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NEW EVIDENCE FOR THE SIGMA MESON

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A B S T R A C T

Assuming only dispersion relations, backward πp elastic scattering experiments give an F ratio probability of $10^8 : 1$ for the presence of a $\pi\pi$ S_0 wave, and require it to have a broad σ resonance.

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Most existing evidence on the pion-pion interaction comes from the analysis of $\pi N \rightarrow \pi \pi N$ by the one-pion-exchange model. Though this is often called "experimental", it depends on a model which nobody believes exact, whose limits of validity are uncertain, and which is known from the Treiman-Yang test frequently to break down. All the other usual sources of information on the $\pi\pi$ S wave are even more model-dependent (though K_{e4} would cease to be so with ten times more events). This makes it surprising that so little attention has been paid to the backward dispersion relations for pion-nucleon elastic scattering. The only previous evaluation was by Atkinson in 1962 ¹⁾, using πp data which were only accurate up to 120 MeV.

The πp backward dispersion relations follow from the Mandelstam representation. They do not involve the double spectral region, and are physical at both $\pm\infty$. If we evaluate them experimentally for the (+) isospin combination ²⁾, and extrapolate to the left-hand cut, we get the $T = 0$ backward amplitude for $\bar{N}N \rightarrow \pi\pi$. This has a big unphysical region between the $\pi\pi$ and $\bar{N}N$ thresholds. The phase there can be identified with the $\pi\pi$ S_0 phase ²⁾, subject only to two limitations :

- a - The $\pi\pi$ mass at which the phase identification is made must be low enough for the process $2\pi \rightarrow 4\pi$ to be negligible. This is certainly so below 558 MeV (its threshold) and probably below 900 MeV (from comparison of 2π and 4π mass spectra in various experiments).
- b - The $\pi\pi$ mass must also be low enough for the $T = 0$ D wave to be neglected. The centrifugal barrier ensures this near the $\pi\pi$ threshold. From the width of the f^0 it may be true up to ~ 1100 MeV.

The (-) isospin combination gives the $\pi\pi$ P_1 phase under similar conditions.

We write the πp backward dispersion relations in the variable k^2 (the πp centre-of-mass momentum squared). Their singularities are then : the right-hand cut from 0 to $+\infty$, the nucleon pole at $k^2 = -0.9945 \mu^2$, and the left-hand cut from $-\infty$ to $-\mu^2$. Regge pole arguments and high energy experiments both indicate that they should easily converge without subtractions.

The $\pi\pi$ phase at a mass m is given by the phase at $k^2 = -m^2/4$ on the left-hand cut. Though there is an extrapolation from the πp physical region, the distance is short compared to the extent of the experiments. However $\pi\pi$ phases at different masses are got by extrapolating different distances. Therefore, reconstructing them from πp experiments is a complicated process, rather like the Fourier analysis of a wave-form ³⁾. Thus, though the theory is firm and the experiments copious, the numerical calculation is delicate. We sought to overcome this by doing the calculation in many different ways, as a check on systematic errors. In experimental terms, we have excellent normalization, but poor energy resolution, and our efficiency decreases with increasing $\pi\pi$ mass. This is the opposite of the peripheral method.

There is no optical theorem for backward scattering, so πp phase-shift analysis is needed to separate the experiments into their real and imaginary parts. Phase-shift error matrices always have large off-diagonal elements. This means that certain combinations will be much better determined than others (a fact whose importance for the evaluation of dispersion relations has not previously been appreciated). In particular the no-flip amplitudes in the forward and backward directions are always better determined than most individual partial waves, and usually an order of magnitude better than the corresponding flip amplitudes. This is because of their direct relation to observables (see Eq. (2) below). We therefore used the no-flip backward dispersion relation, for the amplitudes

$$\begin{aligned}
 F^{(+)}(k^2) &= \frac{1}{M} \left[A^{(+)}(s, -4k^2) + \frac{M\omega}{E} B^{(+)}(s, -4k^2) \right], \\
 F^{(-)}(k^2) &= \frac{E}{M\omega} \left[A^{(-)}(s, -4k^2) + \frac{M\omega}{E} B^{(-)}(s, -4k^2) \right]
 \end{aligned}
 \tag{1}$$

(the notations on the right are standard ²⁾). They can be proved free of kinematic singularities, and finite at all thresholds. Their errors were calculated omitting the partial wave dispersion relation constraints ⁴⁾ from the phase-shift analysis, and using the full error matrix. They are related to the experimental backward πN differential cross-sections (DCS's) by

$$\frac{d\sigma}{d\Omega}(180^\circ)(\pi^\pm p \rightarrow \pi^\pm p) = \frac{E^2 M^2}{(4\pi)^2 s} \left| F^{(+)}(k^2) \mp \frac{\omega}{E} F^{(-)}(k^2) \right|^2,$$

$$\frac{d\sigma}{d\Omega}(180^\circ)(\pi^- p \rightarrow \pi^0 n) = \frac{2\omega^2 M^2}{(4\pi)^2 s} \left| F^{(-)}(k^2) \right|^2.
 \tag{2}$$

In most of the fits, we only used the phase shift data up to 600 MeV. Above this, where there are still ambiguities ⁵⁾, the raw backward DCS's were used to give non-linear constraints by Eq. (2). This DCS data extends up to 7.9 GeV.

The $F^{(\pm)}$ dispersion relations were therefore evaluated together. We used a conformal mapping method ³⁾, applied to both

cuts separately. This approximates by a series of functions for each of which the dispersion integral can be performed exactly, which have the correct threshold behaviour, and which can be shown to converge under very general conditions. As a check on the numerical errors, the fit was done in 184 different ways (the statistical errors being calculated on each). Among other things, we varied the numbers of right and left-hand fitting functions, moved the positions of their conformal zeros³⁾, added terms with cusps at various two particle thresholds, varied the energy at which phase shift data was replaced by DCS data from 300 to 1311 MeV, tried omitting individual pieces of data, using different πp phase shifts, increasing the error on the πp coupling constant, fixing the ρ at its observed mass, including $p\bar{p} \rightarrow \pi\pi$ annihilation data, etc. Though our dispersion relations were unsubtracted, different fits were found to differ violently at high energies above the last experimental point, so this uncertainty is also allowed for.

Figure 1 shows the $\pi\pi$ S_0 phase shifts resulting from a few of our 184 fits. Every one of them goes through 90° . The same is true of all the other fits not shown - whenever the $\pi\pi$ S wave was allowed to resonate it did so, and when this was not allowed an enormous increase in χ^2 resulted. Remembering that this graph is actually the phase of the backward $T = 0$ amplitude for $N\bar{N} \rightarrow \pi\pi$, which is a sum of S, D, G, etc. waves, the second resonance near 1300 MeV in some fits is probably the f^0 .

As some people still doubt the existence of a $\pi\pi$ S wave, we also tried fits with the left-hand cut of the $F^{(+)}$ dispersion relation omitted. χ^2 thereupon changed from 379 to ≥ 2234 (in fits identical in all other respects). The F ratio test gives the probability for the presence of a $\pi\pi$ S wave as over $10^8 : 1$. This evidence is plainly of a higher statistical weight than that from all other sources put together. The increase in χ^2 came almost entirely

from the πp $F^{(+)}$ data below 600 MeV, which shows it is not due to the omission of the distant f^0 . We also tried constraining the $\pi\pi$ S wave phase to forms with no resonance, or with just a narrow Σ resonance, but could only do so with a drastic increase in χ^2 (thus the resonance is not due to our parametrization).

Increasing the number of left-hand variables beyond that required statistically, gave more $\pi\pi$ resonances. So we cannot exclude the possibility of two narrow resonances instead of one broad one.

χ^2/N_f in our various fits ranges from 1.6 to 6.4 (with up to 53 variables). Figure 1 thus shows pessimistic limits on the mass and width of the Σ^- . We can get better values by using only those solutions which fit the $F^{(+)}$ data up to 600 MeV really well. These give two sorts of $\pi\pi$ S waves (excluding ones with several resonances) :

- a - The scattering length is positive ($\sim 0.7 \mu^{-1}$) with a resonance at 430 ± 70 MeV of width 400^{+400}_{-150} MeV.
- b - The scattering length is negative (and sometimes very large), the phase passes through 0° near 350 MeV, and resonates at 680 ± 85 MeV, with a width 750 ± 50 MeV.

The ρ in our $F^{(-)}$ fits is less well determined than our Σ , due mainly to its higher mass.

The evidence favouring a Σ resonance, rather than just a large scattering length, seems to come chiefly from πp data on the upper slope of the $N^*(1236)$ resonance, which explains why it was not obtained in earlier analyses ^{1),6)} (though they did find the ambiguity in the scattering length). A $\pi\pi$ S wave resonance was suggested by Isaev et al. ⁷⁾ but this was weak, being based on the static model and very low energy πp data only. We think the present work puts the matter beyond reasonable doubt.

Compared to ours, all other existing evidence on the $\pi\pi$ S wave is very model-dependent, and of little statistical weight (note our 10^8 F ratio probability). Many peripheral experiments indicate a large S wave at the ρ mass, which is compatible with our results. However, any attempt to obtain S waves at lower energies peripherally⁸⁾, depends crucially on the factor used to correct the normalization. As is well known, such a factor is needed to give the ρ its correct height, and there is no reason to suppose it energy-independent or the same for S and P waves.

As for K_{e4} decay, Kacser et al.⁹⁾, on the basis of only 69 events and a very model-dependent calculation, claimed to exclude a σ resonance. However, their work assumed a narrow σ , and even taking it at face-value, their fit appears to improve rapidly with increasing width.

Low energy pion production experiments also favour a σ ¹⁰⁾. The absence of other unambiguous evidence is not surprising, since an S wave resonance 400 MeV broad would be very difficult to pick out from phase space.

All dynamical calculations on the $\pi\pi$ interaction have shown that an S wave resonance requires a CDD pole¹¹⁾. This means it is elementary as far as the $\pi\pi$ system is concerned, though it could still be a bound state in other channels (e.g., $N\bar{N}$, in which a model of Ball et al.¹²⁾ gives both π and σ as bound states). In either case, it has the same degree of elementarity as the pion itself. The present work is therefore the first clear evidence for a new elementary particle.

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FIGURE CAPTION

Figure 1 - The $\pi\pi$ S_0 phase shift resulting from a few of our fits.