CERN LIBRARIES, GENEVA

CM-P00057285

IMPLICATIONS OF I^2 AND I_3 CONSERVATION

Norman M. Kroll *)

CERN -- Geneva

*) NSF Senior Postdoctoral Fellow. On leave from the
University of California, San Diego, La Jolla, U.S.A.

In a recent publication ¹⁾, Vander Velde has suggested the possibility that violation of G conservation in elementary particle reactions may be substantially more severe than that of I^2 and I_3 . Since G conservation is an immediate consequence of charge conjugation invariance and $\underline{I}$ (i.e., full SU_2 invariance) invariance, one is led to ask to what extent I^2 and I_3 invariance ("Partial Charge Independence - abbreviated PCI) is a weaker constraint than full SU_2 invariance. We shall show below that any S matrix which satisfies the most elementary physical requirements, including, in particular, the cluster decomposition property ²⁾, and commutes with I^2 and I_3 , also commutes with $\underline{I}$.

We note first of all that the introduction of a mass difference among the various components of an isospin multiplet into a Hamiltonian is itself a violation of PCI in that such a term does not commute with I^2 (except, of course, for the trivial case of a one-particle theory). This observation suggests the assertion: particles belonging to the same isospin multiplet in a PCI theory must have the same mass. We wish to show that this is the case without reference to a Hamiltonian. In order that PCI be defined for a theory, it is necessary that the one-particle states be grouped into multiplets which transform among themselves in the conventional way under the operations of the full SU_2 group. If the masses within a multiplet were equal, one would naturally take these transformations to connect states having the same momentum, and, therefore, also the same energy and velocity. Once the masses are different, however, one cannot have all of them the same, and it is easy to see that the only choice consistent with either Gallilean or Lorentz invariance is to connect states of the same velocity. It is also necessary to define the effect of the SU_2 transformations on asymptotically multiparticle states, and it is clear that the conventional meaning of I^2 and I_3 conservation requires that these transform as products of the transformations for the constituent one-particle states. It is then easy to see that energy and momentum conservation in collisions leads to non-conservation of I^2 unless the masses of the different I spin components

2.

of the constituent particles are the same. This may perhaps be made clear most easily by means of an example. Consider the collision of two π mesons with an α particle. An incident $\pi^+ \pi^-$ and $\pi^0 \pi^0$ state can be combined to form an $I^2=0$ state. In this mixture we take the neutral and charged π 's to have the same velocity (i.e., p/E). If, however, the neutral and charged masses are different, the velocities in the final $(\pi^+ \pi^- \alpha)$ and $(\pi^0 \pi^0 \alpha)$ states must be different so that the final state is not an eigenstate of I^2 .

For our purposes the cluster decomposition property (CDP) can be most conveniently stated as follows. Let α and α' refer to two distinct sets of particles in some specified initial states, β and β' to distinct sets in final states. Let $S_{\alpha, \beta}$, $S_{\alpha', \beta'}$ be the S matrix elements describing the indicated transitions. Then the S matrix element describing the transition from an initial state, $\alpha \alpha'$ to a final state, $\beta \beta'$ satisfies :

$$S_{\alpha \alpha', \beta \beta'} = S_{\alpha \beta} S_{\alpha' \beta'} + \Delta S$$

where ΔS contains no combination of S functions guaranteeing the separate conservation of energy and momentum in the $\alpha \beta$ and $\alpha' \beta'$ transitions. We assert that an S matrix which satisfies PCI in the sense indicated in the preceding paragraph, and CDP as well, is fully charge independent. Proof : since S satisfies PCI it may be written

$$S = \sum_{i=0}^{\infty} \sum_{\mu=-i}^i S(i, \mu)$$

where the submatrices $S(i, \mu)$ are diagonal in I^2 and I_3 . Now consider a pair of reactions $\alpha \rightarrow \beta$: $\alpha' \rightarrow \beta'$ belonging to a particular i and i' : to these correspond $S_{\alpha, \beta}(i, \mu)$: $S_{\alpha', \beta'}(i', \mu')$

According to CDP,

$$\sum_{\alpha\alpha', \beta\beta'} (i, \nu; i', \nu') (i, \mu; i', \mu') = \delta_{\mu\nu} \delta_{\mu'\nu'} S_{\alpha\beta} (i, \mu) S_{\alpha'\beta'} (i', \mu') + \Delta S$$

According to PCI the left-hand side, regarded as a matrix in $\nu\nu'; \mu\mu'$ must commute with I^2 and I_3 . This means that

$$\sum_{\nu\nu', \mu\mu'} C_{i_2, \nu\nu'}^{ii'} C_{i_1, \mu\mu'}^{ii'} S_{\alpha\alpha', \beta\beta'} (i\nu, i'\nu') (i\mu, i'\mu') \delta_{\mu, \mu+\mu'} = 0$$

for $i_1 \neq i_2$, where the coefficients are the usual Clebsch-Gordan coupling coefficients. Because of the particular way in which energy momentum conserving δ functions appear in the two terms in the right-hand side, they must satisfy a similar condition separately, i.e.,

$$\sum_{\mu\mu'} C_{i_2, \mu\mu'}^{ii'} C_{i_1, \mu\mu'}^{ii'} \delta_{\mu, \mu+\mu'} S_{\alpha\beta} (i, \mu) S_{\alpha'\beta'} (i', \mu') = 0$$

for $i_1 \neq i_2$. It follows immediately from the property

$$\sum_{\mu} C_{i_1, \mu, \mu-\mu}^{ii'} C_{i_2, \mu, \mu+\mu}^{ii'} = \delta_{i_1, i_2}$$

that

$$S_{\alpha\beta} (i, \mu) S_{\alpha'\beta'} (i', \mu') = S_{\alpha\beta} (i, \mu+\nu) S_{\alpha'\beta'} (i', \mu'-\nu) \quad (1)$$

for all ν such that $|\mu+\nu| \leq i$, $|\mu'-\nu| \leq i'$.

Since this formula must hold for all possible choices of the states α, β and α', β' it is essentially obvious that it must imply

$$S_{\alpha\beta}(i, \mu) = S_{\alpha'\beta'}(i, \mu') \quad (2)$$

It is especially obvious if the theory contains an $i = \frac{1}{2}$ particle. One then takes $S_{\alpha', \beta'}$ to be the trivial one-particle S matrix for that particle. Then $S_{\alpha', \beta'}(\frac{1}{2}, \frac{1}{2}) = S_{\alpha', \beta'}(\frac{1}{2}, -\frac{1}{2})$ is an immediate consequence of energy momentum conservation and unitarity. Equation (1) then yields Eq. (2) immediately. Since Eq. (2) is equivalent to full SU_2 invariance, the proof is now complete.

We take the view that any physically acceptable S matrix satisfies CDP. If the theory admits a Hamiltonian description and the Hamiltonian satisfies PCI, then our discussion shows that the S matrix is fully charge independent. We note, in this connection, that the addition of an arbitrary function of I_3 to the Hamiltonian has no effect on the S matrix. [Because such a term superficially destroys the relativistic invariance of the theory, one must make some obvious modifications in the construction of the Lorentz transformation operators, and define particle velocity by $\partial E / \partial p_i$.] It seems very likely that one cannot construct any non-constant, local, Lorentz invariant, PCI Hamiltonian density that is not fully charge independent.

The results of this note may be less conclusive for the actual physical situation than they are for the kinds of models one conventionally uses. There is no experimental evidence supporting PCI for the entire S matrix. The experimental evidence supports PCI for the cluster amplitudes ²⁾ only. The S matrix as a whole becomes involved if one attributes PCI in the cluster amplitudes to PCI in the Hamiltonian, for then PCI is implied for the entire S matrix. Thus our arguments do not bear upon the possibility of a theory with cluster amplitudes that

are PCI but not fully charge independent (perhaps, for example, as the result of dynamical enhancement). This restricted form of PCI still requires equal masses for members of the same multiplet, and we note that there is no experimental evidence against this requirement. The reactions proposed by Vander Velde do imply quite different fast decay reactions for members of the same charge multiplet. The fact that such behaviour is not easy to reconcile with equal masses may have some bearing on the plausibility of the suggestion, but is certainly not conclusive. We wish, therefore, to emphasize that the observation of a large G conservation violation would have most important implications on the origins of charge independence.

The author wishes to express his appreciation for the opportunity to work in the CERN Theoretical Study Division.

REFERENCES

- 1) J.C. Vander Velde, Phys.Rev.Letters 16, 69 (1966).
- 2) E.H. Wichmann and J.H. Crichton, Phys.Rev. 132, 2788 (1963).