow formula, we subtract Eq. (4.16a) from (4.16c). Then, on the right-hand side we again have the difference of matrix elements between states of the same V spin multiplet. Neglecting this term, we obtain

$$m(\Delta_S^0) - m(\Delta_S^+) + m(\Sigma_S^+) - m(\Sigma_S^-) + m(\Xi_S^-) - m(\Xi_S^0) = 0 \quad (4.17)$$

With the same procedure applied to Eqs. (4.16b) and (4.16c), we get the difference of the matrix elements between states of the same I spin multiplet. Therefore the formula we obtain, dropping this term, seems to be highly accurate :

$$m(\Delta_S^+) + m(\Delta_S^-) - 2m(\Delta_S^0) = m(\Sigma_S^-) + m(\Sigma_S^+) - 2m(\Sigma_S^0). \quad (4.18)$$

Relations (4.17) and (4.18) are not independent of Eqs. (4.7a) and (4.7b). In fact, subtracting Eq. (4.7b) from Eq. (4.7a), we get, using the same arguments as before :

$$m(\Xi_S^-) - m(\Xi_S^0) + 2[m(\Sigma_S^0) - m(\Sigma_S^-)] + m(\Delta_S^-) - m(\Delta_S^0) = 0$$

But this relation can also be obtained from Eqs. (4.17) and (4.18) or from (4.16a) and (4.16b), (with the same accuracy).

5. DISCUSSION OF THE RESULTS

Introduction of the auxiliary quantity $Q(k)$ provides a simple and unambiguous way of deriving Fubini-Furlan dispersion relations for the matrix elements of current commutators. The point is that in this approach we avoid dealing with asymptotic values of "charges" at $t = i\infty$, because of the presence of the exponential damping factor.

The second remark concerns the mass formulae involving electromagnetic mass differences. It is usually assumed that in order to obtain such formulae, one is to neglect the interference of electromagnetic and medium-strong interactions. For example, this assumption forms the basis of the so-called "parallelogram law" ⁴⁾.

However, as we already have seen above, both sides of Eqs. (4.13) and (4.16) are of the order of (λe^2) and consequently do include the above-mentioned interference. So, in order to get the proper mass formulae, it is necessary to make the appropriate combinations of these equations (second differences) where the right-hand side is of a higher order of smallness. Therefore it seems that our formulae including electromagnetic mass splittings for the baryon decuplet are more accurate than those previously obtained ^{4),5),9)}. In this connection, it is worth mentioning that the model based on the use of equal time commutators gives us a specific form of the above-mentioned interference. The interference term of the general form has been considered by Okubo ⁹⁾. In this case the Coleman-Glashow formula (4.15) for the baryon octet becomes invalid.

The mass operator for the baryon decuplet (and in general for any triangular representation) has the form ⁹⁾

$$M = a_0 + a_1 Y + b_1 Q + b_2 Q^2 + c_1 QY + c_2 Q^2 Y \quad (5.1)$$

This equation gives us two equal spacing formulae (4.7). Now, the mass formula (4.17) can be obtained from (5.1) if, in addition, we put $c_2 = 0$. Thus it seems plausible that the approach based on current algebra gives us some additional dynamical information compared with the purely group theoretic method.

Available experimental data ¹⁰⁾ on electromagnetic mass differences within the baryon decuplet are :

$$\begin{aligned}
m(\Sigma_8^-) - m(\Sigma_8^+) &= 2.6 \pm 1.1 \text{ MeV} \\
m(\Xi_8^-) - m(\Xi_8^0) &= 6.2 \pm 2.4 \text{ MeV} \\
m(\Delta_8^-) - m(\Delta_8^{++}) &= 7.9 \pm 6.8 \text{ MeV} \\
m(\Delta_8^0) - m(\Delta_8^{++}) &= 0.45 \pm 0.85 \text{ MeV}
\end{aligned}
\tag{5.2}$$

Now, making use of Eq. (4.11), we can also get

$$\begin{aligned}
m(\Delta_8^0) - m(\Delta_8^+) &= \frac{1}{3} [m(\Delta_8^-) - m(\Delta_8^{++})] = \\
&= 2.6 \pm 2.3 \text{ MeV} .
\end{aligned}
\tag{5.3}$$

Thus we have all the mass differences which enter Eq. (4.17). Substituting the numbers, we obtain for the right-hand side of Eq. (4.17) the value $(6.2 \pm 3.5) \text{ MeV}$. Large experimental errors prevent from making any definite conclusions on the validity of Eq. (4.17).

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MULTIPLETS OF THE U AND V SPIN

Multi- plets Spin	Singlets $k = L = 0$	Doublets $k = L = \frac{1}{2}$	Triplets $k = L = 1$	Quadruplets $k = L = \frac{3}{2}$
U	$\frac{1}{2}(\sqrt{3} \Sigma^0 + \Lambda)$ Δ_{δ}^{++}	$(p; \Sigma^+) (\Sigma^-; E^-)$ $(\Delta_{\delta}^+; \Sigma_{\delta}^+)$	$(n; \frac{1}{2}(\sqrt{3} \Lambda - \Sigma^0); E^0)$ $(\Delta_{\delta}^0; \Sigma_{\delta}^0; E_{\delta}^0)$	$(\Delta_{\delta}^-; \Sigma_{\delta}^-; E_{\delta}^-; \Omega^-)$
V	$\frac{1}{2}(\sqrt{3} \Sigma^0 - \Lambda)$ Δ_{δ}^-	$(\Sigma^+; E^0) (n; \Sigma^-)$ $(\Delta_{\delta}^0; \Sigma_{\delta}^-)$	$(p; \frac{1}{2}(\Sigma^0 + \sqrt{3} \Lambda); E^-)$ $(\Delta_{\delta}^+; \Sigma_{\delta}^0; E_{\delta}^-)$	$(\Delta_{\delta}^{++}; \Sigma_{\delta}^+; E_{\delta}^0; \Omega^-)$

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