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QUARK MODEL AND RELATIONS BETWEEN
HIGH ENERGY SCATTERING AMPLITUDES

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A B S T R A C T

Further consequences are discussed of a simple quark model for high energy scattering based on the additivity assumption for quark amplitudes.

Recently, a very simple quark model for high energy scattering of hadrons, based on the assumption of additivity for the two-body quark amplitudes, has met with considerable success ^{1),2),3)}. In this note we briefly show that further consequences follow from this model when combined with certain dynamical assumptions about the quark amplitudes. Similar ideas were put forward in a recent note by Lipkin ⁴⁾.

The additivity assumption enables one to write the amplitude $T_{AB}(s,t)$ for elastic scattering of hadrons A and B in the following form ³⁾ :

$$T_{AB}(s,t) = \sum_{ij} f_i^A(t) f_j^B(t) T_{ij}(s,t) , \quad (1)$$

in which T_{ij} is the amplitude for elastic scattering of quark i in hadron A with quark j in hadron B. The $f(t)$ have the significance of form factors; $f_i^A(t)$ describes the distribution of the i^{th} quark in hadron A. It should be stressed that $T_{ij}(s,t)$ for given $\sqrt{s}$ and $\sqrt{-t}$, the c.m. energy and momentum transfer for the complete A + B collision, is in general not the same depending on whether one has a meson-nucleon or a nucleon-nucleon collision ³⁾. This affects the predictions which compare these two types of reactions.

We consider first scattering in the forward direction ($t = 0$), and denote ^{*)} $\text{Im } T_{AB}(s, 0)$ by (AB) . It was first shown in Ref. ²⁾ that if in addition to additivity, i.e., Eq. (1), one assumes SU(3)

*) We consider only the imaginary parts of the amplitudes, because at high energies and small momentum transfers elastic amplitudes are believed to be dominantly imaginary.

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symmetry, the Johnson-Treiman relations

$$\frac{1}{2}[(K^+P) - (K^-P)] = (\pi^+P) - (\pi^-P) = (K^+N) - (K^-N) \quad , \quad (2)$$

are obtained. We wish to point out, however, that (2) follows from (1) under conditions weaker than $SU(3)$ symmetry. Necessary and sufficient for the Johnson-Treiman relations to follow from the additivity assumption is, apart from isospin symmetry, the condition

$$(\bar{n}p) - (np) = (\bar{\lambda}p) - (\lambda p) \quad . \quad (3)$$

Here n , p and λ denote the quarks ; n and p forming the isodoublet of strangeness zero, λ the isosinglet of strangeness -1 . Condition (3) states that the deviations from the Pomeranchuk limit for quark scattering, where $(np) = (\bar{n}p)$ and $(\lambda p) = (\bar{\lambda}p)$, for the np and λp systems should be equal. It is not necessary that the Pomeranchuk limit for these processes be reached and that charge exchange be negligible, as was stated by Lipkin ⁴⁾. Condition (3) can also be expressed by saying that the influence of annihilation channels should be as strong for $\bar{n}p$ as for $\bar{\lambda}p$ scattering.

Further relations can be obtained by making assumptions on the energy behaviour of the various quark-antiquark annihilation reactions. If one assumes that for the pn system only isosinglet annihilation is important at high energies, i.e.,

$$(\bar{n}p) - (np) \ll (\bar{p}p) - (pp) \quad , \quad (4)$$

one obtains the Freund relations ^{4), 5)}

$$(\bar{P}P) - (PP) = \frac{5}{4} [(\bar{N}P) - (NP)] = 5 [(\pi^-P) - (\pi^+P)] \quad . \quad (5)$$

For a careful comparison of the above relations with experiment, we refer to Refs. 2) and 4).

We next discuss the case of non-zero momentum transfers ($t \neq 0$). We restrict ourselves to those particular cases where the additivity formula (1) can be written in the simple form

$$T_{AB}(s, t) = F(t) \sum_{ij} T_{ij}(s, t) \quad , \quad (6)$$

where $F(t)$ does not depend on the labels A, B, i and j . We shall discuss below examples of situations in which (6) is valid. It is clear that under validity of (6) all relations between amplitudes which follow for the forward direction also hold for $t \neq 0$. In particular, the generalizations to finite t of the Johnson-Treiman relations (2) and the Freund relations (5) follow from (6) if we generalize the conditions (3) and (4) respectively to non-zero t . Condition (3) then becomes a statement referring to deviations from the generalized Pommeranchuk theorem for diffraction scattering⁶⁾ of quarks. If the conditions (3) and (4) are satisfied simultaneously for finite t , one has

$$\frac{1}{2} \left[(K^+P)_t - (K^-P)_t \right] = (\pi^+P)_t - (\pi^-P)_t = \frac{1}{5} \left[(PP)_t - (\bar{P}P)_t \right] \quad (7)$$

A possible way to check these relations (and this applies separately to the generalized Johnson-Treiman and Freund relations) is to consider the cross-over points of the various pairs of diffraction curves. It is an experimental fact⁷⁾ that for given energy each pair of differential cross-sections $d\sigma(AP \rightarrow AP)/dt$ and $d\sigma(\bar{A}P \rightarrow \bar{A}P)/dt$ with $A = P, \pi^+, K^+$, has a cross-over point in the interval $0.05 \lesssim -t \lesssim 0.25 \text{ (GeV/c)}^2$, where the difference between the cross-sections changes sign. Relations (7) are consistent with this fact. Indeed, if we assume the amplitude within the diffraction

region to be purely imaginary, which seems to be true to good approximation at high energies, then a cross-over point in one pair, giving $(AP)_t = (\bar{A}P)_t$ in that point, implies by virtue of (7) cross-over points at the same value of t for the other pairs as well, including that where $A = N$ as well as $K^\pm N$ scattering [see (2) and (5)]. In general, however, the amplitudes will have small real parts which have the effect of shifting the positions of the cross over-points. They could therefore appear at slightly different t values for the various reactions ^{*)}.

We finally discuss two cases in which (6) is valid. Both are extreme cases, distinguished by their different behaviour in the asymptotic limit ^{3),8)}. Case I is obtained when in the asymptotic limit the entire t dependence of $T_{AB}(s,t)$ given by (1) comes from $\lim_{s \rightarrow \infty} T_{ij}^{(I)}(s,t)$; i.e., in the diffraction peak the f 's are approximately constant and equal to one, hence $F(t) = 1$. This case corresponds to very transparent quarks with spatial dimensions of the same order of magnitude as those for the hadrons ³⁾.

Case II is the opposite of case I. Now the t dependence of the asymptotic total amplitude is assumed to be entirely determined by the form factors, i.e., $\lim_{s \rightarrow \infty} T_{ij}^{(II)}(s,t)$ is constant in t . In this case the quarks have spatial dimensions which are small compared to those of the corresponding hadrons. Consider first the case where A and B do not contain strange quarks. It follows from isospin invariance that in this case the form factors $f_i^A(t)$ and $f_j^B(t)$ do not depend on the quark labels i and j . The experimental fact that the slopes of the πP and PP diffraction curves tend to about

*) Even if the amplitudes are purely imaginary relations (7) do not necessarily predict the positions of the cross-over points for a given energy to be at the same t value for meson-nucleon and nucleon-nucleon scattering. This is due to the effect mentioned in the text following Eq. (1). This effect, however, is not very important because the positions of the cross-over points will not change very much with the energy.

the same asymptotic limit then implies that $f^\pi = f^P = f$. Hence we obtain (6) with $F(t) = [f(t)]^2$; from this the Freund relations for non-zero t follow.

To include strange particles in case II, as is required to obtain the generalized Johnson-Treiman relations, one has to make an additional assumption, namely that the form factors f_i^A do not depend on i even when i refers to the λ quarks. This is not necessarily so unless $SU(3)$ symmetry is assumed for the form factors. Making this assumption, one then concludes $f^\pi = f^K$ from the near equality of the asymptotic slopes of the πP and KP diffraction curves. This leads to (6) for all hadrons.

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R E F E R E N C E S

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