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ANTIPROTON-PROTON ELASTIC SCATTERING AT HIGH ENERGIES
IN THE UNCORRELATED JET MODEL

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A B S T R A C T

The uncorrelated jet model proposed by Van Hove as a phenomenological description of hadron collisions at high energies is applied to antiproton-proton scattering. An analysis is made of the experimental elastic scattering data at laboratory momenta above 4 GeV/c. A modification to the model is introduced, made necessary by the presence of $\bar{p}p$ - annihilation channels at finite energies. The modified model is shown to reproduce excellently the existing data concerning total cross-sections and small angle diffraction scattering.

1. INTRODUCTION

Many experimental results have become available recently concerning high energy collisions of hadrons (strongly interacting particles). Among others, the following characteristic properties have emerged from the experimental data :

- a) near constancy of the total cross sections σ_T for increasing energy,
- b) the smallness of the real part of the elastic scattering amplitude as compared to its imaginary part,
- c) a sharp exponential diffraction peak at small momentum transfers ($-t \lesssim 1 \text{ (GeV/c)}^2$, $t = -(\text{c.m. momentum transfer})^2$), which is energy independent for meson-proton elastic scattering, shrinks slightly with increasing energy for proton-proton elastic scattering, whereas it shows a weak expansion for antiproton-proton elastic scattering.

As long as a dynamical theory for the strong interactions is lacking, it is fruitful to try to analyse the experiments in a phenomenological way in terms of tentative models. One such model was proposed recently by Van Hove ¹⁾. In this model, it is assumed that

- i) elastic scattering at high energies is the shadow of inelastic collisions, i.e., the scattering amplitude is purely imaginary,
- ii) the most common inelastic channels produce a large number of secondaries having no other correlations with each other than the ones imposed by momentum and energy conservation (uncorrelated jet model).

Using these assumptions together with the experimental fact that the transverse momentum distribution of secondaries in inelastic collisions is energy independent and limited to small transverse momentum values, Van Hove was able to explain the exponential dependence

of $d\sigma_{el}/dt$ on t . A model in which assumption (ii) has been replaced by one in which it is taken that inelastic collisions consist entirely of isobar excitation of one or both incident particles, has been discussed by Bialas, Ruijgrok and Van Hove²⁾. A combination of both models finally has been studied in a paper by Bialas and Van Hove³⁾.

The uncorrelated jet model was applied recently by Cottingham and Peierls⁴⁾ to π^-p scattering and pp scattering. For the first process it was found that the model gives excellent results; the data for the second reaction could only be reproduced after introducing a small real part in the scattering amplitude, of a size compatible with the values measured in near-forward directions.

In this paper we will analyse antiproton-proton elastic scattering data in terms of the uncorrelated jet model. Sufficiently accurate experimental results exist for laboratory momenta p_L of 4⁵⁾, 8.9 and 12 GeV/c⁶⁾. As is well known, in the region around 10 GeV/c there is still a big difference between the total cross sections σ_T^{pp} and $\sigma_T^{\bar{p}p}$ for proton-proton and antiproton-proton scattering. This indicates that here we have not yet reached the asymptotic region of almost constant and equal cross sections. The most important cause for this difference in σ_T^{pp} and $\sigma_T^{\bar{p}p}$ lies in the presence of those inelastic channels that are open for the $\bar{p}p$ -system, but closed for the pp -system. They consist mainly of annihilation channels. With increasing energy the contribution of these channels to the inelastic cross section decreases rapidly. This is reflected in the behaviour of σ_T^{pp} and $\sigma_T^{\bar{p}p}$ as functions of the energy, the latter decreasing much more rapidly with increasing energy in the experimentally known region than the former. At very high energies σ_T^{pp} and $\sigma_T^{\bar{p}p}$ are expected to approach the same constant limit (Pomeranchuk theorem).

The presence of the above-mentioned additional channels and the rather strong energy dependence of their reaction cross sections make it necessary to modify the Van Hove model for finite energies and to adapt it to this particular situation. Roughly speaking, the modification we have used in this paper consists in treating the annihilation channels separately from the remaining channels. We preserve a purely imaginary scattering amplitude, in agreement with theoretical predictions on $\bar{p}p$ scattering ^{7), 8)}.

The model, modified in this way, gives very good results concerning total cross sections, elastic cross sections and small-angle diffraction scattering. It seems to be less useful for a description of elastic collisions at high momentum transfers.

Sections 2 and 3 describe the general formalism. As in Ref. ⁴⁾, we reformulate the Van Hove model in the impact parameter representation, a method which is very useful for treating high energy collisions. In section 4 we apply the model to $\bar{p}p$ scattering. We introduce the modification which accounts for the presence of annihilation channels and present the results of the calculations. The modification of the model is discussed in more detail in the last section.

4.

2. THE UNITARITY CONDITION

We consider the elastic collision ^{*)} of two hadrons, $a+b \rightarrow a+b$, neglecting their spins. The unitarity condition for the S operator describing this collision can be written as

$$i(T^\dagger - T) = T^\dagger T, \quad (2.1a)$$

with

$$S = 1 + iT. \quad (2.1b)$$

When taken between initial and final two-particle states, this condition becomes :

$$i \langle \vec{k}, -\vec{k} | T^\dagger - T | \vec{k}, -\vec{k} \rangle = \sum_n \langle \vec{k}, -\vec{k} | T^\dagger | n \rangle \langle n | T | \vec{k}, -\vec{k} \rangle. \quad (2.2)$$

Here $|\vec{k}, -\vec{k}\rangle$ and $|\vec{k}', -\vec{k}'\rangle$ are the initial and final states of the two colliding hadrons in their centre-of-mass system; $\vec{k}$ and $\vec{k}'$ denote the initial and final centre-of-mass momenta ($\vec{k}_a = -\vec{k}_b = \vec{k}$, etc). The sum over n on the right-hand side of (2.2) runs over a complete set of states $|n\rangle$, which, like $|\vec{k}, -\vec{k}\rangle$ and $|\vec{k}', -\vec{k}'\rangle$, are considered to be eigenstates of the total four-momentum operator P . Taking out the delta function describing energy-momentum conservation :

$$\langle i | T | j \rangle = \delta^{(4)}(p_i^\mu - p_j^\mu) \langle i | \tau | j \rangle, \quad (2.3)$$

^{*)} A collision is called elastic when the final and initial particles have the same internal quantum numbers.

where $P^\mu |i\rangle = p_i^\mu |i\rangle$, etc, and separating the sum over intermediate states $|n\rangle$ into two sums, one corresponding to the elastic channels $|\vec{k}_a'', \vec{k}_b''\rangle$, and one corresponding to the inelastic channels $|n, \text{inel.}\rangle$, we get

$$i\langle \vec{k}', -\vec{k}' | \tau^+ - \tau | \vec{k}, -\vec{k} \rangle = \int d^3k_a'' \int d^3k_b'' \delta^{(4)}(k_a'' + k_b'' - k_a - k_b) \cdot \\ \langle \vec{k}', -\vec{k}' | \tau^+ | \vec{k}_a'', \vec{k}_b'' \rangle \langle \vec{k}_a'', \vec{k}_b'' | \tau | \vec{k}, -\vec{k} \rangle + \\ \sum_n \langle \vec{k}', -\vec{k}' | \tau^+ | n, \text{inel.} \rangle \delta^{(4)}(q_n - k_a - k_b) \langle n, \text{inel.} | \tau | \vec{k}, -\vec{k} \rangle. \quad (2.4)$$

Defining

$$|f(\vec{k})\rangle = \sum_n |n, \text{inel.}\rangle \langle n, \text{inel.} | \tau | \vec{k}, -\vec{k} \rangle, \quad (2.5)$$

Equation (2.4) can be written in the form

$$i\langle \vec{k}', -\vec{k}' | \tau^+ - \tau | \vec{k}, -\vec{k} \rangle = \int d^3k_a'' \int d^3k_b'' \delta^{(4)}(k_a'' + k_b'' - k_a - k_b) \cdot \\ \langle \vec{k}', -\vec{k}' | \tau^+ | \vec{k}_a'', \vec{k}_b'' \rangle \langle \vec{k}_a'', \vec{k}_b'' | \tau | \vec{k}, -\vec{k} \rangle + \\ \langle f(\vec{k}') | \delta^{(4)}(P - k_a - k_b) | f(\vec{k}) \rangle. \quad (2.6)$$

It can easily be seen ¹⁾ that, apart from a delta-function, $|f(\vec{k})\rangle$ is the wave function of the final inelastic state resulting from the collision of two particles with momenta $\vec{k}$ and $-\vec{k}$. The last term on the right of (2.6), which will be denoted by F , is called the overlap function ¹⁾, because it expresses the overlap between the states $|f(\vec{k})\rangle$ and $|f(\vec{k}')\rangle$. Note that $|f(\vec{k})\rangle$ and $|f(\vec{k}')\rangle$ differ only by an over-all rotation over θ , the angle between $\vec{k}$ and $\vec{k}'$. The overlap function F is real and depends only upon two variables, namely k and $\cos\theta$, or equivalently, k and $t = -2k^2(1 - \cos\theta)$.

6.

We define the relativistic elastic amplitude $T(s, t)$ by the following equation

$$T(s, t) = 2\pi \varepsilon_a \varepsilon_b \langle \vec{k}', -\vec{k}' | \tau | \vec{k}, -\vec{k} \rangle. \quad (2.7)$$

Here ε_a and ε_b are the energies of the particles a and b in the centre-of-mass system, $\varepsilon_1 = (\vec{k}_1^2 + m_1^2)^{\frac{1}{2}} = (k^2 + m_1^2)^{\frac{1}{2}}$, while $s = (\varepsilon_a + \varepsilon_b)^2$. Inserting this into the unitarity condition and putting

$$g(s, t) = \frac{4\pi^2 \varepsilon_a \varepsilon_b}{k s^{\frac{1}{2}} \sigma_{inel}} F(s, t), \quad (2.8)$$

one gets, assuming time reversal invariance,

$$\begin{aligned} \text{Im } T(s, t) = & \frac{k}{4\pi s^{\frac{1}{2}}} \int d\Omega' T^*(s, \cos \theta') T(s, \cos \theta'') \\ & + \frac{k s^{\frac{1}{2}}}{4\pi} \sigma_{inel} g(s, t), \end{aligned} \quad (2.9)$$

with

$$\begin{aligned} d\Omega' &= \sin \theta' d\theta' d\varphi' \\ \cos \theta'' &= \cos \theta \cos \theta' + \sin \theta \sin \theta' \cos \varphi'. \end{aligned}$$

σ_{inel} is the total inelastic cross section at momentum k . The optical theorem implies that the function $g(s, t)$, which is proportional to the overlap function, has the value one in the forward direction.

Following Cottingham and Peierls ⁴⁾, we now write the unitarity condition (2.9) in the impact parameter representation. For that purpose, we define the impact parameter expansion of $T(s, t)$ by the relation ⁴⁾ ($\Lambda = \sqrt{-t_{max}}$):

$$h(b) = \frac{1}{2k s^{\frac{1}{2}}} \int_0^\Lambda T(s, t) J_0(b\sqrt{-t}) \sqrt{-t} d\sqrt{-t}. \quad (2.10)$$

From this relation one has for the amplitude by inversion :

$$T(s, t) = 2 k s^{1/2} \int_0^{\infty} h(b) J_0(b\sqrt{-t}) b db, \quad 0 \leq t \leq t_{\max}. \quad (2.11)$$

In Eqs. (2.10) and (2.11), J_0 represents the zeroth order Bessel function. Although the function $h(b)$ in general depends on the energy, we have not indicated this explicitly.

In the same way, we define the impact parameter expansion for the overlap function $g(s, t)$, as follows :

$$\rho(b) = \frac{\sigma_{inel}}{2\pi} \int_0^{\infty} g(s, t) J_0(b\sqrt{-t}) \sqrt{-t} d\sqrt{-t}, \quad (2.12)$$

$$g(s, t) = \frac{2\pi}{\sigma_{inel}} \int_0^{\infty} \rho(b) J_0(b\sqrt{-t}) b db, \quad 0 \leq t \leq t_{\max}. \quad (2.13)$$

Inserting these expansions into Eq. (2.9), one obtains ⁴⁾ the following unitarity condition for $h(b)$:

$$\begin{aligned} \text{Im } h(b) = & \frac{1}{4} \rho(b) + |h(b)|^2 \\ & + \frac{1}{4k^2} \frac{d}{db^2} \left(b^2 \left| \frac{dh}{db} \right|^2 \right) + O(k^{-4}). \end{aligned} \quad (2.14)$$

In the limit of very high energies and for small momentum transfers, the terms of order k^{-2} , etc., give only a very small correction to the right-hand side of (2.14) (see section 4). Asymptotically, one has the simple result

$$\text{Im } h(b) = \frac{1}{4} \rho(b) + |h(b)|^2. \quad (2.15)$$

Before proceeding we first write down the formulae for the various cross sections in terms of the amplitudes $h(b)$.

Total cross section :

$$\sigma_T = \frac{4\pi}{k^2 s^{1/2}} \operatorname{Im} T(s, 0) = 8\pi \int_0^\infty \operatorname{Im} h(b) b db, \quad (2.16)$$

Total elastic cross section :

$$\sigma_{el} = 8\pi \int_0^\infty |h(b)|^2 b db, \quad (2.17)$$

Total inelastic cross section :

$$\sigma_{inel} = 2\pi \int_0^\infty p(b) b db, \quad (2.18)$$

Differential elastic cross section :

$$\frac{d\sigma_{el}}{dt} = \frac{\pi}{k^2 s} |T(s, t)|^2 = 4\pi \left| \int_0^\infty h(b) J_0(b\sqrt{-t}) b db \right|^2, \quad (2.19)$$

$$\left(\frac{d\sigma_{el}}{dt} \right)_{t=0} = \frac{\sigma_T^2}{16\pi} (1 + \alpha^2), \quad \alpha = \frac{\operatorname{Re} T(s, 0)}{\operatorname{Im} T(s, 0)}. \quad (2.20)$$

3. THE UNCORRELATED JET MODEL

In the limit of very high energies, the amplitude $T(s,t)$, and consequently $h(b)$, is believed to be purely or almost purely imaginary⁹⁾. As a consequence, we have from (2.15) in this limit

$$\text{Im } h(b) = \frac{1}{2} \left[1 - (1 - \wp(b))^{1/2} \right] , \quad (3.1)$$

where we have used the requirement that, due to the finite range of the forces, both $\wp(b)$ and $h_I(b)$ should go to zero for very large b . Equation (3.1) shows that the (imaginary) elastic amplitude is uniquely determined by the function $\wp(b)$. Or, expressed alternatively, it tells how shadow scattering is determined by the inelastic collisions, the latter being described by the function $\wp(b)$.

From its definition (see Eqs. (2.6), (2.8), (2.12)), it is clear that we know $\wp(b)$ once we know the wave function $|f(\vec{k})\rangle$ for the final inelastic state. Van Hove¹⁾ has considered a class of models in which it is assumed that the particles in the inelastic final state (secondaries) are numerous and are only weakly correlated, apart from the correlation imposed by energy-momentum conservation (uncorrelated jet models). The wave functions for these models, which have simple product forms, lead to the following form for the overlap function

$$F(k,t) = F(k,0) e^{\frac{1}{2} A' t} \quad (3.2)$$

This equation holds for high energies and small angles. From Eqs. (2.8) and (2.12), we see that this form for $F(k,t)$ results in a Gaussian for the function $\wp(b)$:

$$\wp(b) = A \exp(-b^2/b_0^2) , \quad (3.3)$$

with $b_0^2 = 2A^2$. The unitarity condition gives the restriction

$$A \leq 1 \quad (3.4)$$

Combining Eqs. (3.1), (3.3), (2.16) and (2.17), one finds in the asymptotic limit

$$\frac{\sigma_{el}}{\sigma_T} = 1 - \frac{A}{4} \left\{ 1 - (1-A)^{1/2} - \ln \left[\frac{2 - 2(1-A)^{1/2}}{A} \right] \right\}^{-1}, \quad (3.5)$$

$$\left(1 - \frac{\sigma_{el}}{\sigma_T} \right) \sigma_T = \pi A b_0^2. \quad (3.6)$$

Hence the parameters A and b_0^2 are uniquely determined by the quantities σ_T and σ_{el}/σ_T . The model predicts a maximum value for σ_{el}/σ_T corresponding to $A = 1$: $(\sigma_{el}/\sigma_T)_{\max} = 0.185$.

Cottingham and Peierls ⁴⁾ have applied this model to pion-proton elastic scattering. In this case for laboratory momenta p_L of about 10 GeV/c and larger σ_T and σ_{el}/σ_T are approximately constant, with $\sigma_{el}/\sigma_T \simeq 0.16$. With the values of the parameters determined from (3.5) and (3.6) to be $A = 0.91$ and $b_0^2 = 7.56$ mb, they obtain a very good fit for the differential elastic cross section at $p_L = 17$ GeV/c and $|t| \lesssim 1(\text{GeV}/c)^2$. That is simple two parameter model, based on a purely imaginary amplitude, works so well in this case is in agreement with the experimental fact ¹⁰⁾ that the ratio $\text{Re } T(s,0)/\text{Im } T(s,0)$ for πp scattering at high energies decreases rapidly with increasing energy and is very small for laboratory momenta around 17 GeV/c.

The same authors also discussed proton-proton scattering. In this case the Van Hove model in its simple form (purely imaginary amplitude, Gaussian overlap function) cannot explain the data because the values of σ_{el}/σ_T in the energy range considered exceed the model limit $(\sigma_{el}/\sigma_T)_{\max} = 0.185$. However, Cottingham and Peierls

were able to fit the data between 12 and 20 GeV/c by taking an amplitude with a small energy dependent real part, preserving the Gaussian form for the overlap function. This is in accordance with experiment : the existence of such a real part in small angle pp scattering has recently been demonstrated ¹¹⁾. In their model, the shrinking of the pp diffraction peak is mainly due to this real part.

4. ANTIPROTON - PROTON ELASTIC SCATTERING

We will analyse the experimental data on antiproton-proton scattering in terms of the uncorrelated jet model. However, as in the case of proton-proton scattering, it is impossible to use the Van Hove model in its simplest form, because in the energy region where experimental data are available, the values of the ratio σ_{el}/σ_T , although smaller than the corresponding values for pp scattering, still exceed the limit 0.185. This can be seen from Table I where we have compiled a few data on high energy $\bar{p}p$ scattering. The listed data at $p_L = 4, 8.9$ and 12 GeV/c are the only data which are sufficiently accurate to be compared with theoretical predictions.

- Table I -

Lab. momentum p_L (GeV/c)	σ_T (mb)	σ_{el}/σ_T	$\left(\frac{d\sigma_{el}}{dt}\right)_{t=0}$ (mb/(GeV/c) ²)
4.0 5)	71.0 ± 1.0	0.278 ± 0.010	252 ± 18.0
8.9 6)	59.5 ± 1.5	0.238 ± 0.009	178 ± 5.1
12.0 6)	54.4 ± 1.7	0.213 ± 0.010	147 ± 6.4
∞ 5)	37.0 ± 0.5	0.178 ± 0.006	-

Experimental data for $\bar{p}p$ scattering at various energies. $(d\sigma_{el}/dt)_{t=0}$ is the result of extrapolation of the measured values of $d\sigma_{el}/dt$ to $t=0$. The last line gives the results of an extrapolation to infinite incident energy of both pp and $\bar{p}p$ data, made by Czyzewski, et al. 5).

There are various ways to modify the Van Hove model,

- a) by introducing a real part, while preserving the pure Gaussian form for the overlap function, as was done in Ref. ⁴⁾ for treating pp scattering,
- b) by keeping a purely imaginary amplitude, but allowing slight deviations from a pure Gaussian behaviour of the overlap function,
- c) by a combination of both.

Concerning the first possibility, we remark that there are good reasons to believe that the real part of the scattering amplitude for $\bar{p}p$ - scattering at high energies and small scattering angles will be much smaller than that for pp scattering. This follows from the theoretical work by Söding ⁷⁾ and Biaľas ⁸⁾, and is also in agreement with the fact that the extrapolations for various energies of the experimental values of $d\sigma_{el}/dt$ to $t=0$ (compare Table I), coincide within the experimental errors with the optical theorem values $(d\sigma_{el}/dt)_{opt} = \sigma_T^2/16\pi \text{ mb}^2 = \sigma_T^2/19.6 \text{ mb}/(\text{GeV}/c)^2$ (see Eq.(2.20)). Nevertheless, we have tried to fit the data according to point a) by assuming various forms for the function $\text{Re } h(b)$. It was impossible to obtain reasonable results. We therefore reject possibility a) and consider instead possibility b), i.e., we will take a non-Gaussian form for the overlap function, thereby assuming that an eventual real part is small enough to be negligible. It will be seen in the last section how this way of modifying the Van Hove model, and in particular the form we assume for the overlap function, can be intuitively understood, to a certain degree, in terms of $\bar{p}p$ annihilation reactions.

We proceed as follows. First we consider the asymptotic limit $p_L \rightarrow \infty$. Recently, Czyzewski, et al. ⁵⁾ have made an extrapolation to infinite incident energy of the total cross sections and

the slopes of the diffraction curves for both $p\bar{p}$ and $\bar{p}p$ scattering assuming that the asymptotic limits of these quantities are the same for pp and $\bar{p}p$ reactions. (Pomeranchuk theorem¹²⁾ and its extension to diffraction scattering¹³⁾). This enabled them to calculate the asymptotic limit of the ratio σ_{el}/σ_T (assuming a purely imaginary asymptotic amplitude), which then, of course, is also the same for pp and $\bar{p}p$ scattering. The results of this extrapolation are contained in Table I. We see the interesting result that the asymptotic limit of σ_{el}/σ_T lies below the limit imposed by the Van Hove model. This suggests that asymptotically this model is applicable for the description of inelastic collisions in both pp and $\bar{p}p$ reactions. We therefore start from the assumption that for $p_L \rightarrow \infty$ the overlap function is a Gaussian with parameters A and b_0^2 determined from Eqs. (3.5) and (3.6) with $\sigma_T^{as} = 37$ mb and $(\sigma_{el}/\sigma_T)^{as} = 0.178$. We find

$$\rho^{as}(b) = A \exp(-b^2/b_0^2) = 0.98 \exp(-b^2/10) , \quad (4.1)$$

$(b^2 \text{ in mb}) ,$

indicating that there is almost complete absorption for $b = 0$ (s waves).

For finite energies, we now write for the overlap function

$$\rho(b) = \rho^{as}(b) + \bar{\rho}(b) , \quad (4.2)$$

where $\bar{\rho}(b)$ describes the deviation from $\rho^{as}(b)$. For $p_L \rightarrow \infty$, $\bar{\rho}(b)$ goes to zero. The presence of $\bar{\rho}(b)$ is a reflection of the presence of proton-antiproton annihilation channels, which at finite energies, i.e., when the asymptotic region is not yet reached, are the main cause for the big difference between the proton-proton and antiproton-proton total cross sections. We will come back to the

question of trying to understand $\bar{\rho}(b)$ in terms of annihilation processes in the next section ; here we restrict ourselves to a discussion of the results of the calculations performed on the basis of Eq. (4.2) and with $\bar{\rho}(b)$ of the form given by (4.3) below.

It turns out that very good fits to the experimental data, i.e., cross sections and diffraction curves, at $p_L = 4$ GeV/c, 8.9 GeV/c and 12 GeV/c, can be obtained with a $\bar{\rho}(b)$ of the following form

$$\bar{\rho}(b) = B \left[\exp(-b^2/10) - \lambda \exp(-b^2/5) \right], \quad (4.3)$$

$(b^2 \text{ in mb})$.

The positive parameters B and λ vary with the energy. However, λ varies in the energy range considered only between 0.983 and 1.0, i.e., $\bar{\rho}(0)$ is very close to zero and never negative. This is very reasonable, although perhaps not strictly necessary ; it means that the absorption at $b = 0$, as in the asymptotic limit, remains almost complete, even more than at $p_L = \infty$. The parameter B becomes zero at $p_L = \infty$. Unitarity, i.e., $\rho(b) = \rho^{\text{as}}(b) + \bar{\rho}(b) \leq 1$ for all b , imposes the following restriction on B :

$$B \leq B_{\text{max}} = 2\lambda - 0.98 + 2(\lambda^2 - 0.98\lambda)^{1/2}, \quad (4.4)$$

or $\lambda = 1, \quad B_{\text{max}} = 1.304$

$$\lambda = 0.983, \quad B_{\text{max}} = 1.094$$

Since, as we shall see below (Table II), the data at $p_L = 4$ GeV/c can be reproduced with the values $B = 1.3$, $\lambda = 1$, the condition (4.4) implies that the form for $\bar{\rho}(b)$ given by Eq. (4.3)

cannot be used for laboratory momenta lower than 4 GeV/c. This is, however, just about the limit below which we do not expect high energy models of the type under study to be reliable.

Using (4.1) - (4.3), (2.16) - (2.19) and the asymptotic form of the unitarity condition, Eq. (3.1), we have, for various values of λ , calculated σ_T , σ_{el}/σ_T and $d\sigma_{el}/dt$ for B ranging from zero to B_{max} . Best fits for σ_T , σ_{el}/σ_T and the forward diffraction peak ($t \lesssim 0.6 (\text{GeV}/c)^2$), for the above-mentioned three values of p_L , were obtained with the values of the parameters B and λ listed in Table II. In the same table, we have written down the corresponding calculated values of σ_T and σ_{el}/σ_T . It is seen that B varies strongly with the energy, roughly as k^{-1} , while λ , as said above, remains almost constant and very close to 1.

- Table II -

p_L (GeV/c)	λ	B	σ_T (mb)	σ_{el}/σ_T	$\sigma_T^{(c)}$ (mb)	$(\sigma_{el}/\sigma_T)^{(c)}$
4.0	1.0	1.30	70.43	0.273	71.48	0.284
8.9	0.983	0.90	58.99	0.234	59.07	0.235
12.0	1.0	0.72	53.52	0.214	53.56	0.214

The values of the parameters B and λ , which give best fits to the data for the three indicated values of the laboratory momentum. The 4th and 5th column contain the corresponding calculated values of σ_T and σ_{el}/σ_T using the asymptotic form of the unitarity condition. The last two columns are the values of σ_T and σ_{el}/σ_T obtained by including the corrections of order $1/k^2$ to the asymptotic unitarity condition.

Figures 1 and 2 show the theoretical curves σ_T v. B and σ_{el}/σ_T v. B for two values of λ , $\lambda = 0.983$ and $\lambda = 1$, and B ranging from 0 to B_{max} . The experimental points in these figures are taken from Table I, the corresponding B values from Table II.

In Fig 3, we have drawn the theoretical diffraction curves : $\ln X(t) = \ln [d\sigma_{el}/dt / (d\sigma_{el}/dt)_{t=0}]$ v. $-t$ for $\lambda = 1$ and various values of B . We see a set of curves expanding with increasing energy (decreasing B) to the asymptotic diffraction curve ($B = 0$). According to our assumptions, this asymptotic curve is the common limit for both the $\bar{p}p$ and the pp curves, the $\bar{p}p$ curves expanding to this limit, the pp curves shrinking to it. Curve 4 is the best fit to the $\bar{p}p$ data at 12 GeV/c (see Fig. 5). For comparison, we have included in Fig. 3 the proton-proton diffraction curve at 12.8 GeV/c according to Ref. 4.

When considering Fig. 3, we observe that the curves corresponding to higher B values (for instance, curves 4 and 5) are almost straight lines. The curves with low values of B on the contrary have considerable curvature ; the same is true for the pp curve. This reflects very well the experimental facts. It is known that, for $|t| \lesssim 0.6 (\text{GeV}/c)^2$, the pp curves between 10 and 20 GeV/c are curved with negative curvature, while the $\bar{p}p$ curves around 10 GeV/c are almost straight lines (Figs. 5 and 6).

The fact that our theoretical curves become straight for the higher B values, is caused by the presence of the negative term in $\bar{\rho}(b)$, see Eq. (4.3). We will see in the next section that this term can be interpreted as describing the mutual suppression of the annihilation channels and the non-annihilation channels, each set of channels being damped by the other through absorption corrections. This damping is very strong around $b = 0$, but decreases rapidly for higher b values, going to zero for large b . Consequently for finite energies the total absorption at $b = 0$ is almost the same as in the asymptotic case, but for higher b values, it is larger, causing the diffraction

curves (small t) to fall off with t more rapidly. Towards lower energies (larger values of B) the annihilation becomes stronger, therefore also the absorption for the higher b values, causing steeper diffraction peaks. Or conversely, with increasing energy, the $\bar{p}p$ peaks expand and become more and more curved.

In proton-proton scattering, no such mechanism is operating. On the contrary, here we have a real part at finite energies, which has the effect of reducing the absorption as compared to the asymptotic situation. This follows from the unitarity condition. The result is a diffraction peak which is expanded with respect to the asymptotic curve.

In Figs. 4 and 5, we have shown the theoretical diffraction curves for small t at $p_L = 8.9$ GeV/c and 12 GeV/c respectively, together with the experimental points from Ref. 6). The curves are almost exactly straight lines with slopes 12.80 (GeV/c) $^{-2}$ and 12.75 (GeV/c) $^{-2}$, respectively. The expansion is only very small for this difference in laboratory momenta. Also the calculated curve for $p_L = 4$ GeV/c is straight for $-t \lesssim 0.3$ (GeV/c) 2 with slope 12.87 (GeV/c) $^{-2}$, in good agreement with the experimental results of Ref. 5). Figure 6 shows the overlap function $F(k,t)$, normalized to 1 at $t = 0$, for $p_L = 4$ GeV/c, 12 GeV/c and ∞ .

For each of the three values of p_L in Table II, we have computed the $1/k^2$ correction to the asymptotic unitarity condition, using Eq. (2.14). The corrected values of the total cross sections and the ratios σ_{el}/σ_T are shown in the last two columns of Table II. It is seen that for $p_L = 8.9$ and 12 GeV/c the corrections are negligible. This is also true for the small - angle diffraction scattering. For $p_L = 4$ GeV/c the corrections are somewhat bigger, although still not more than a few percent. The corrected diffraction curves approach the corresponding uncorrected curves from above. The values for the slopes quoted above are the corrected values.

We finally devote a few words to elastic scattering at the larger momentum transfers. Except at 4 GeV/c⁵⁾, there exist no experimental data on large angle elastic antiproton-proton scattering in the high energy region. We nevertheless mention the predictions of this model for the higher momentum transfers. In Fig. 7 are given the theoretical curves of the differential elastic cross sections up to momentum transfers squared of $8 (\text{GeV}/c)^2$ for $p_L = 4 \text{ GeV}/c$, $12 \text{ GeV}/c$ and ∞ . For $p_L = 4 \text{ GeV}/c$ also the corrected curve is drawn. We see that the corrections become very big at large t . This persists for the higher energies (8.9 and $12 \text{ GeV}/c$). Note the sharp minima in the $4 \text{ GeV}/c$ curves around $-t = 0.7 (\text{GeV}/c)^2$. Here $\text{Im } T(s,t)$ goes through zero, and hence $d\sigma_{el}/dt$ becomes zero. Similar zeros probably occur for the curves corresponding to higher energies, although shifted to larger t values. In the asymptotic curve the zeros have disappeared. The positions of these zeros are extremely sensitive to the values of the parameters B and λ , and may also differ greatly for an uncorrected curve and the corresponding corrected one. A change of λ from 1 to 0.99, for instance, may therefore cause very big differences in the values of $d\sigma_{el}/dt$ for large t . This instability in the behaviour of the curves for large t due to the occurrence of zeros in $\text{Im } T(s,t)$ as a function of t , makes our model very unreliable in the region of large scattering angles, at least for momenta not higher than $12 \text{ GeV}/c$. The introduction of a real part in the amplitude would have a stabilizing effect: compare in this connection the work of Cottingham and Peierls⁴⁾ on pp scattering. At present, the existence of such a real part in $\bar{p}p$ scattering at large values of t cannot be excluded.

Nevertheless, except for the region around $-t = 0.7 (\text{GeV}/c)^2$ the curves corresponding to $p_L = 4 \text{ GeV}/c$ are in rough agreement with the large angle data of Ref.⁵⁾ (see Fig. 7). This may be accidental; however, it indicates that shadow scattering may produce a considerable

part, if not all, of the elastic scattering at high momentum transfers.

Our conclusion is that the uncorrelated jet model with the modification introduced in this paper reproduces excellently the experimental data on $\bar{p}p$ scattering between 4 and 12 GeV/c as far as total cross sections, total elastic cross sections and small angle diffraction scattering are concerned. For large angle scattering it works less well mainly because of instabilities due to zeros in $\text{Im } T$, the positions of which depend sensitively on the parameters B and λ , i.e., on the form of $\bar{\rho}(b)$. Moreover, the assumption of a purely imaginary amplitude also for the higher values of t is probably not justified.

5. DISCUSSION OF THE FUNCTION $\bar{\rho}(b)$

The results of the calculations presented above, which appear to reproduce the experimental data very well, are based on the particular form of the overlap function defined in Eqs. (4.1)-(4.3). In this section we wish to make a few remarks about the function $\bar{\rho}(b)$; in particular we shall try to understand the form of this function and see whether any conclusions can be drawn about the properties of the jets in inelastic antiproton-proton reactions.

Consider first the case of infinite incident energy. In that case the total cross sections σ_T^{pp} and $\sigma_T^{\bar{p}p}$ for proton-proton and antiproton-proton scattering are supposed to be equal, and both pp and $\bar{p}p$ inelastic scattering processes are described by the same Gaussian overlap function ρ^{as} , given by Eq. (4.1).

At finite energies the cross sections for pp and $\bar{p}p$ scattering become different. The reason is, as said before, that now the additional inelastic channels for the $\bar{p}p$ system which have no counterpart for the pp - system, give non-negligible contributions to the cross section. At infinite energies these contributions are believed to vanish. The bulk of these channels, which we will denote by P , consists of annihilation. Taken by themselves, i.e., the reactions $\bar{p} + p \rightarrow P$, they should in the framework of the Van Hove model be described by a Gaussian overlap function:

$$\rho_P(b) = \beta \exp(-b^2/b_1^2) \quad . \quad (5.1)$$

The remaining (non-annihilation) channels, which we call R , are at very high energies those which contribute most of the total inelastic cross section. With very good approximation they can be

assumed to be common to $\bar{p}p$ and pp scattering, except for the replacement of the antinucleon by a nucleon. In the absence of the P channels they would be described by an overlap function which, as suggested by the analysis of pp collisions, depends very little on energy. We therefore take for this function

$$\rho_R(b) \equiv \rho^{as}(b) = A \exp(-b^2/b_0^2) . \quad (5.2)$$

We have seen already (Eq. (4.1)) that $A = 0.98$ and $b_0^2 = 10$ mb. The total set of inelastic channels is made up of the sets P and R. However, the total overlap function is not simply the sum of ρ_P and ρ_R , because there will be mutual damping of one set of inelastic channels by the other. This is clearly required since $\rho_R + \rho_P$ would violate the unitarity condition unless ρ_P is practically negligible. This mutual suppression will depend on the impact parameter b . The above expressions for the two overlap functions ρ_P and ρ_R , which refer to the situation where damping of channels P by channels R and conversely is not yet taken into account, implies that the corresponding absorption is strongest for $b = 0$ and decreases with increasing b . From this we expect the mutual damping also to be strongest for $b = 0$ and to fall off rapidly towards higher b values, more rapidly than the overlap functions. This is because the damping is expected to vanish as soon as $\rho_P(b) + \rho_R(b)$ is well below the unitarity limit of one.

We describe the mutual damping of channels P and R by subtracting from the left hand sides of Eqs. (5.1) and (5.2) b dependent damping terms. These damping terms will, of course, depend also on the energy. A guess for the form of the damping terms of large values of b can be made by invoking arguments similar to those used in the theory of the absorption mode ¹⁴⁾ to explain the

sharp peaking of the angular distribution of production in quasi-two-body reactions. The difference is that in quasi-two-body reactions one has to take into account absorption by competing inelastic channels in both the incoming and the outgoing two-particle states. In our case absorption occurs only in the incoming two-particle states in reactions of the type $\bar{p} + p \rightarrow R$ and $\bar{p} + p \rightarrow P$.

Anticipating the results (5.6) below, namely that from our calculations it follows that $b_1^2 = b_0^2$, we find in this way that to very good approximation both damping terms behave for large b as $\exp(-2b^2/b_0^2)$. We assume that this Gaussian dependence on b persists for the lower b values. On the basis of this, we thus write

$$\bar{\rho}_P(b) = \beta \exp(-b^2/b_1^2) - \alpha_1 \exp(-2b^2/b_0^2) \quad , \quad (5.3)$$

$$\bar{\rho}_R(b) = A \exp(-b^2/b_0^2) - \alpha_2 \exp(-2b^2/b_0^2) \quad , \quad (5.4)$$

in which α_1 , α_2 and β are all positive.

The total overlap function is the sum of these two functions,

$$\rho(b) = \bar{\rho}_P(b) + \bar{\rho}_R(b) \quad (5.5)$$

This function becomes identical with the one given by Eqs. (4.1) - (4.3) if we make the following identifications :

$$\begin{aligned} b_1^2 &= b_0^2 = 10 \text{ mb} \quad , \quad \text{for all } p_L \geq 4 \text{ GeV}/c \quad , \\ \beta &= B \quad , \\ \alpha_1 + \alpha_2 &= B \lambda \quad . \end{aligned} \quad (5.6)$$

The total overlap function $\mathcal{P}(b)$ has, due to the damping terms, a non-Gaussian form. In the frame work of the Van Hove model this has to be interpreted as resulting from rather strong correlation effects in the inelastic final states of $\bar{p}p$ scattering when both annihilation and non-annihilation channels are present.

We can estimate the ratio α_1/α_2 by calculating the contribution σ_P of the P channels to the total inelastic cross section for antiproton-proton scattering at a given energy, using the formula

$$\sigma_P = 2\pi \int_0^{\infty} \bar{\rho}_P(b) b db, \quad (5.7)$$

in conjunction with (5.6). The main part of σ_P is the cross section for $\bar{p}p$ annihilation. Extrapolating experimental data on annihilation cross sections listed by Ferbel ¹⁵⁾ to $p_L = 12$ GeV/c, and using the result that for this value of the laboratory momentum we have $B = 0.72$, $\lambda = 1$ (see Table II) : we obtain

$$\frac{\alpha_1}{\alpha_2} \approx 6, \quad p_L = 12 \text{ GeV/c}$$

This shows that the P channels are much more strongly suppressed by the R channels than the reverse process. This is reasonable, since the R channels are the dominant ones at high energy.

A final remark concerns the first identification made in (5.6), namely $b_1 = b_0$, or more explicitly the fact that our calculations lead to good agreement with experiment when the overlap functions $\mathcal{P}_P(b)$ and $\mathcal{P}_R(b) \equiv \mathcal{P}^{as}(b)$ are proportional. This implies

that the parameters A' , defined in Eq. (3.2), are equal for the two sets of channels (each set considered in the absence of the other) :

$$A'_P = A'_R = \frac{1}{2} b_0^2, \quad (5.8)$$

independent of the energy for $p_L \gtrsim 4 \text{ GeV/c}$.

Can we draw any conclusions from this result ? Van Hove¹⁶⁾ has calculated the quantity A' for a general process in terms of model wave functions for the inelastic final state $|f(\vec{k})\rangle$ defined by Eq. (2.5). On the basis of general assumptions he estimates

$$A' = (84/N) [\langle q_0^2 \rangle / \langle q_0 \rangle^2], \quad (5.9)$$

in which N is the multiplicity (the number of secondaries), q_0 is the energy of a secondary of momentum $\vec{q}$ and ψ is determined by the transverse momentum distribution of the secondaries. The quantities within square brackets are averages over the energy distribution of the secondaries. This formula tells us that the quantity A' is essentially determined by the general structure of the jet : multiplicity and energy-momentum distributions. Equality (5.8) seems to imply that the jets corresponding to channels P and R have a very similar structure, or at least have dominant parts with the same structure. To be more explicit, consider the following reactions (N = nucleons, B = baryons) :

$$p + p \rightarrow N + N + \text{mesons} + B\bar{B} - \text{pairs} + \dots, \quad (5.10a)$$

$$\bar{p} + p \rightarrow \bar{N} + N + \text{mesons} + B\bar{B} - \text{pairs} + \dots, \quad (5.10b)$$

$$\bar{p} + p \rightarrow \text{mesons} + B\bar{B} - \text{pairs} + \dots. \quad (5.10c)$$

Reactions (a) and (b), which correspond to channels R, are believed to be the dominant inelastic channels in the asymptotic limit for pp and $\bar{p}p$ scattering respectively. As said before, except for the first particle on the right of (5.10a) and (5.10b), we believe the jets corresponding to these two reactions to be nearly identical. The third reaction is that corresponding to channels P. It is expected that at high energies the $B\bar{B}$ pairs in the above reactions do not play a significant role in determining the properties of the jets.

We have reached the conclusion in this paper that experimental data are well accounted for when the sets of channels (a) (b) and (c) give rise to the same value of the quantity A' . Accepting this conclusion does it reveal anything about the properties of the jets? The answer to this question can, in view of our present very limited knowledge about the structure of jets, both theoretically and experimentally, be only highly speculative. However, one possible way to understand why the jets in the reactions (5.10) produce the same value for the quantity A' , would be to ascribe to the jets the following properties: (i) the part "mesons + $B\bar{B}$ - pairs + ..." of the inelastic final state of reaction (c) has about the same structure, i.e., multiplicity and energy-momentum distribution, as the corresponding parts of reactions (a) and (b), and (ii) the part "mesons + $B\bar{B}$ - pairs + ..." is in all three reactions mainly responsible for the value of A' , i.e., the presence of the two nucleons (leading particles) in the final states of reactions (a) and (b) is of no considerable influence on A' . According to a discussion by Biaľas¹⁷⁾ the second property would imply that the average transverse momentum of the nucleons ($N, \bar{N}$) in reactions (a) and (b) is big as compared with the average transverse momentum of the mesons.

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FIGURE CAPTIONS

- Figure 1 Calculated values of the total cross section for $\bar{p}p$ scattering as a function of the parameter B for two values of the parameter λ , obtained from the asymptotic form of the unitarity condition. The experimental points are inserted according to Tables I and II.
- Figure 2 Calculated values of the ratio σ_{el}/σ_T for $\bar{p}p$ scattering as a function of the parameter B for two values of the parameter λ , obtained from the asymptotic form of the unitarity condition. The experimental points are inserted according to Tables I and II.
- Figure 3 Theoretical diffraction curves for $\lambda = 1$ and various values of B , calculated from the asymptotic unitarity condition. Curve 1 : $B = 0$, asymptotic curve ; Curve 2 : $B = 0.3$; Curve 3 : $B = 0.5$; Curve 4 : $B = 0.72$; Curve 5 : $B = 0.9$; Curve 6 : Best fit to proton-proton diffraction curve at 12.8 GeV/c according to Ref. ⁴).

$$X(t) = (d\sigma_{el}/dt) / (d\sigma_{el}/dt)_{t=0}.$$
- Figure 4 Elastic $\bar{p}p$ - scattering with small momentum transfers at $p_L = 8.9$ GeV/c. The experimental points are taken from Ref ⁶). The theoretical curve ($B = 0.9$, $\lambda = 0.983$) includes the $1/k^2$ correction to the asymptotic unitarity condition.

Figure 5 Elastic $\bar{p}p$ scattering with small momentum transfers at $p_L = 12.0$ GeV/c. The experimental points are taken from Ref.⁶⁾. The theoretical curve ($B = 0.72$, $\lambda = 1$) includes the $1/k^2$ correction to the asymptotic unitarity condition.

Figure 6 The overlap function $F(k, t)$ for $B = 0$ (curve 1, $p_L = \infty$), $B = 0.72$, $\lambda = 1$ (curve 2, $p_L = 12$ GeV/c) and $B = 1.3$, $\lambda = 1$ (curve 3, $p_L = 4$ GeV/c).

Figure 7 Large-angle $\bar{p}p$ scattering, $X = (d\sigma_{el}/dt) / (d\sigma_{el}/dt)_{t=0}$. The experimental points are those for $p_L = 4$ GeV/c and are taken from Ref.⁵⁾. The theoretical curves correspond to $B = 0$, $p_L = \infty$ (curve 1); $B = 0.72$, $\lambda = 1$, $p_L = 12$ GeV/c (curve 2); $B = 1.3$, $\lambda = 1$, $p_L = 4$ GeV/c (curve 3); these curves are obtained from the asymptotic unitarity condition. Curve 4 differs from curve 3 by taking into account the $1/k^2$ corrections to the unitarity condition.

FIG. 1

FIG. 2

FIG.3

FIG. 4

FIG. 5

FIG. 6

FIG. 7