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HIGHER ORDER CORRECTIONS TO THE Λ BINDING ENERGY
IN NUCLEAR MATTER

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A B S T R A C T

The convergence of the usual expansion for the well depth D of a Λ particle in nuclear matter is examined for a pion-exchange potential and a square well potential. Terms of second order in the Λ nucleon potential tail are found to give important contributions to D .

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1. INTRODUCTION

The nuclear well depth D of a Λ particle determines the Λ binding energy in nuclear matter. It is a quantity of some recent interest ¹⁾. The experimental value of D is ²⁾

$$D \approx (28 \pm 3) \text{ MeV} , \quad ^3)$$

The theoretical calculations which use hard core type Bethe-Goldstone wave functions but treat the tail only in first order ^{4),5),6)} give values between 30 and 40 MeV ; they are based on potentials fitted to the Λ nucleon interaction in light hyperfragments. These D values are roughly 5 to 10 MeV too high. However - as pointed out by Dalitz ¹⁾ recently - the calculated D values depend on many still uncertain quantities, e.g., the effective Λ and nucleon mass and the coupling between the Λ nucleon and the Σ nucleon channel, the last showing a slight influence on the Λ nucleon potential.

Dabrowski and Köhler ⁷⁾ draw attention to the rearrangement effect which reduces D by approximately 4 to 6 MeV [see also ¹⁾]. As the P state contribution is very large, a strong suppression of the interaction in odd ℓ states was proposed ⁴⁾ to reduce the theoretical value of D ; this suppression was not confirmed by the analysis of the low energy Λ nucleon scattering data ⁸⁾.

In the nucleon-nucleon case Hoenig and Lomon ⁹⁾ found contributions to the nucleon well depth D_N of roughly +10 MeV from second order terms in the potential tail and overlap contributions between the core and the tail.

Here we present a second order calculation for a low energy effective Λ nucleon potential ¹⁰⁾ which takes implicitly into account the presence of the closed Σ nucleon channel. It is a pion-exchange potential. Its tensor part was found to give negligible contributions to D ⁶⁾. The second order values for a square well potential ⁵⁾ are also computed. Both potentials have a hard core.

2.

For the effective masses and the Fermi momentum the values of Downs and Ware ⁵⁾ are taken.

Both potentials considered lead to large positive second order contributions ^{II}D. This increases the discrepancy of the value of D deduced from measured heavy hyperfragments and that calculated from two-body potentials adjusted to the data of light hyperfragments (expressed as a scattering length and an effective range which still have very large errors).

In Sections 2 and 3 we describe the general procedure ; in Section 4 a Green's function respecting the boundary conditions is introduced and in Section 5 the third order Born terms are considered. The numerical results are presented in Section 6 and are discussed in the last Section.

2. GENERAL PROCEDURE

The well depth D of a Λ particle in nuclear matter is usually calculated by the first order Born approximation ^{4),5),6)} in the potential tail. The starting point is the two-body cluster approximation

$$D = -\frac{4}{(2\pi)^3} \int_0^{k_F} d^3 k_N \int e^{-i\vec{k} \cdot \vec{r}} V(r) \psi_{BG}(\vec{r}, \vec{k}) d^3 r \quad (2.1)$$

Here $V(r)$ is the two-particle Λ nucleon potential. Only its central part will be considered here. It is

$$V(r) = \frac{1}{4} V_{\text{singl.}}(r) + \frac{3}{4} V_{\text{tripl.}}(r)$$

The wave function $\psi_{BG}(\vec{k}, \vec{r})$ is the Bethe-Goldstone ¹¹⁾ function, $\vec{r}$ and $\vec{k}$ are the relative co-ordinate and relative momentum of the Λ nucleon pair considered; $\vec{k}_N$ is the nucleon momentum and k_F is the Fermi momentum. Because we are examining the ground state energy the Λ particle is considered here to be at rest ($\vec{k}_\Lambda = 0$), although the centre-of-mass momentum $\vec{P}$ of a Λ nucleon pair is set equal to zero in the approximation used for $\psi_{BG}(\vec{k}, \vec{r})$. If the potentials - as those to be considered here - have a hard core D is

$$\begin{aligned} D &= -D_c + D_A \\ &= -\frac{4}{(2\pi)^3} \left(\frac{M_N^*}{\mu^*} \right)^3 \int_0^{\beta = \frac{\mu^* k_F}{M_N^*}} d^3 k \int e^{-i\vec{k} \cdot \vec{r}} (V_c(r) + V_A(r)) \psi_{BG}(\vec{r}, \vec{k}) d^3 r. \end{aligned} \quad (2.2)$$

The mass factor, containing the effective nucleon mass M_N^* and the effective reduced Λ nucleon mass μ^* , comes from the transformation of $\vec{k}_N$ to $\vec{k}$.

The quoted calculations are first order in the potential tail. The Bethe-Goldstone function we use here is - in fact - only a good numerical approximation to the Bethe-Goldstone function for the hard core ; i.e., for our zeroth order approximation to $\psi_{BG}(\vec{r}, \vec{k})$ we use $\psi_{BG}(\vec{r}, \vec{k}) \approx \psi_{BG}^{core}(\vec{r}, \vec{k})$. The radial S wave part of $\psi_{BG}^{core}(\vec{r}, \vec{k})$ is $\psi_{BG}^{core}(5), 11)$

$$R_{BG}^o(k, r) = \frac{A^o(k)}{kr} \left\{ \sin k(r-c) + \frac{1}{\pi} \left(\int_0^r \sin k(r-r') \left[\frac{\sin k_F(r'+c)}{r'+c} - \frac{\sin k_F(r'-c)}{r'-c} \right] dr' \right) \right\} \quad (2.3a)$$

$$A^o(k) = \left\{ \cos kc + \frac{1}{\pi} \left[\sin kc \left(Ci[c(k_F+k)] - Ci[c(k_F-k)] \right) - \cos kc \left(Si[c(k_F+k)] - Si[c(k_F-k)] \right) \right] \right\}^{-1} \quad (2.3b)$$

The function $\psi_{BG}^{core}(\vec{r}, \vec{k})$ has the approximate behaviour of a function ⁵⁾

$$\psi_{BG}(\vec{r}, \vec{k}) = f(r; N, a) e^{i\vec{k} \cdot \vec{r}} \quad (2.4)$$

$$f(r; N, a) = N(1 - e^{-2(r-c)/a}) \quad (2.5)$$

where N and a are parameters depending on the potential $V(r)$ ¹²⁾. Moreover, it is

$$\psi_{BG}(|\vec{r}|=c; \vec{k}) = 0$$

The function $\psi_{BG}(\vec{r}, \vec{k})$ obeys the integral equation

$$\psi_{BG}(\vec{r}, \vec{k}) = \psi_0(\vec{r}, \vec{k}) + \frac{M}{\hbar^2} \int G(\vec{r}, \vec{r}') (V_c(r') + V_A(r')) \psi_{BG}(\vec{r}', \vec{k}) d^3r' \quad (2.6)$$

which can be iterated. $G(\vec{r}, \vec{r}')$ is the Green's function.

We consider the hard core problem as solved exactly and perform a perturbation expansion in the attractive tail of the potential. To calculate the second and third order approximations to D we use the first order approximation $\psi_{BG}^I(\vec{r}, \vec{k})$ to the function $\psi_{BG}(\vec{r}, \vec{k})$ which takes the potential tail into account

$$\begin{aligned}\psi_{BG}^I(\vec{r}, \vec{k}) &= \varphi_{BG}(\vec{r}, \vec{k}) + \varphi^I(\vec{r}, \vec{k}) \\ &= \varphi_{BG}(\vec{r}, \vec{k}) + \frac{\mu^*}{\hbar^2} \int G(\vec{r}, \vec{r}') V_A(r') \varphi_{BG}(\vec{r}', \vec{k}) d^3r' \quad (2.7)\end{aligned}$$

$$G(\vec{r}, \vec{r}') = \frac{1}{(2\pi)^3} \int_{k_F}^{\infty} \frac{e^{i\vec{k}' \cdot (\vec{r} - \vec{r}')}}{k^2 - k'^2} d^3k' \quad (2.8)$$

The function $\psi_{BG}^I(\vec{r}, \vec{k})$ is an approximate solution of the Bethe-Goldstone equation ^{11), 13), 16)} that includes the pure core effects to all orders (with sufficient numerical accuracy) and the potential tail to first order.

In (2.8) $\vec{k}'$ is the relative momentum of a Λ nucleon pair whose nucleon is excited out of the Fermi sea. Choosing $|\vec{k}'| = k_F$ as the lower limit of the integral (2.8) means keeping the centre-of-mass momentum $\vec{P}$ of the Λ nucleon pair strictly to zero: $\vec{P} \equiv 0$. This is consistent with the neglect of the $\vec{P}$ dependence of the Bethe-Goldstone equation ^{9), 11), 14)}. It was found ^{9), 14)} that the inclusion of the $\vec{P}$ dependence of the Bethe-Goldstone equation in first order calculations does not change the results by more than a few MeV. By varying the lower limit of the k' integral one can obtain an idea of the importance of the Pauli principle and indirectly of the neglect of the $\vec{P}$ dependence in the problem considered.

In spite of $\psi_{BG}(\vec{r}', \vec{k})$ respecting the boundary condition at the hard core $r' = c$

$$\psi_{BG}(|\vec{r}'| = c; \vec{k}) = 0 \quad (2.9)$$

the function $\psi_{BG}^I(\vec{r}, \vec{k})$ does not have this property for $|\vec{r}| = c$. It gives

$$\psi_{BG}^I(|\vec{r}| = c; \vec{k}) \neq 0 \quad (2.10)$$

because of the form of the r dependence of the Green's function (2.8).

The function $\psi_{BG}^I(\vec{r}', \vec{k})$ Eq. (2.7) inserted in the integral equation (2.6) gives

$$\begin{aligned} \psi_{BG}(\vec{r}, \vec{k}) = \psi_0(\vec{r}, \vec{k}) + \frac{\mu^*}{\hbar^2} \int G(\vec{r}, \vec{r}') (V_C(r') + V_A(r')) \psi_{BG}^{core}(\vec{r}', \vec{k}) d^3 r' + \\ + \left(\frac{\mu^*}{\hbar^2} \right)^2 \int G(\vec{r}, \vec{r}') (V_C(r') + V_A(r')) \psi_{BG}^I(\vec{r}', \vec{k}) d^3 r'. \end{aligned} \quad (2.11)$$

The $V_C(r')$ part of the second term together with $\psi_0(\vec{r}, \vec{k})$ is replaced by $\psi_{BG}^{core}(\vec{r}, \vec{k})$ 11)

$$\psi_{BG}^{core}(\vec{r}, \vec{k}) = \psi_0(\vec{r}, \vec{k}) + \frac{\mu^*}{\hbar^2} \int G(\vec{r}, \vec{r}') V_C(r') \psi_{BG}^{core}(\vec{r}', \vec{k}) d^3 r'. \quad (2.12)$$

With $\psi_{BG}^{core}(\vec{r}, \vec{k}) \approx \psi_{BG}(\vec{r}, \vec{k})$ we have

$$\begin{aligned} \psi_{BG}(\vec{r}, \vec{k}) = \psi_{BG}(\vec{r}, \vec{k}) + \frac{\mu^*}{\hbar^2} \int G(\vec{r}, \vec{r}') V_A(r') \psi_{BG}(\vec{r}', \vec{k}) d^3 r' + \\ + \left(\frac{\mu^*}{\hbar^2} \right)^2 \int G(\vec{r}, \vec{r}') (V_C(r') + V_A(r')) G(\vec{r}', \vec{r}'') V_A(r'') \psi_{BG}(\vec{r}'', \vec{k}) d^3 r' d^3 r''. \end{aligned} \quad (2.13)$$

The function (2.13) when used in the expression (2.2) for the well depth gives rise to several terms in the Born series

$$\begin{aligned}
 D &= \langle k | V_C(r) | \psi_{BQ}(k) \rangle + \langle k | V_A(r) | \psi_{BQ}(k) \rangle + \\
 &+ \langle k | V_C(r) | \psi(k') \rangle \langle \psi(k') | V_A(r') | \psi_{BQ}(k) \rangle + \\
 &+ \langle k | V_A(r) | \psi(k') \rangle \langle \psi(k') | V_A(r') | \psi_{BQ}(k) \rangle + \\
 &+ \langle k | V_C(r) | \psi(k') \rangle \langle \psi(k') | V_C(r') | \psi(k'') \rangle \langle \psi(k'') | V_A(r) | \psi_{BQ}(k) \rangle + \\
 &+ \langle k | V_C(r) | \psi(k') \rangle \langle \psi(k') | V_A(r') | \psi(k'') \rangle \langle \psi(k'') | V_A(r) | \psi_{BQ}(k) \rangle + (2.14) \\
 &+ \langle k | V_A(r) | \psi(k') \rangle \langle \psi(k') | V_C(r') | \psi(k'') \rangle \langle \psi(k'') | V_A(r) | \psi_{BQ}(k) \rangle + \\
 &+ \langle k | V_A(r) | \psi(k') \rangle \langle \psi(k') | V_A(r') | \psi(k'') \rangle \langle \psi(k'') | V_A(r) | \psi_{BQ}(k) \rangle \\
 &= (D_C + D_A) + (D_{CA} + D_{AA}) + (D_{CCA} + D_{CAA} + D_{ACA} + D_{AAA}) \\
 &= \text{I} D + \text{II} D + \text{III} D
 \end{aligned}$$

The terms D_{CA} , D_{CCA} , D_{ACA} , and D_{CAA} are the overlap terms between core and attractive well.

3. THE SECOND ORDER BORN TERMS

The term D_{AA} is - after performing the angular integration - given by

$$D_{AA} = \sum_{\ell} D_{AA}^{\ell} \quad (3.1)$$

$$D_{AA}^{\ell} = -\frac{4}{(2\pi)^6} (4\pi)^4 \left(\frac{M_N^*}{M^*}\right)^3 \frac{M^*}{\hbar^2} (2\ell+1) \int_0^{\beta} k^2 dk \int r^2 dr j_{\ell}(kr) V_A^{\ell}(r).$$

$$\cdot \int_{k_F}^{\infty} \frac{k'^2 dk'}{k^2 - k'^2} \int j_{\ell}(k'r) j_{\ell}(k'r') f(r') j_{\ell}(k'r') V_A^{\ell}(r') r'^2 dr' \quad (3.2)$$

where D_{AA}^{ℓ} is the contribution of the ℓ^{th} partial wave. $R_{BG}^{\ell}(k, r')$ was used in the form (2.4) after the partial wave expansion of (2.4). For S waves we have found this term to be very big. Its importance is exaggerated by the violation of the boundary condition at $r = c$ by the first order wave function. This is especially bad for the S wave because its radial part is proportional to the zeroth order spherical Bessel function $j_0(k'r)$ which behaves like

$$j_0(k'r \rightarrow 0) \rightarrow 1, \quad r \rightarrow 0.$$

For small k' the $j_0(k'r)$ functions fall off rather slowly with r , for higher k' they go down more rapidly. Thus, at the hard core radius c ($c \sim 0.4f$ to $0.5f$) and small k' values the S wave function is about 1 instead of 0. Therefore the function (2.7) by use of the Green's function (2.8) used to calculate the well depth D in second order exaggerates the effect of the innermost, steep region of the potential in the neighbourhood of the hard core (see the Figure). This argument holds, of course, also for higher partial waves ; but there the situation is not as bad : the radial part behaves in this case like $j_{\ell}(k'r)$ and

$$j_{\ell}(k'r \rightarrow 0) \rightarrow 0 \quad \ell > 0, \quad r \rightarrow 0.$$

At the hard core the functions $j_\ell(k'r = k'c)$ ($\ell > 0$) are not zero but they are small. Therefore the wave function (2.7) by use of the Green's function (2.8) is reasonable for expansions of higher partial waves or the effect of the tensor potential ; although the results obtained in this way ⁶⁾ should be considered as upper bounds.

Furthermore, overlap terms, such as D_{CA} , Eq. (2.14), etc., are infinite in this approximation. The usual construction of the first order correction to the wave function, as shown above, does not permit any estimate of overlap effects.

4.

A GREEN'S FUNCTION RESPECTING THE BOUNDARY CONDITION

Because of the reasons discussed in Section 3, we need a function similar to $\psi_{BG}^I(\vec{r}, \vec{k})$ Eq. (2.7) but which respects the boundary condition for r at the hard core. Such a function can be constructed by use of a Green's function with the property

$$G(k; r=c, r'=c) = 0.$$

This Green's function will be

$$G \rightarrow \frac{1}{(2\pi)^3} \int \frac{|\psi\rangle \langle \psi|}{k^2 - k'^2} d^3k' \quad (4.1)$$

where the functions $|\psi\rangle$ form an orthogonal, complete set of eigenfunctions of the equation

$$(\nabla^2 + k^2) \psi(\vec{r}) = 0 \quad r > c \quad (4.2)$$

with the boundary condition

$$\psi(|\vec{r}|=c) = 0. \quad (4.3)$$

In writing down Eqs. (4.2) and (4.3) we ignore for the moment the Pauli principle (which adds an extra term to form the Bethe-Goldstone¹¹⁾ equation) and consider the problem as a scattering of hard spheres without any further interaction.

We only consider S waves. The radial S wave part of the eigenfunction $\psi(\vec{r})$ in (4.2), (4.3) is

$$R(k', r)|_{r=c} = \frac{\sin k'(r-c)}{k'r} \quad (4.4)$$

These functions form an orthogonal set

$$\int_0^{\infty} r^2 \frac{\sin k(r-c)}{kr} \frac{\sin k'(r-c)}{k'r} dr = \frac{\pi}{2k^2} \delta(k-k')$$

and - from general arguments - also a complete set ¹⁵⁾ so that we can expand the radial part of the Green's function in terms of the function (4.4)

$$G(k; r, r') \Big|_{\ell=0} = \frac{4\pi}{(2\pi)^3} \int_{k_F}^{\infty} \frac{\frac{\sin k'(r-c)}{k'r} \frac{\sin k'(r'-c)}{k'r'}}{k^2 - k'^2} k'^2 dk' \quad (4.5)$$

In choosing the lower limit k_F in the k' integral (4.5) we treat the Pauli effect as if for plane waves, because

$$\sin k'(r-c) = \sin k'r \cos k'c - \sin k'r \cos k'r$$

has only phase shifted plane wave components of momentum k' .

Inserting (4.5) in (2.7) we find for the term $D_{AA}^{\ell=0}$

$$D_{AA}^0 = -\frac{4}{(2\pi)^6} (4\pi)^4 \left(\frac{M_N^*}{\mu^*} \right)^3 \frac{\mu^*}{\hbar^2} \int_0^{\beta} k^2 dk \int r^2 dr \frac{\sin kr}{kr} V_A^0(r) \cdot \int_{k_F}^{\infty} \frac{k'^2 dk'}{k^2 - k'^2} \int \frac{\sin k'(r-c)}{k'r} \frac{\sin k'(r'-c)}{k'r'} V_A^0(r') R_{BG}^0(k, r') r'^2 dr' \quad (4.6)$$

The Green's function (4.5) used in $\Psi_{BG}^I(\vec{r}, \vec{k})$ Eq. (2.7) allows us to calculate the overlap term D_{CA}^0 between core and attractive tail

$$D_{CA}^0 = -\frac{4}{(2\pi)^6} (4\pi)^4 \left(\frac{M_N^*}{\mu^*}\right)^3 \frac{\mu^*}{f^2} \int_0^\beta k^2 dk \left(\frac{\sin kr}{kr} dr \int_{k_F}^\infty \frac{k'^2 dk'}{k^2 - k'^2} \cdot \right. \\ \left. \cdot \int r'^2 V_c(r) r^2 \frac{\sin k'(r-c)}{kr} \frac{\sin k'(r'-c)}{k'r'} V_A^0(r') R_{BG}^0(k, r') dr' \right). \quad (4.7)$$

Although the r dependent part of the integrand in (4.7) vanishes at the hard core $r = c$, the contribution to D_{CA}^0 does not vanish because of the discontinuity of the integrand's first derivative at $r = c$ (16), 5). The r dependent part is $\sin k'(r-c)$ and we have

$$V_c(r) \frac{\sin k'(r-c)}{k'} \approx \frac{f^2}{2\mu^*} \lim_{\epsilon \rightarrow 0} \left(\frac{\partial}{\partial r} \frac{\sin k'(r-c)}{k'} \right)_{r=c+\epsilon} \delta(r-c) = \quad (4.8)$$

$$= \frac{f^2}{2\mu^*} \lim_{\epsilon \rightarrow 0} \cos k\epsilon \delta(r-c) = \\ = \frac{f^2}{2\mu^*} \delta(r-c). \quad (4.9)$$

Then D_{CA}^0 becomes

$$D_{CA}^0 = -\frac{4}{(2\pi)^6} (4\pi)^4 \left(\frac{M_N^*}{\mu^*}\right)^3 \frac{1}{2} \int_0^\beta dk k \sin kc \int_{k_F}^\infty \frac{k' dk'}{k^2 - k'^2} \cdot \\ \cdot \int \sin k'(r-c) V_A^0(r') R_{BG}^0(k, r') r' dr'. \quad (4.10)$$

This term is repulsive and expected to be large.

5. THE THIRD ORDER BORN TERMS

With the Green's function (4.5) the third order term for the attractive part of the potential is for S waves

$$\begin{aligned}
 D_{AAA}^0 = & -\frac{4}{(2\pi)^9} (4\pi)^6 \left(\frac{M_N^*}{\mu^*}\right)^3 \left(\frac{\mu^*}{f^2}\right)^2 \iiint_0^B k^2 dk \frac{\sin kr}{kr} V_A^0(r) \\
 & \cdot \int_{k_F}^{\infty} \frac{k'^2 dk'}{k^2 - k'^2} \frac{\sin k'(r-c)}{k'r} \frac{\sin k'(r'-c)}{k'r'} V_A^0(r') \int_{k_F}^{\infty} \frac{k''^2 dk''}{k^2 - k''^2} \\
 & \cdot \frac{\sin k''(r'-c)}{k''r'} \frac{\sin k''(r''-c)}{k''r''} V_A^0(r'') R_{Bq}^0(k, r'') r^2 r'^2 r''^2 dr dr' dr'' . \quad (5.1)
 \end{aligned}$$

The overlap term D_{CAA}^0 can be calculated using (4.8) and (4.9). It is

$$\begin{aligned}
 D_{CAA}^0 = & -\frac{4}{(2\pi)^9} (4\pi)^6 \left(\frac{M_N^*}{\mu^*}\right)^3 \left(\frac{\mu^*}{f^2}\right)^2 \frac{t^2}{2\mu^*} \int k dk \sin kc \int_{k_F}^{\infty} \frac{k' dk'}{k^2 - k'^2} \\
 & \cdot \iint \frac{\sin k'(r'-c)}{k'r'} V_A^0(r') \int_{k_F}^{\infty} \frac{k''^2 dk''}{k^2 - k''^2} \frac{\sin k''(r'-c)}{k''r'} \frac{\sin k''(r''-c)}{k''r''} \\
 & \cdot V_A^0(r'') R_{Bq}^0(k, r'') r'^2 r''^2 dr' dr'' . \quad (5.2)
 \end{aligned}$$

The overlap term

$$D_{CCA}^0 = \langle \psi(k) | V_c(r) | \psi(k') \rangle \langle \psi(k') | V_c(r') | \psi(k'') \rangle \langle \psi(k'') | V_A(r'') | \psi(k) \rangle \quad (5.3)$$

vanishes as can be seen as follows : the integrand of D_{CCA}^0 has a vanishing r and r' part. These are by use of (4.8)

$$V_c(r) \frac{\sin k'(r-c)}{k'} \approx \frac{\hbar^2}{2\mu^*} \delta(r-c)$$

and

$$\begin{aligned} V_c(r') \frac{\sin k'(r'-c)}{k'} \frac{\sin k''(r'-c)}{k''} &\approx \\ &\approx \frac{\hbar^2}{2\mu^*} \lim_{\epsilon \rightarrow 0} \left[\cos k'\epsilon \frac{\sin k''\epsilon}{k''} + \frac{\sin k'\epsilon}{k'} \cos k''\epsilon \right] \delta(r-c) \xrightarrow{\epsilon \rightarrow 0} 0 \end{aligned} \quad (5.4)$$

and we have $D_{CCA}^0 = 0$. The term

$$D_{ACA}^0 = \langle k | V_A(r) | \psi(k') \rangle \langle \psi(k') | V_c(r') | \psi(k'') \rangle \langle \psi(k'') | V_A(r) | \varphi_{BQ}(k) \rangle \quad (5.5)$$

also involves the r' dependent part (5.4) and vanishes therefore, too.

6. THE NUMERICAL EVALUATION AND THE RESULTS

The terms D_{AA}^{ℓ} ($\ell = 0, 1, 2$) Eq. (3.2), D_{AA}^0 Eq. (4.6), and D_{CA}^0 (4.10) as well as D_{AAA}^0 (5.1) and D_{CAA}^0 (5.2) have been evaluated numerically. The k' integrals involved in the integrals (3.2), (4.6), and (4.10) were performed analytically⁶⁾. To find the P and D wave contributions to D_{AA} we used for $k' \geq 2k_F$ the asymptotic formulae for the spherical Bessel functions $j_{\ell}(k'r)$ and $j_{\ell}(k'r')$ ($\ell = 1, 2$) and also neglected k^2 in the denominator ($k^2 \ll k'^2$) in the same k' region. The integrals were evaluated by the Monte Carlo method on a high speed computer.

The results are shown in Tables 1 and 2¹⁷⁾. We always compare the results for the effective Λ nucleon potential¹⁰⁾ with those for the square well potential⁵⁾

$$\begin{aligned} V &= \infty & r < C & & C = 0.4 \text{ f} \\ V &= -44.24 \text{ MeV} & C \leq r \leq 1.5 \text{ f} & & \\ V &= 0 & r > 1.5 \text{ f} & & \end{aligned} \quad (6.1)$$

In first order calculation this square well potential gives^{5), 18)}
 $I_D = 33.1 \text{ MeV}$ with the partial wave contributions

$$I_D^0 = 18.7 \text{ MeV} \quad I_D^1 = 13.5 \text{ MeV} \quad I_D^2 = 0.9 \text{ MeV}.$$

The effective Λ nucleon potential¹⁰⁾ gives in first order the value
 $I_D = 38.1 \text{ MeV}$ with the partial wave contributions of the

i) singlet potential

$$I_D^0 = 48.1 \text{ MeV} \quad I_D^1 = 11.6 \text{ MeV} \quad I_D^2 = 1.1 \text{ MeV}$$

ii) triplet potential

$$I_D^0 = 15.2 \text{ MeV} \quad I_D^1 = 14.5 \text{ MeV} \quad I_D^2 = 1.4 \text{ MeV}.$$

Table 1 shows the results for D_{AA}^{ℓ} ($\ell = 0, 1, 2$) calculated according to (3.2), that is with a wave function which violates the boundary condition at $r = c$. In Table 2 the results for D_{AA}^0 and D_{CA}^0 Eqs. (4.6) and (4.10) using the modified Green's function (satisfying the boundary condition) are listed. In Table 3 only a rough estimate is presented for the third order terms D_{AAA}^0 and D_{CAA}^0 , the k^2 being neglected in the whole k' region ; these values give a lower limit and are only correct up to the order of magnitude. The values of $D_{AA}^{\ell=1}$ and $D_{AA}^{\ell=2}$ of Table 1 also have a systematic error of about 10% which comes from the approximations for $k' > 2k_F$.

All errors quoted are the statistical root mean square errors of the Monte Carlo calculation ; uncertainties of the potential are not included.

7. DISCUSSION

A comparison of the second order results ^{II}D (Tables 1 and 2) obtained with the function $\psi_{BG}^I(\vec{r}, \vec{k})$ using the unmodified Green's function (2.8) and those using the modified Green's function (4.5) shows that the Green's function (4.5) respecting the boundary condition gives much smaller values D_{AA}^0 than the function (2.8) for the pion exchange potential. Moreover, by use of the modified Green's function (4.5) the total second order contribution $^{II}D^{\ell=0}$ becomes still lower than $D_{AA}^{\ell=0}$ because (4.5) allows a calculation of the overlap terms between the hard core and the attractive tail of the potential which is not the case for the unmodified Green's function (2.8). Still, the values calculated by use of the modified Green's function (4.5) show that the higher order terms cannot be neglected ; they are large.

The singlet part $V_C(^1S_0)$ of the effective Λ nucleon potential gives very high contributions to D_{AA}^0 and D_{AAA}^0 compared to those of the triplet potential $V_C(^3S_1)$ and the square well potential (6.1) : in the neighbourhood of the hard core the singlet potential is deeper than the triplet potential ¹⁰⁾ (as it is expected for all Λ nucleon potentials from the analysis of the light hyperfragments) and especially small r and r' give the main contributions to D_{AA}^0 . So, the D value depends very strongly on the behaviour of the Λ nucleon potential near the hard core ; the hard core radii themselves are not very precisely known because of the uncertainties of the scattering length a .

The convergence of the perturbation expansion of such steep potentials is very poor, the D_A^0 and D_{AA}^0 contributions to first and second order are

$V_C(^1S_0)$	$D_A^0 = 110.2 \text{ MeV}$	$D_{AA}^0 = 55.0 \text{ MeV}$
$V_C(^3S_1)$	$D_A^0 = 92.4 \text{ MeV}$	$D_{AA}^0 = 34.1 \text{ MeV}$
square well (6.1)	$D_A^0 = 73.2 \text{ MeV}$	$D_{AA}^0 = 9.9 \text{ MeV.}$

As in first order also in higher orders the well depth is essentially the sum of a large positive and a large negative term, so the errors are big.

In third order, the square well potential gives a small D_{AAA}^0 , the values of D_{AAA}^0 for the meson potential are very large. As a check the method used for calculating D_{AAA}^0 was also applied to D_{AA}^0 . There it was found to give values too small by about a factor of 2. Therefore, the figures of Table 3 are only a very rough estimate and by a more careful calculation they may very well turn out a factor of 2 larger. Of course, the integrals become very difficult to compute. The overlap term in third order D_{CAA}^0 is calculated with the same approximation. It cancels part of D_{AAA}^0 .

In view of the large higher order contributions of S waves it does not seem worth while to consider P waves in second order in as much detail, the $D_{AA}^{\ell=1}$ (Table 1) is small compared to $D_{AA}^{\ell=0}$ (of course, respecting the boundary condition at $r' = c$ on the first order wave function does not have so much influence on D_{AA}^1 as on D_{AA}^0 , but the consideration of the overlap terms will lower the final value $II_D=1$ and $II_D=2$, too).

As pointed out in Section 1, first order calculations with reasonable Λ nucleon potentials give results for the well depth D which are about 5 to 10 MeV too high in comparison with experiment. The second order calculation makes the disagreement with the experiment even more striking.

In first and second order the well depth calculated from the potentials considered is

a) for the square well potential

$$D = 33.1 + 2.8 \text{ MeV} = 35.9 \text{ MeV}$$

b) for the effective Λ nucleon potential

$$D = 38.1 + 18.6 \text{ MeV} = 56.7 \text{ MeV}.$$

Because of the bad convergence of the Born series these numbers for D are not very meaningful, higher order terms can still play a role. The use of more refined two-body cluster calculation methods ^{9),19),20)} as are proposed for the nucleon-nucleon interaction in nuclear matter might be appropriate.

We always have neglected the $\vec{P}$ dependence ($\vec{P}$ is the centre-of-mass momentum of a Λ nucleon pair) of the Bethe-Goldstone equation. In taking $\vec{P} \neq 0$ into account all partial waves are coupled ^{5),14)}. By changing the lower limit α of the k' integral of the Green's function between $\alpha = \beta$ and $\alpha = k_F$ we found a variation of II_D of about 9 MeV for the effective Λ nucleon potential and about 4 MeV for the square well potential, $\alpha = \beta$ giving the largest value (by use of the Green's function (2.8) which does not respect the boundary condition the dependence of D_{AA}^0 on α is much larger). From this we draw the conclusion that the results in second order depend strongly on Pauli exclusion, allowing an important $\vec{P}$ dependence of the Bethe-Goldstone equation.

Very recently Downs and Grypeos ²¹⁾ tried a variational calculation for the Λ binding energy in nuclear matter. For different trial functions the binding energy found in this calculation was higher ³⁾ than the independent pair value calculated in first order with the same potential. This is in agreement with our results.

The rearrangement effect mentioned in Section 1 [Ref. ⁷⁾] will lower the theoretical expectation by approximately 4 to 6 MeV. A smaller effective Λ mass will also lower D by a few MeV ⁷⁾.

It is questionable if such steep potentials like the effective Λ nucleon potential ⁹⁾ (especially because of its singlet part) represent the Λ nucleon interaction in nuclear matter or if the simple pair approximation can be applied in this case. Three-body forces and three-body clusters could play a significant role.

Concerning the binding energies of light hyperfragments it would be interesting to see how well the experimental values agree with those calculated from the effective Λ nucleon potential.

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The second order contributions D_{AA}^{ℓ} ($\ell = 0, 1, 2$) [Eq. (3.2)] for the effective Λ nucleon potential and the square well potential (6.1) (in MeV).

	effective Λ N potential		square well potential (6.1)
	Singlet V_C	Triplet V_C	
D_{AA}^0	98.5 ± 2.2	46.5 ± 1.0	9.0 ± 0.3
II_D^0	59.5 ± 1.3		
D_{AA}^1	2.6 ± 0.2	2.0 ± 0.2	1.3 ± 0.1
II_D^1	2.2 ± 0.2		
D_{AA}^2	< 0.1	< 0.1	< 0.1
II_D^2	< 0.1		

T A B L E 1

Modified second order terms D_{AA}^0 and D_{CA}^0 [Eqs. (4.6) and (4.10)] for the effective Λ nucleon potential and the square well potential (6.1) (in MeV).

	effective Λ N potential		square well potential (6.1)
	Singlet $V_C(^1S_0)$	Triplet $V_C(^3S_1)$	
D_{AA}^0	55.0 ± 1.9	34.1 ± 0.9	9.9 ± 0.3
D_{CA}^0	-26.1 ± 0.3	-18.9 ± 0.2	-7.1 ± 0.2
II_D^0	18.6 ± 1.4		2.8 ± 0.5

T A B L E 2

Rough estimate for the third order terms D_{AAA}^0 [Eq. (5.1)] and D_{CAA}^0 [Eq. (5.2)] (in MeV).

	effective Λ N potential		square well potential (6.1)
	Singlet $V_C(^1S_0)$	Triplet $V_C(^3S_1)$	
D_{AAA}^0	52.8	12.0	3.0
D_{CAA}^0	-9.3	-4.2	-1.2
III_D^0	16.7		1.8

T A B L E 3

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