CM-P00057387

C AND T INVARIANCE AND NUMBER OF SU(3) AMPLITUDES

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A B S T R A C T

The implications of C and T invariance, together with P invariance and statistics, for the number of SU(3) amplitudes, are re-examined, especially when the particles involved have spin and some of them baryon number. Meson-meson octet scattering, annihilation of baryon and antibaryon octets into two mesons and into baryon-antibaryon, and baryon scattering are given as examples. It is shown that crossing does not give any new restriction. The crossing of the three "unusual" partial wave amplitudes occurring in the scattering of four spin $\frac{1}{2}$ particles is discussed. Some experimental implications are given.

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In the octet model¹⁾ of $SU(3)$, the interaction of two octets producing two octets will be described, for a given total angular momentum j , by the $SU(3)$ invariant, partial-wave amplitudes

$$A(\nu_f \ell_f s_f; \nu_i \ell_i s_i; \alpha) \quad (1)$$

ν_f stands for the $SU(3)$ representation to which the final state belongs, ℓ_f for its orbital angular momentum and s_f its spin, ν_i, ℓ_i, s_i being the corresponding quantum numbers of the initial state, and α the remaining diagonal quantum numbers. ν_f and ν_i belong to the set obtained by multiplying the two octet representations:

$$8 \otimes 8 = 1 \oplus 8_s \oplus 27 \oplus 8_a \oplus 10 \oplus \overline{10} \quad (2)$$

where the subscript $s(a)$ means symmetric (antisymmetric) under the interchange of the I, I_3 and T quantum numbers of the particles belonging to the two octets being multiplied in (2). For fixed values of ℓ, s and α , there are eight $SU(3)$ invariant amplitudes, namely four amplitudes with

$$\nu_f = \nu_i \text{ for } \nu = 1, 10, \overline{10}, 27$$

which will be abbreviated by A_ν and the four amplitudes

$$A(8_s \ell_f s_f; 8_s \ell_i s_i; \alpha) \equiv A_{ss}$$

$$A(8_a \ell_f s_f; 8_a \ell_i s_i; \alpha) \equiv A_{aa}$$

$$A(8_s \ell_f s_f; 8_a \ell_i s_i; \alpha) \equiv A_{sa}$$

$$A(8_a \ell_f s_f; 8_s \ell_i s_i; \alpha) \equiv A_{as}$$

2.

In the special case²⁾ of the scattering of a spin $\frac{1}{2}$ baryon (B) with a spin 0 meson (P), where $s_i = s_f$, parity conservation implies $\ell_i = \ell_f$, and therefore time reversal invariance implies

$$A_{as} = A_{sa} \quad (3)$$

It has been incorrectly assumed³⁾ that (3) holds in more general situations. When all particles have spin, and some of them have baryon number, it is possible to construct amplitudes which are simultaneously odd under T in spin space and unitary spin space and hence invalidate (3). We therefore find it useful to review the various possible cases. First, the implication of C and CP invariance, as well as statistics, are discussed for the processes $X + \bar{X} \rightarrow Y + \bar{Y}$, X and Y being octets. This is applied to $M + M \rightarrow M' + M'$, M and M' being mesons, $\bar{B} + B \rightarrow P + P$, $\bar{B} + B \rightarrow V + V$, V being a spin 1 meson. Finally, the process $\bar{B} + B \rightarrow \bar{B} + B$ is studied in more detail, together with the crossed channel $B + B \rightarrow B + B$. An explicit example of a crossing symmetric interaction, which satisfies C and T invariance and which describes pure triplet-singlet baryon-baryon scattering, is given. Some experimental tests are indicated.

1. $X + \bar{X} \rightarrow Y + \bar{Y}$. X and Y are octets.

It is shown in the Appendix that C invariance and statistics imply for a partial wave amplitude

$$A(\bar{\nu}_f \ell_f s_f; \bar{\nu}_i \ell_i s_i, \alpha) = (-)^{\ell_i + s_i + \ell_f + s_f} A(\nu_f \ell_f s_f; \nu_i \ell_i s_i, \alpha) \quad (4)$$

where $\bar{\nu}$ is the conjugate representation to ν , and $\bar{\nu} \neq \nu$ only for $\nu = 10$ and $\nu = \bar{10}$.

From parity conservation it follows for the reaction

$$\bar{B}B \rightarrow MM : (-)^{\ell_i + 1} = (-)^{\ell_f} \quad (5)$$

and for

$$\overline{BB} \rightarrow \overline{BB} \text{ or } MM \rightarrow M'M' : (-)^{\ell_i} = (-)^{\ell_f} \quad (6)$$

Hence, CP implies for

$$\overline{BB} \rightarrow MM :$$

$$A(\overline{\nu}_f \ell_f s_f; \overline{\nu}_i \ell_i s_i; \alpha) = (-)^{s_i + s_f + 1} A(\nu_f \ell_f s_f; \nu_i \ell_i s_i; \alpha) \quad (7)$$

and for

$$\overline{BB} \rightarrow \overline{BB} \text{ or } MM \rightarrow M'M' :$$

$$A(\overline{\nu}_f \ell_f s_f; \overline{\nu}_i \ell_i s_i; \alpha) = (-)^{s_i - s_f} A(\nu_f \ell_f s_f; \nu_i \ell_i s_i; \alpha) \quad (8)$$

Finally, Bose statistics implies for the state

$$MM : (-)^{\ell + s} = \varepsilon_v = \begin{cases} +1 & v = 1, 8_s, 27 \\ -1 & v = 8_a, 10, \overline{10} \end{cases} \quad (9)$$

2. MM → M'M'

From (4) and (9), one gets:

$$A_{as} = A_{sa} = 0; \quad A_{10} = \overline{A_{10}} \quad (10)$$

This of course means that C invariance implies here R invariance¹⁾.

3. BB → PP

Since $s_f = 0$, $j = \ell_f$ and since $(-)^{\ell_f} = (-)^{\ell_i + 1}$, from (5), one gets $s_i = 1$. Hence (7) gives:

$$\overline{A_{10}} = A_{10}; \quad s_i = 1 \quad (11)$$

4.

Using crossing symmetry, (11) implies (3) for $BP \rightarrow BP$ (see Table III).

4. $\overline{BB} \rightarrow \overline{VV}$

Equation (7) implies:

$$\left. \begin{aligned} S_i + S_f &= \text{odd} & \text{for } \nu = 1, 8, 27 \text{ and } A_{10} = A_{\overline{10}} \\ S_i + S_f &= \text{even} & \text{for } A_{10} = -A_{\overline{10}} \end{aligned} \right\} \quad (12)$$

5. $\overline{BB} \rightarrow \overline{BB}$

We discuss this case in more detail. Recall that for definite total angular momentum j a system of two spin- $\frac{1}{2}$ objects can take on orbital and spin angular momentum quantum numbers given by the sets $(\ell, s) = (j+1, 1), (j, 1), (j-1, 1)$ and $(j, 0)$. For definite j there are eight distinct transitions $(\ell, s) \rightarrow (\ell', s')$ consistent with the parity invariance requirement (6). Call the corresponding partial wave amplitudes $A(\ell', s'; \ell, s)$. Five of these amplitudes are the "classical" ones present, for example, in proton-proton scattering. The three remaining ones are:

$$A(j+1, 1; j-1, 1) - A(j-1, 1; j+1, 1) \equiv (\Delta\ell = 2)_{\text{odd}} \quad (13)$$

$$A(j, 1; j, 0) - A(j, 0; j, 1) \equiv ts - st \quad (14)$$

$$A(j, 1; j, 0) + A(j, 0; j, 1) \equiv ts + st \quad (15)$$

(13) and (14) describe amplitudes odd under T (in spin space) and (14) and (15) describe triplet-singlet scattering.

From (8) one sees that:

$$\left. \begin{aligned} S_i + S_f &= \text{even for } \nu = 1, 8, 27 \text{ and } A_{10} = A_{\overline{10}} \\ S_i + S_f &= \text{odd for } A_{10} = -A_{\overline{10}} \end{aligned} \right\} \quad (16)$$

Therefore, triplet-singlet scattering is described by the $SU(3)$ amplitude $(A_{10} - A_{\overline{10}})$.

Using now T invariance for the scattering $\overline{B}B \rightarrow \overline{B}B$, one finds that the amplitudes (13) and (14), odd under T in spin space, are described by $A_{as} - A_{sa}$. However, (16) implies for the latter $s_i + s_f = \text{even}$. Therefore, (14) is forbidden by $SU(3)$ and T invariance.

The five "classical" amplitudes are described by six $SU(3)$ amplitudes even under T (with $A_{10} = A_{\overline{10}}$) and $ts + st$ is described by $(A_{10} - A_{\overline{10}})$. It should be noticed that the relative sign of ts and st depends on the ordering of the particles.

Summarising, there are altogether $5 \times 6 + 1 + 1 = 32$ amplitudes in spin and unitary spin space. This is shown in Table I.

Table I $\overline{B}B \rightarrow \overline{B}B$

spin space			unitary spin space	
$s_i + s_f$	T	name	allowed	forbidden
even	even	five "classical"	$A_1, A_{ss}, A_{aa}, A_{27}$ $(A_{as} + A_{sa}), (A_{10} + A_{\overline{10}})$	$(A_{10} - A_{\overline{10}})$ $(A_{as} - A_{sa})$
odd	even	$ts + st$	$(A_{10} - A_{\overline{10}})$	all others
even	odd	$(\Delta \ell = 2)_{\text{odd}}$	$(A_{as} - A_{sa})$	all others
odd	odd	$ts - st$	none	all

6.

6. BB → BB

All particles belong to the same octet. The results can be obtained from the preceding ones using crossing. Under this operation, the five "classical" amplitudes transform amongst themselves, the same being true for the three remaining ones. To find this relationship, we use the explicit, crossing symmetric, interactions

$$\mathcal{L}_1 = i \bar{\Psi}_3 \gamma^\mu \Psi_1 \bar{\Psi}_4 \overleftrightarrow{\partial}_\mu \Psi_2 - i \bar{\Psi}_3 \overleftrightarrow{\partial}_\mu \Psi_1 \bar{\Psi}_4 \gamma^\mu \Psi_2 + h.c. \quad (17)$$

$$\mathcal{L}_2 = i \bar{\Psi}_3 \gamma^\mu \gamma_5 \Psi_1 \bar{\Psi}_4 \gamma_5 \overleftrightarrow{\partial}_\mu \Psi_2 + \epsilon i \bar{\Psi}_3 \gamma_5 \overleftrightarrow{\partial}_\mu \Psi_1 \bar{\Psi}_4 \gamma^\mu \gamma_5 \Psi_2 + h.c. \quad (18)$$

where

$$\bar{\Psi} \overleftrightarrow{\partial}_\mu \Psi = \bar{\Psi} \partial_\mu \Psi - (\partial_\mu \bar{\Psi}) \Psi$$

For the process $B_1 B_2 \rightarrow B_3 B_4$ (17) describes pure ts+st transition, (18) describes pure $(\Delta l = 2)_{\text{odd}}$ for $\epsilon = +1$ and pure ts - st transition for $\epsilon = -1$, in the centre of mass system. Table II gives the crossing matrix, where invariant functions of CM angle and energy are omitted. In channel II, $\bar{B}_3$ has same spin and space labels as B_2 in channel I, etc.⁴⁾.

Table II

I $B_1 B_2 \rightarrow B_3 B_4$	$(\Delta\ell = 2)_{\text{odd}}$	ts - st	ts + st
II $B_1 \bar{B}_3 \rightarrow \bar{B}_2 B_4$	ts + st	ts - st	$(\Delta\ell = 2)_{\text{odd}}$
III $B_1 \bar{B}_4 \rightarrow B_3 \bar{B}_2$	ts - st	$(\Delta\ell = 2)_{\text{odd}}$	ts + st

Table III gives the corresponding $SU(3)$ crossing matrix⁵⁾, omitting numerical factors.

Table III

I $B_1 B_2 \rightarrow B_3 B_4$	$10 - \bar{10}$	as - sa	as + sa
II $B_1 \bar{B}_3 \rightarrow \bar{B}_2 B_4$	as - sa	$10 - \bar{10}$	as + sa
III $B_1 \bar{B}_4 \rightarrow B_3 \bar{B}_2$	as + sa	as - sa	$10 - \bar{10}$

8.

Table I applies to channel III of Tables II and III. Therefore, one can immediately write down the results⁶⁾ for $BB \rightarrow BB$ which are given in Table IV.

Table IV $BB \rightarrow BB$

Spin space			unitary spin space	
$s_i + s_f$	T	name	allowed	forbidden
even	even	five "classical"	A_1, A_{ss}, A_{aa} $A_{10}, A_{\overline{10}}, A_{27}$	A_{as}, A_{sa}
odd	even	ts + st	$A_{as} + A_{sa}$	all others
even	odd	$(A^2 = 2)_{\text{odd}}$	none	all
odd	odd	ts - st	$A_{as} - A_{sa}$	all others

A somewhat faster but completely equivalent way of finding forbidden reactions⁷⁾ is to use I-spin, U-spin and V-spin invariance⁸⁾.

We can use the equations and discussions of the preceding section, but applied now to the group $SU(2)$. The new features are the absence of amplitudes odd under T, because $SU(2)$ is a simply reducible group, and the

absence of amplitudes odd under C, because all representation of SU(2) are selfconjugate.

Let us use the notation $B_1 \approx B_2$, for particles belonging to the same representation of the SU(2) I-spin, U-spin or V-spin. Then for the reactions $B_1 B_2 \rightarrow B_3 B_4$ and $B_1 \bar{B}_2 \rightarrow B_3 \bar{B}_4$, the Pauli principle combined in the second case with charge conjugation, implies, if $B_1 \approx B_2$ and $B_3 \approx B_4$, that ts and st is forbidden. For the same reaction, time reversal invariance implies if $B_1 \approx B_3$ and $B_2 \approx B_4$, that ts - st and ($\Delta\ell = 2$) odd is forbidden.

Finally, we give an explicit example of two interactions, which are SU(3), C and T invariant, and crossing symmetric, and which give rise to triplet-singlet scattering for channel $B_1 B_2 \rightarrow B_3 B_4$:

$$\mathcal{L}_1 = i\bar{\Psi}_{ab} \gamma^\mu \Psi_{da} \bar{\Psi}_{bc} \vec{\gamma}_\mu \Psi_{cd} - i\bar{\Psi}_{bc} \gamma^\mu \Psi_{cd} \bar{\Psi}_{ab} \vec{\gamma}_\mu \Psi_{da} \quad (19)$$

$$\mathcal{L}_2 = i\bar{\Psi}_{ab} \gamma^\mu \gamma_5 \Psi_{cd} \bar{\Psi}_{bc} \vec{\gamma}_\mu \gamma_5 \Psi_{da} - i\bar{\Psi}_{dc} \gamma^\mu \gamma_5 \Psi_{ba} \bar{\Psi}_{ad} \vec{\gamma}_\mu \gamma_5 \Psi_{cb} \quad (20)$$

where Ψ is the usual Dirac spinor field, and besides a 3×3 matrix in unitary space, whose matrix elements are labelled by the indices a, b, c, d.

$$\psi = \begin{pmatrix} \frac{1}{\sqrt{6}} \Lambda + \frac{1}{\sqrt{2}} \Sigma^0 & \Sigma^+ & p \\ \Sigma^- & \frac{1}{\sqrt{6}} \Lambda - \frac{1}{\sqrt{2}} \Sigma^0 & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}} \Lambda \end{pmatrix} \quad (21)$$

Note that under charge conjugation, in the Majorana representation

$$\psi_{ab}^c = \psi_{ba}^* \quad (22)$$

Comparison of (20) with (18) shows that $SU(3)$ and T invariance forbid $(\Delta\ell = 2)_{\text{odd}}$ in the $BB \rightarrow BB$ channel, which confirms the previous results. We have thus shown that crossing symmetry does not lead to a further reduction of the number of independent amplitudes in a given channel.

Table V gives a certain number of examples. $f(SU(3))$ means: forbidden by $SU(3)$ and T (or C) invariance; $f(T)$ means: forbidden by T invariance only.

In order to detect the presence of the listed terms, it is necessary to measure the polarization of at least two particles⁷⁾.

Table V

	$(\Delta \ell = 2)_{\text{odd}}$	ts - st	ts + st
$p \Sigma^+ \rightarrow p \Sigma^+$	f(T)	f(T)	f(SU ₃)
$p \bar{p} \rightarrow \bar{\Sigma}^+ \Sigma^+$	f(SU ₃)	f(C)	f(C)
$p \Sigma^- \rightarrow p \Sigma^-$	f(T)	f(T)	a
$p \bar{p} \rightarrow \bar{\Sigma}^- \Sigma^-$	a	f(C)	f(C)
$p \Lambda \rightarrow p \Lambda$	f(T)	f(T)	a
$p \bar{p} \rightarrow \bar{\Lambda} \Lambda$	a	f(C)	f(C)
$p \Xi^- \rightarrow p \Xi^-$	f(T)	f(T)	f(SU ₃)
$p \bar{p} \rightarrow \bar{\Xi}^- \Xi^-$	f(SU ₃)	f(C)	f(C)
$p \Xi^0 \rightarrow p \Xi^0$	f(T)	f(T)	a
$p \bar{p} \rightarrow \bar{\Xi}^0 \Xi^0$	a	f(C)	f(C)
$n \Xi^0 \rightarrow p \Xi^-$	f(I)	f(I)	a
$n \bar{p} \rightarrow \bar{\Xi}^0 \Xi^-$	a	f(I)	f(I)
$p \Sigma^0 \rightarrow n \Sigma^+$	f(I)	f(I)	a
$\bar{p} n \rightarrow \bar{\Sigma}^0 \Sigma^+$	a	f(I)	f(I)
$p \Lambda \rightarrow n \Sigma^+$	f(SU ₃)	a	a
$\bar{p} n \rightarrow \bar{\Lambda} \Sigma^+$	a	a	f(SU ₃)
$p \Lambda \rightarrow p \Sigma^0$	f(SU ₃)	a	a
$p \bar{p} \rightarrow \bar{\Lambda} \Sigma^0$	a	a	f(SU ₃)

f(T): forbidden by T; f(C): forbidden by C; f(SU₃): forbidden by SU₃;

a: allowed; f(I): forbidden by I-spin.

ACKNOWLEDGEMENTS

The author thanks Prof. J.S. Bell for asking the right question and for constant stimulation of this work, and Prof. M. Gourdin, J. Prentki and L. Wolfenstein for discussions.

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Proof of equation (4)

Consider the two-particle states $| I I_3 Y B X, I' I'_3 Y' - B X' \rangle$, where B is the baryon number and X and X' stand for spin and space variables.

Further quantum numbers which may be needed are not specified. Define charge conjugation C and the operations E and R by the following equations:

$$C | I I_3 Y B X, I' I'_3 Y' - B X' \rangle = | I - I_3 - Y - B X, I' - I'_3 - Y' B X' \rangle \quad (A1)$$

$$E | I I_3 Y B X, I' I'_3 Y' - B X' \rangle = | I' I'_3 Y' B X, I I_3 Y - B X' \rangle \quad (A2)$$

$$R | I I_3 Y B X, I' I'_3 Y' - B X' \rangle = | I - I_3 - Y B X, I' - I'_3 - Y' - B X' \rangle \quad (A3)$$

$$= (-)^{\eta} | I' - I'_3 - Y' - B X', I - I_3 - Y B X \rangle \quad (A4)$$

where $\eta = \begin{cases} +1 & \text{for bosons} \\ -1 & \text{for fermions} \end{cases}$

After projection on the partial wave $| j \ell s \rangle$ one finds³⁾:

$$\begin{aligned} \langle j \ell s | C | I I_3 Y B X, I' I'_3 Y' - B X' \rangle = \\ (-)^{\ell+s} \langle j \ell s | E R | I I_3 Y B X, I' I'_3 Y' - B X' \rangle \end{aligned} \quad (A5)$$

C invariance implies for a reaction

$$\langle f | S | i \rangle = \langle C f | S | C i \rangle \quad (A6)$$

If ν is an irreducible representation of $SU(3)$:

$$\begin{aligned} \langle f | S | i \rangle &= \langle f | \nu_f j_f s_f \rangle \langle \nu_i j_i s_i \rangle A(\nu_f j_f s_f, \nu_i j_i s_i, \alpha) \\ &= (-)^{l_i + s_i + l_f + s_f} \langle R f | \nu_f j_f s_f \rangle \langle \nu_i j_i s_i | R e \rangle A(\nu_f j_f s_f, \nu_i j_i s_i, \alpha) \end{aligned} \quad (A7)$$

where $A(\dots, \dots)$ is the reduced matrix element $\langle \nu_f j_f s_f || S || \nu_i j_i s_i \rangle$

$$\langle E f | \nu_f j_f s_f \rangle = \epsilon(\nu_f) \langle f | \nu_f j_f s_f \rangle \quad (A8)$$

According to the rules of the $SU(3)$ tensor calculus,

$$\epsilon(\nu) = \begin{cases} +1 & \nu = 1, 8_s, 27 \\ -1 & \nu = 8_a, 10, \bar{10} \end{cases} \quad (A9)$$

$$\langle R f | \nu_f j_f s_f \rangle = \rho(\nu_f) \langle f | \bar{\nu}_f j_f s_f \rangle \quad (A10)$$

with $\rho(\nu_f) = \pm 1$. For selfconjugate states of selfconjugate representations, $\rho(\nu) = \epsilon(\nu)$. Using definition(A3), this can be extended to non-selfconjugate states. Therefore

$$\epsilon(\nu_i) \rho(\nu_i) \epsilon(\nu_f) \rho(\nu_f) = 1 \quad (A11)$$

This equation remains true for non-selfconjugate representations with arbitrary phase conventions, since, in this case, $\nu_i = \nu_f$. (A7), (A8), (A10) and (A11) imply:

$$A(\bar{\nu}_f \ell_f s_f, \bar{\nu}_i s_i \ell_i, \alpha) = (-)^{\ell_i + s_i + \ell_f + s_f} A(\nu_f \ell_f s_f, \nu_i s_i \ell_i, \alpha) \quad (4)$$

R E F E R E N C E S

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- 3) K. Itabashi and K. Tanaka, Phys.Rev. 135, B452 (1964).
- 4) When expressed in terms of Pauli spinors, the amplitudes are defined
in such a way that spinor indices are saturated between B_1 and B_3 ,
 B_2 and B_4 in channel I, between B_1 and $\bar{B}_2$, $\bar{B}_3$ and B_4 in channel ¹II,
and between B_1 and B_3 , $\bar{B}_4$ and $\bar{B}_2$ in channel III. This fixes the
relative sign of ts and st .
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and the Pauli principle. For example, due to T, $(\Delta\ell = 2)_{\text{odd}}$
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