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CONSISTENCY OF PION-PION PHASE SHIFTS WITH ANALYTICITY

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Recently, a phase shift analysis of pion-pion scattering has been proposed ¹⁾. We wish to point out that the usual analyticity assumptions, in particular the Mandelstam representation, but also much weaker analyticity assumptions proposed by Jin and the author ^{2),3)} indicate that the low energy behaviour of the D wave phase shifts may have to be corrected.

The analyticity assumption is that the nearest singularities to the physical region are due to two pion exchange. Technically this means that the $\pi_0 \pi_0 \rightarrow \pi_0 \pi_0$ amplitude, which is the one where the contradiction might occur, has the following properties :

- 1) $F_{\pi_0 \pi_0}(s, t, u)$ with s = square of c.m. energy for $s > 4\mu^2$, $t = -2k^2(1 - \cos\theta)$, k momentum, θ scattering angle, and $s+t+u = 4\mu^2$, satisfies fixed t dispersion relations, not only for $t \leq 0$ but also for $|t| \leq 4\mu^2$.
- 2) the scattering amplitude for s real $> 4\mu^2$ can be prolonged from $t=0$ to $t=4\mu^2$, inside a complex neighbourhood of the real t axis;
- 3) due to crossing symmetry this holds for any permutation of s, t, u .

These assumptions are obviously satisfied if Mandelstam representation is valid. Now, from these assumptions, Jin and the author deduced that fixed s dispersion relations hold with two subtractions for $0 \leq s < 4\mu^2$, i.e., below the s threshold. This means that the Froissart-Gribov representation ⁴⁾ for partial waves holds for $\ell \geq 2$ $0 \leq s < 4\mu^2$:

$$f_2^{\pi_0 \pi_0}(s) = \frac{4}{\pi(4-s)} \int_4^\infty A_t(s, t') Q_2 \left[\frac{2t'}{4-s} - 1 \right] dt' \quad (1)$$

where u - t crossing symmetry has been used and A_t is the absorptive part in the t channel. Now, notice that $A_t(s, t')$ is positive for

2.

$0 \leq s \leq 4\mu^2$ because its Legendre polynomial expansion in the t channel converges, from assumptions 1) and 2) and has positive coefficients, due to unitarity.

In the neighbourhood of $s = 4\mu^2$, α_2 can be replaced by its asymptotic expression :

$$f_2^{\pi_0\pi_0}(s) \simeq \frac{(s-4)^2}{2\pi} \frac{\Gamma(3)}{\Gamma(7/2)} \int_4^\infty \frac{A_t(s', t') dt'}{(2t')^3} \quad (2)$$

Now two things can happen : either the integral has finite positive limit as $s \rightarrow 4\mu^2$ or it goes to $+\infty$. The second possibility can be excluded if the S waves have normal threshold behaviour, by using fixed t dispersion relations and elastic unitarity of the $T=0$ and $T=2$ components. The argument is rather lengthy and will be presented elsewhere. The conclusion therefore is that if the S waves have normal threshold behaviour

$$\lim_{s \rightarrow 4} \frac{f_2^{\pi_0\pi_0}(s)}{(s-4)^2}$$

exists and is positive.

Therefore

$$\lim_{s \rightarrow 4} \frac{\frac{1}{3} f_2^{T=0}(s) + \frac{2}{3} f_2^{T=2}(s)}{(s-4)^2} > 0 \quad (3)$$

This condition is not obviously violated by the analysis of Ref. 1) in the sense that the D wave phase shifts are too small to be estimated in the region $0.28 \text{ GeV} < \sqrt{s} < 0.58 \text{ GeV}$. However, from 0.58 GeV to 1 GeV , the combination $\frac{1}{3}\delta_2^0 + \frac{2}{3}\delta_2^2$ is negative and becomes later on positive for $\sqrt{s} > 1 \text{ GeV}$. Therefore if one sticks to the analysis, the only way to reconcile it with the theory is to assume that $\frac{1}{3}\delta_2^0 + \frac{2}{3}\delta_2^2$ vanishes twice in the interval $0.28 \text{ GeV} < s < 1.2 \text{ GeV}$. This, we think, is difficult to believe because one expects $\frac{1}{3}\text{Re}f_2^0 + \frac{2}{3}\text{Re}f_2^2$ to stay positive for some time above threshold. Indeed the representation (1), if there is anything true in the application of Regge's ideas to elementary particle scattering (remember that here we are in the low energy region where the potential description can make sense), is expected to hold as long as we do not meet a $J=2, T=0$ or $T=2$ resonance. Then in (1) the nearest part of the integrand dominates, and we know from unitarity that $A_t(s, t')$ is positive for t' close to $4\mu^2$ [at $s=16\mu^2$, $A_t(s, t')$ is certainly positive up to $t'=16\mu^2$]. So we believe that the "signs of the D waves should be slightly improved".

To conclude, let us indicate some more stringent conditions on the D wave scattering lengths once the S and P waves are known up to some energy s_0 . These come from the positivity properties of the absorptive parts and from the positivity of certain crossing matrix coefficients :

$$1) \lim_{s \rightarrow 4} \frac{\frac{1}{3}f_2^0(s) + \frac{2}{3}f_2^2(s)}{(s-4)^2} > \frac{\Gamma(3)}{2\pi\Gamma(7/2)} \int_4^{s_0} \frac{\frac{1}{3}\text{Im}f_0^0(s') + \frac{2}{3}\text{Im}f_0^2(s')}{(2s')^3} ds'$$

$$2) \lim_{s \rightarrow 4} \frac{f_2^{T=0}(s)}{(s-4)^2} > \frac{\Gamma(3)}{2\pi\Gamma(7/2)} \int_4^{s_0} \frac{\frac{1}{3}\text{Im}f_0^0(s') + \frac{5}{3}\text{Im}f_0^2(s') + \left[1 + \frac{8}{s'-4}\right]\text{Im}f_1^1(s')}{(2s')^3} ds'$$

Numerical estimates will be published later.

R E F E R E N C E S

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