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DIRECT TESTS OF CP INVARIANCE

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E R R A T A

Page 2 - Line 3 : P should be p
" " - " 4 : γ " " β
" 5 - " 2 from below : (5) should be 5)
" 7 - " 13 : (7) " " (17)

Matts Roos : "Direct tests of CP invariance".

Although it is now commonly felt that the discovery ¹⁾ of the long-lived $\pi^+\pi^-$ decay mode of the K^0 meson demonstrates the absence of CP symmetry, the question is not entirely settled until one has disposed of the remaining undefeated CP invariant explanations to this decay. The purpose of the present letter is to exploit the $\pi^+\pi^-$ decay mode to make four quantitative predictions, by which the theories of Nishijima and Saffouri ²⁾, Kabir and Lewis ³⁾ and Laurent and Roos ^{4),5)} can be simultaneously disproved. The authors of the first two theories ^{2),3)} do not themselves offer ample means for disproof; two of our predictions have been discussed qualitatively before ⁵⁾.

Let the total decay rate of the long-lived (short-lived) state $K_L(K_S)$ be $\Gamma_L(\Gamma_S)$, and let the rate of decay into $\pi^+\pi^-$ be $\Gamma_{L\pi}(\Gamma_{S\pi})$.

The amplitude for the decay

$$K^0 \rightarrow \pi^+ + \pi^- \quad (1)$$

is, at the time-of-flight t of the K^0 meson, given (up to a phase) by

$$A(t) = p \left(\Gamma_{S\pi}^{1/2} e^{-\alpha t} + \Gamma_{L\pi}^{1/2} e^{i\phi} e^{-\beta t} \right) \quad (2)$$

in a linear CPT invariant theory, and by

$$A(t) = \Gamma_{S\pi}^{1/2} g(t) e^{-\alpha t} \quad (3)$$

in a non-linear theory ^{4),5)}. Here

$$\alpha \equiv \frac{1}{2} \Gamma_S + i m_S, \quad (4)$$

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where m_S is the K_S mass. Further
 ϕ is an arbitrary real phase,
 p is a real number of order $\sqrt{\frac{1}{2}}$,
 γ is an arbitrary complex number, and
 $g(t)$ is an arbitrary ⁵⁾ complex function.

Since most of the conclusions we are aiming at are common to linear and non-linear theories, we shall in the following neglect Eq.(3) and only use Eq.(2), unless otherwise specified.

The conclusions of our discussion will not be altered if we assume, for simplicity, that $p^2 = \frac{1}{2}$ and that

$$\beta = \frac{1}{2} \Gamma_L + i(m_S + \Delta m) \quad (5)$$

where Δm is the $K_L - K_S$ mass difference.

The CPT invariant amplitude for the decay

$$\bar{K}^0 \rightarrow \pi^+ + \pi^- \quad (6)$$

may now be written

$$\bar{A}(t) = \Gamma_{S\pi}^{1/2} e^{-\alpha t} \pm \Gamma_{L\pi}^{1/2} e^{i\phi} e^{-i\beta t} \quad (7)$$

In CP invariant (CP violating) theories, the second term on the right is chosen with the sign $+(-)$. It follows, as pointed out before ⁵⁾, that the time distributions $|A(t)|^2$ and $|\bar{A}(t)|^2$ for the decays (1) and (6), respectively, are identical (different)

in CP invariant (CP violating) theories.

Suppose we are given a beam of incoherently mixed K^0 and $\bar{K}^0$ mesons, in the proportion $Z : 1 - Z$. It then follows from Eqs. (2) and (7) that the $\pi^+ \pi^-$ decay rate of such a beam will be

$$\Gamma_{\pi^+ \pi^-}(t, Z) = Z |A(t)|^2 + (1 - Z) |\bar{A}(t)|^2.$$

Differentiating with respect to Z , we find the change in the decay rate due to a change in the proportions of the mixture :

$$\frac{d\Gamma_{\pi^+ \pi^-}(t, Z)}{dZ} = \begin{cases} 0 & \text{for CP invariance} \\ 4\sqrt{\Gamma_{S\pi}\Gamma_{L\pi}} \cos(\Delta m t - \phi) e^{-\frac{t}{2}(\Gamma_L + \Gamma_S)} & \text{for CP violation} \end{cases} \quad (8)$$

Incoherent mixtures of K^0 and $\bar{K}^0$ are produced in high-energy $p - p$ and p -nucleon collisions. Although there exists no ready knowledge of how to vary the ratio of $K^0 : \bar{K}^0$, it seems a reasonable (and testable) proposition that if the ratio $K^+ : K^-$ shows strong momentum dependence, there will be some momentum dependence in $K^0 : \bar{K}^0$, too. For incident protons of momenta 19 and 23 GeV/c, the $K^+ : K^-$ ratio has been measured ⁶⁾ in the forward direction as a function of secondary (kaon) momentum. The ratio is found to vary from about 1.65 ± 0.35 at 4 GeV/c secondary momentum to about 11 ± 3 at 12 GeV/c, corresponding to a power law of

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(almost constant) power ≈ 1.7 . Even a smaller variation with secondary momentum in the $K^0 : \bar{K}^0$ ratio could certainly be exploited for the purpose of Eq. (8).

Let us next turn to the behaviour of K^0 and $\bar{K}^0$ beams, or incoherent mixtures, which are passed through a regenerator at a time when only the K_L components remain. Let the intensity of a K^0 beam be 1 at the surface of the regenerator.

The intensity of that long-lived component which decays into $\pi^+ \pi^-$ is then $\Gamma_{L\pi} / \Gamma_L$, and its decay amplitude

$$\Gamma_{L\pi}^{1/2} e^{i\phi} e^{-\beta t}, \quad (9)$$

where t now is measured from the surface of the regenerator. In the CP violating theories the amplitude for the regenerated short-lived $\pi^+ \pi^-$ decays is

$$\Gamma_{S\pi}^{1/2} \varrho e^{-\alpha t}, \quad (10)$$

where ϱ is a complex function of the regenerator material and thickness (for a detailed derivation of ϱ , cf. Christenson ⁷).

In the linear CP invariant theories, the amplitude corresponding to (10) is

$$\left(1 - \sqrt{\frac{\Gamma_{L\pi}}{\Gamma_S}}\right) \varrho \left(\Gamma_{S\pi}^{1/2} e^{-\alpha t} + \Gamma_{L\pi}^{1/2} e^{i\phi} e^{-\beta t} \right). \quad (11)$$

Here, we may safely drop the root term in the first bracket.

With the same phase convention as before, the effect of considering a $\bar{K}^0$ beam instead of a K^0 beam now amounts to multiplying (10) and (11) by -1 , and (9) by ± 1 , where the upper (lower) sign corresponds to CP invariance (CP violation). Adding coherent terms we obtain the following decay amplitudes after regeneration :

$$A_{\pi^+\pi^-}^{CP} = \rho \pi_{S\pi}^{1/2} e^{-\alpha t} + (1+\rho) \pi_{L\pi}^{1/2} e^{i\phi} e^{-\beta t} \quad (12)$$

$$\bar{A}_{\pi^+\pi^-}^{CP} = -\rho \pi_{S\pi}^{1/2} e^{-\alpha t} + (1-\rho) \pi_{L\pi}^{1/2} e^{i\phi} e^{-\beta t} \quad (13)$$

$$A_{\pi^+\pi^-}^{CP} = \rho \pi_{S\pi}^{1/2} e^{-\alpha t} + \pi_{L\pi}^{1/2} e^{i\phi} e^{-\beta t} \quad (14)$$

$$\bar{A}_{\pi^+\pi^-}^{CP} = -\rho \pi_{S\pi}^{1/2} e^{-\alpha t} - \pi_{L\pi}^{1/2} e^{i\phi} e^{-\beta t} \quad (15)$$

From these equations, we immediately see that the decay rates

$$|A_{\pi^+\pi^-}^{CP}|^2 \quad \text{and} \quad |\bar{A}_{\pi^+\pi^-}^{CP}|^2$$

are identical, so that the decay rate of an incoherent mixture of ZK_L from K^0 and $(1-Z)K_L$ from $\bar{K}^0$ will be independent of the mixing parameter Z , whereas the decay rates

$$|A_{\pi^+\pi^-}^{CP}|^2 \quad \text{and} \quad |\bar{A}_{\pi^+\pi^-}^{CP}|^2$$

are different and will, in general, produce a Z dependent decay rate in an incoherently mixed beam (5). In the linear CP invariant case, the decay rate becomes

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$$\Gamma_{\pi^+\pi^-}^{CP} = |\varrho|^2 \Gamma_{S\pi} e^{-\Gamma_S t} + [1 + 2(2Z-1)|\varrho| \cos \psi + |\varrho|^2] \Gamma_{L\pi} e^{-\Gamma_L t} +$$

$$+ 2|\varrho| [|\varrho| \cos(\Delta m t - \phi) + (2Z-1) \cos(\Delta m t + \psi - \phi)] \sqrt{\Gamma_{L\pi} \Gamma_{S\pi}} e^{-\frac{t}{2}(\Gamma_S + \Gamma_L)}$$

(16)

where $\psi \equiv \arg \varrho$. We can then write our second prediction as

$$\frac{d\Gamma_{\pi^+\pi^-}^{\text{regen.}}(t, Z)}{dZ} = \begin{cases} 4|\varrho| \Gamma_{L\pi} \left[\cos \psi e^{-\Gamma_L t} + \sqrt{\frac{\Gamma_{S\pi}}{\Gamma_{L\pi}}} \cos(\Delta m t + \psi - \phi) e^{-\frac{t}{2}(\Gamma_S + \Gamma_L)} \right] & \text{for CP invariance (linear theories)} \\ 0 & \text{for CP violation} \end{cases}$$

(17)

It is interesting that at large times, when the oscillating second term in the first line of Eq. (17) is small, there still survives a non-oscillating term. This may be used to make a third prediction: if a regenerator is inserted in a K^0 beam far (several K_S lifetimes) from the source and far upstream from the detector, the branching ratio $\Gamma_{L\pi}/\Gamma_L$ should change by a factor

$$1 + 2|\varrho| \cos \psi + |\varrho|^2, \quad (18)$$

if linear CP invariant theories apply. In CP violating theories no such change is expected.

Note that Kabir and Lewis³⁾ have predicted an increase in the branching ratio due to a difference in the absorption cross-section of K_2 and the long-lived K_1 , but apparently neglected the factor (18) which also should appear in their theory. Therefore, should an increase be observed, it can not be considered as a support of Kabir and Lewis' theory rather than of any other CP invariant theory, unless the increase is much larger than the factor (18) with $\psi = 0$, or until study of the detailed $|\varphi|$ dependence of the factor has produced supporting evidence.

Qualitatively non-linear CP invariant theories make the same predictions as linear theories in Eq. (7) but the detailed expression may of course not be written down unless a specific model is chosen. The question whether non-linear theories predict factors like (18) different from unity is again model-dependent.

Let us finally turn to the $K_1 K_2$ two particle system, produced by annihilation at rest

$$\bar{p} + p \rightarrow \bar{K}^0 + K^0 = K_1 + K_2 . \quad (19)$$

As is well known, the final $\bar{K}^0 K^0$ pair must be in a $C = -1$ state, if C and CP are conserved in the production process and if the $\bar{p}$ captures take place mainly from atomic S states⁸⁾. If Bose statistics is valid, the final states $K_1 K_1$ and $K_2 K_2$ are excluded and the particular combination of K_1 and K_2 wave functions which is odd under C is

$$\psi = \frac{1}{2^{1/2}} (\bar{K}_a^0 \bar{K}_b^0 - \bar{K}_a^0 K_b^0) = \frac{1}{2^{1/2}} (K_{2a} K_{1b} - K_{1a} K_{2b}), \quad (20)$$

where a and b stand for the space indices. Since the annihilation takes place at rest, a and b are opposite directions in the laboratory.

In order to study the coherent system of two kaons produced simultaneously, one must set up an apparatus measuring the coincidence (of the production vertices) of two kaons decaying in opposite directions. Let the time at which the decays occur be t_a and t_b , measured in the directions a and b , respectively. The time development of the linear CP invariant state (20) is clearly

$$|\psi(t)\rangle = \frac{1}{2^{1/2}} \left(|K_{2a}, K_{1b}\rangle e^{-\alpha t_b - \beta t_a} - |K_{1a}, K_{2b}\rangle e^{-\alpha t_a - \beta t_b} \right). \quad (21)$$

When a K_2 decay has been found in, say, the a direction, the coincidentally produced kaon in the b direction can only be a K_1 , whether short-lived or long-lived, in a CP invariant theory. This situation is very similar to the one discussed by Einstein, Podolsky and Rosen ⁹⁾, as noted by Inglis ¹⁰⁾ and Day ¹¹⁾ *). Needless to say, no pair of coincident K_1 decays in opposite directions will be found.

*) I am indebted to Professor J.J. Sakurai for having pointed out references ¹⁰⁾ and ¹¹⁾ to me.

Consider now the situation as described by the CP violating, CPT invariant theories. K_1 and K_2 above are now taken to mean K_S and K_L . The state K_2 or K_L will have a small but finite probability for long-lived $\pi^+\pi^-$ decays. In consequence, there will be a finite probability of finding a pair of coincident $\pi^+\pi^-$ decays in opposite directions. The probability of finding a $\pi^+\pi^-$ decay at time t_a per unit time, in coincidence with a $\pi^+\pi^-$ decay anywhere in the b direction (at time $0 \leq t_b \leq \infty$) is then given by

$$\begin{aligned} \Gamma(t) &= \int_0^\infty dt_b \frac{\Gamma_{S\pi}\Gamma_{L\pi}}{2} \left| e^{-\alpha t_b - \beta t_a} - e^{-\alpha t_a - \beta t_b} \right|^2 = \\ &= \frac{\Gamma_{S\pi}\Gamma_{L\pi}}{2} \left[\frac{e^{-\Gamma_S t}}{\Gamma_L} + \frac{e^{-\Gamma_L t}}{\Gamma_S} - \right. \\ &\quad \left. - \frac{(\Gamma_S + \Gamma_L) \cos \Delta m t + 2 \Delta m \sin \Delta m t}{\frac{1}{4} (\Gamma_S + \Gamma_L)^2 + (\Delta m)^2} e^{-\frac{t}{2} (\Gamma_S + \Gamma_L)} \right], \end{aligned} \quad (22)$$

where we have set $t \equiv t_a$. Substituting numerical values, it turns out that the oscillating term may be neglected everywhere. At best one could hope to study this effect close to the production vertex, where only the first term is important. Then if $\Gamma(t)$ would turn out to be zero, one could unambiguously conclude that the decay is CP invariant. If $\Gamma(t)$ would be non-zero, however, one could not conclude anything about the CP invariance, since coincident $\pi^+\pi^-$ decays could be due equally well to CP violation as to a slight violation of the theorem of Day, Snow and Sucher⁸⁾ used to obtain Eq. (20).

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