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AN ADDITIONAL RADIATIVE CORRECTION TO WIDE ANGLE
PAIR PRODUCTION

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ABSTRACT

In pair photoproduction the reaction energy is inferred from the pair energy. We derive the radiative correction arising from the shift of reaction energy when the pair radiates an undetected photon. The effect is significant.

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Recent experiments on the photoproduction of wide angle electron-positron pairs from carbon ^{1),2)} indicate a disagreement with the theoretical calculation ³⁾. As the process is dominated by electromagnetic effects, in particular by the "Bethe-Heitler" mechanism of Fig. 1a, such a disagreement implies a violation of quantum electrodynamics. The violation may be confined to high mass pairs (because of the normalization uncertainty) corresponding to a violation at small distances.

The importance of such a result has led to careful evaluation of smaller electromagnetic ³⁾ and nuclear ⁴⁾ effects. The former were calculated ^{1),3)} using the formula ⁵⁾ :

$$\sigma_{rad} = - \frac{2\alpha}{\pi} \ln \left(\frac{Q_M}{m} \right)^2 \left(\ln(k/2\Delta E) - \frac{13}{12} \right) \sigma_{BH} \quad (1)$$

where $(-Q_M^2)^{\frac{1}{2}}$ is the pair mass, m the electron mass, k is the photon energy, $\Delta E = k_M - k$ (k_M is the peak of the beam spectrum) is the effective resolution ⁶⁾, and $\sigma_{BH}(p_+, p_-)$ is the cross-section of the Bethe-Heitler process.

The validity of Eq. (1) depends on :

- a) the accuracy of the second order calculation ^{7),8)} and
- b) the separability of the Bethe-Heitler amplitude from the radiative contribution ⁷⁾.

Because of the large value of ΔE , condition a) is satisfied. But condition b) is not well satisfied, because of the rapid dependence of σ_{BH} on k and θ (the electron-positron half-angle). In this letter we give an expression for the extra correction due to the latter effect and make a rough numerical evaluation. We find that the effect is significant and tends to bring the theory into line with the experimental normalization and in part with its Q_M dependence. An accurate evaluation depends on the specific "acceptance function" of the apparatus of a given experiment and we have not attempted this here.

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The dominant radiative corrections are due to one-photon corrections to the external electron and positron lines. Those involving real radiation are shown in Figs. 1b and 1c. As the experimental definition is $k = E_+ + E_-$, the Bethe-Heitler production photon is of energy $k + \epsilon$ rather than k ⁹⁾. As the radiated photons are predominantly in the electron or positron direction, we also take the effective four-momenta at the Bethe-Heitler vertex as $p_{+(-)}^{\text{eff}} = p_{+(-)} + \epsilon$ and $p_{-(+)}^{\text{eff}} = p_{-(+)}$ when the photon is emitted by the external positron (electron). This implies no change in angle. The virtual one-photon correction leaves the energy of the Bethe-Heitler vertex unchanged, the virtual vertex correction altering p_+ and p_- only weakly, after integration. As a result, only the real radiative corrections are substantially affected by the variation of σ_{BH} with k . It follows that

$$\sigma_{\text{rad}} = \sigma_{\text{rad}}^s + \sigma'_{\text{rad}} \quad (2)$$

where

$$\sigma'_{\text{rad}} = \int_0^{\Delta \epsilon} d\epsilon \Delta \sigma_{\text{BH}}(p_+, p_-, \epsilon) \left(2\alpha/\pi\epsilon \right) \ln - (Q_M/m)^2 \quad (3)$$

and

$$\Delta \sigma_{\text{BH}} = \frac{1}{2} \sigma_{\text{BH}}(p_+ + \epsilon, p_-) + \frac{1}{2} \sigma_{\text{BH}}(p_+, p_- + \epsilon) - \sigma_{\text{BH}}(p_+, p_-) \quad (4)$$

The factor $(2\alpha/\pi\epsilon) \ln - (Q_M/m)^2$ represents the probability of emitting a photon of energy ϵ .

Because $\lim_{\epsilon \rightarrow 0} \Delta \sigma_{\text{BH}} = 0$, the integral in Eq. (3) is finite. We note that in fact the singularity of the probability factor is weaker than ϵ^{-1} because of many photon corrections ^{7), 8)}. In the present case only a small contribution comes from ϵ small enough to make many photon corrections important.

In a form suitable for present experiments (i.e., for $E_+ \approx E_-$, $\theta_+ \approx \theta_-$ and $\phi \approx 0$) we have ^{1),3)}

$$\sigma_{BH}(p_+, p_-) = \frac{\alpha^3}{2\pi^2} \frac{8}{k^2 \Theta^6} S(k) \left[1 + \frac{\frac{1}{4} \gamma^2}{\left\{ \gamma^2 + \frac{\Theta^2}{4} \right\}^2} \right] \quad (5)$$

where

$$\gamma^2 \equiv \left(\frac{\theta_+ - \theta_-}{2\Theta} + \frac{E_+ - E_-}{E_+ + E_-} \right)^2 + \sin^2 \frac{\phi}{2} \quad (6)$$

determines the asymmetry and $s(k)$ is the beam spectrum. E_+ (E_-) refers to the energy of the positron (electron), θ_+ (θ_-) is the angle of the positron (electron) with respect to the beam; $\Theta \equiv \frac{1}{2}(\theta_+ + \theta_-)$; and ϕ is the angle between the positron beam plane and the electron beam plane.

Taking $s(k+\epsilon) = k(k+\epsilon)^{-1}s(k)$, we have

$$\Delta \sigma_{BH}(\epsilon) = \sigma_{BH}(p_+, p_-) \left[\left(1 + \frac{\epsilon}{k}\right)^{-4} \left\{ 1 + \frac{\frac{1}{4} \gamma^2}{\left(\gamma^2 + \frac{\Theta^2}{4}\right)^2} \right\}^{-1} \left\{ 1 + \frac{\frac{1}{4} \gamma_\epsilon^2}{\left(\gamma_\epsilon^2 + \frac{\Theta^2}{4}\right)^2} \right\} - 1 \right] \quad (7)$$

where

$$\gamma_\epsilon^2 = \gamma^2 + \frac{\epsilon^2}{(k+\epsilon)^2} - \frac{\epsilon(E_+ - E_-)^2}{k(k+\epsilon)^2}$$

A term linear in ϵ has dropped from the asymmetric contribution after averaging over the emission from the two lines.

The rapid dependence of Eq. (7) on $E_+, E_-, \theta_+, \theta_-$ and ϕ shows that the result is sensitive to the mean values and also to the experimental acceptance function of these variables. This is in contrast with σ_{rad}^s which depends logarithmically on k and θ . The term $(1 + \epsilon/k)^{-4}$ arising from the energy dependence of σ_{BH} at symmetry makes a negative contribution to $\Delta \sigma_{\text{BH}}$ ($\epsilon > 0$), while the asymmetry contribution makes a positive contribution near the minimum. For $k_M - k < k$ the latter contribution is the smaller, but is probably of opposite sign to the former one. To obtain the qualitative features of σ'_{rad} we ignore the asymmetry term and consider a narrow acceptance function (probably overestimating the magnitude of the effect, but not necessarily its Q_M dependence)

$$\frac{\sigma'_{\text{rad}}}{\sigma_{\text{BH}}(P_+, P_-)} \approx \frac{2\alpha}{\pi} \ln - (Q_M/m)^2 \int_0^{\Delta E} \frac{d\epsilon}{\epsilon} \left[\left(1 + \frac{\epsilon}{k}\right)^{-4} - 1 \right] \quad (8)$$

Although this is easily evaluated, we again apply $\epsilon < k$ (valid for most of the experimental data) and evaluate to lowest order in $\Delta E/k$.

$$\sigma'_{\text{rad}} / \sigma_{\text{BH}} = -4 \left(\frac{2\alpha}{\pi} \right) \left(\frac{k_M}{k} - 1 \right) \ln - (Q_M/m)^2 \quad (9)$$

Equation (9) expresses the fact that the probability of the pair emitting a photon of energy ϵ is $[(2\alpha k)/(\pi \epsilon)] \ln - (Q_M/m)^2$ while $\Delta \sigma_{\text{BH}} \approx -4(\epsilon/k) \sigma_{\text{BH}}$ in the interval $0 < \epsilon/k < [(k_M/k) - 1]$. It overestimates Eq. (8) by $\sim 25\%$ for $k = \frac{1}{2}k_M$.

In the experiment of Ref. 1), $0.05 < (2\alpha/\pi) \ln - (Q_M/m)^2 < 0.07$ and $1/8 \leq [(k_M/k) - 1] \leq 1.75$. The above estimate of Eq. (8) then gives $-0.3 \leq (\sigma'_{\text{rad}}/\sigma_{\text{BH}}) \leq -0.03$ ¹⁰⁾. This establishes that σ'_{rad} is generally larger than σ_{rad}^s and may be about half the magnitude of the discrepancy noted in Refs. 1) and 2), in the estimate made here. This does not remove the discrepancy but two qualitative features should be noted.

i) Equation (8) decreases or removes the normalization difficulty ^{1),2)} by reducing the predicted value. ii) Larger values of $(-Q_M^2)$ are correlated experimentally ¹⁾ with smaller k_M/k . Consequently the correction $|\sigma_{\text{rad}}/\sigma_{\text{BH}}|$ decreases with increasing $(-Q_M^2)$ for some of the experimental points. This tends to decrease the variation of experiment to theory as a function of $(-Q_M^2)$ or of k . This is consistent with the smaller variation seen in the experiment of Ref. 2), in which k_M/k was not varied.

In conclusion, we have given an expression for a radiative correction σ_{BH} with the energy of a real emitted photon. The expression is only roughly evaluated and the numerical result is not directly comparable to experiment. The effect appears to be significant and may explain discrepancies between experiments and with the theoretical normalization at small $(-Q_M^2)$. It tends to decrease the deviation of the theoretical Q_M dependence from the experiment, but does not remove the discrepancy in the present estimate.

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- 9) If the beam photon energy was experimentally fixed then Eq. (1)
would accurately give the radiative correction, as the Bethe-Heitler
vertex would be determined by a fixed energy. This is presumably
what the derivation of Ref. 3) had in mind.
- 10) The variation in σ'_{rad} is almost entirely due to the variation
of k_M/k in Ref. 1).

8.

FIGURE CAPTION

Photoproduction of an electron-positron pair from a nucleus.

a) The dominant Bethe-Heitler process. b) and c) Lowest order radiative corrections with real photons.