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 $U_3 \times U_3$ SYMMETRY FOR COLLINEAR PROCESSES
H. Ruegg and D.V. Volkov^{*)}

CERN - Geneva

A B S T R A C T

Some implications of $U_3 \times U_3$ symmetry for forward and backward scattering of spin $\frac{1}{2}$ baryons and pseudoscalar mesons are discussed. The $U_3 \times U_3$ group which is considered, is the smallest group which combines SU_3 and spin, and which commutes with a Lorentz transformation along a given direction. A linear combination of the Johnson-Treiman relation still holds, as well as the relation $1/4 \sigma(K^-p \rightarrow K^+\Xi^-) = \sigma(\pi^-p \rightarrow K^+\Sigma^-) = \sigma(K^-p \rightarrow K^0\Xi^0) = \sigma(K^-p \rightarrow K^+\Sigma^-)$ for the scattering cross-section in forward and backward direction.

^{*)} Present address: The Physical-Technical Institute,
Kharkov, U.S.S.R.

I. INTRODUCTION

It has been proposed¹⁾ that particles at rest are classified according to the parity conserving $U_6 \times U_6$ group, whereas vertex functions and the scattering of particles moving in the same direction, obey the symmetry of the subgroup $(U_6)_W$ which commutes with the Lorentz transformation along this direction. The predictions of this scheme may also be obtained using the various relativistic generalizations²⁾ of the U_6 symmetry proposed by Gürsey, Radicati and Sakita³⁾, especially if the inhomogeneous $\tilde{U}(12)$ group is used⁴⁾.

A more modest scheme consists in classifying particles at rest according to the original U_6 group³⁾, and assuming $U_3 \times U_3$ symmetry for vertex functions and collinear processes. This predicts the ratio $-\frac{3}{2}$ for the magnetic form factor of proton and neutron⁵⁾. The purpose of this note is to apply $U_3 \times U_3$ invariance to the forward and backward scattering of octet baryon and pseudoscalar octet meson. A linear combination of the Johnson-Treiman relations⁶⁾ is found to hold, as well as some equalities between cross-section in the forward and backward direction, previously derived under much more restrictive conditions by Blankenbecler et al.⁷⁾. The results of $U_3 \times U_3$ symmetry are the same as those obtained from the inhomogeneous $SL(6,C)$ group⁸⁾, the present scheme having however the advantage of much greater simplicity.

It has been shown that the $\tilde{U}(12)$ symmetry, when irregular couplings are included, is not inconsistent with unitarity⁹⁾, but cannot simultaneously satisfy unitarity and crossing symmetry for the elastic quark-quark and quark-antiquark scattering¹⁰⁾. The same statement is true for the chain $U_6 \times U_6$ at rest $\rightarrow (U_6)_W$ collinear $\rightarrow U_3 \times U_3$ coplanar¹¹⁾. No such contradiction has been demonstrated for the present scheme, which assumes the chain U_6 at rest $\rightarrow U_3 \times U_3$ collinear $\rightarrow U_3$ coplanar¹²⁾.

It should finally be stressed that the collinear $U_3 \times U_3$ group is the smallest group which combines unitary (SU_3) spin with ordinary spin.

II. THE GROUPS U_6 and $U_3 \times U_3$ AND THEIR REPRESENTATIONS

The group U_6 is defined by its action on a SU_3 triplet of spin $\frac{1}{2}$ quarks, its Hermitian generators being the 6×6 matrices

$$\lambda_i \times T_\mu \quad (1)$$

where λ_i ($i = 0 \dots 8$) are the generators of U_3 and T_μ ($\mu = 0 \dots 3$) the Pauli matrices together with the 2×2 unit matrix.

$U_3 \times U_3$ is the subgroup of U_6 which commutes with a Lorentz transformation along a given, say the z -direction (generated by $i T_z$). Its generators are therefore:

$$\lambda_i \frac{(1 \pm T_z)}{2} \quad (2)$$

Using tensor notation, (1) corresponds to

$$\delta q_A = \delta q_{a\alpha} = i (\lambda_i)_a^b (T_\mu)_\alpha^\beta q_{b\beta} \delta \varepsilon_{i\mu} \quad (3)$$

$A = 1 \dots 6 \quad a = 1 \dots 3 \quad \alpha = 1, 2$

and (2) gives rise to

$$\left. \begin{aligned} \delta q_{a1} &= i (\lambda_i)_a^b q_{b1} \delta \varepsilon_{i1} \\ \delta q_{a2} &= i (\lambda_i)_a^b q_{b2} \delta \varepsilon_{i2} \end{aligned} \right\} \quad (4)$$

If one chooses the z -direction as the quantization axis, the two U_3 groups transform the two helicity states separately. In the following, we use the notation

$$\left. \begin{aligned} q_{a1} &\equiv q_a \in (3, 1) & q_{a2} &\equiv q_{\bar{a}} \in (1, 3) \\ q^{a1} &\equiv q^a \in (\bar{3}, 1) & q^{a2} &\equiv q^{\bar{a}} \in (1, \bar{3}) \end{aligned} \right\} \quad (5)$$

Parity relates q_a to $q^{\bar{a}}$.

The representation 35 of U_6 contains as usual 8 0^- and 9 1^- mesons.

$$M_B^A = \frac{1}{\sqrt{2}} P_\theta^a \delta_\beta^\alpha + V_\theta^a (\chi_8)_\beta^\alpha + \frac{1}{\sqrt{3}} \delta_\theta^a (\chi_1)_\beta^\alpha \quad (6)$$

where χ_8 is the spin 1 wave function for the vector octet, etc.
Under $U_3 \times U_3$, 35 splits into $(8,1) \oplus (1,8) \oplus (3,\bar{3}) \oplus (\bar{3},3) \oplus (1,1)$, namely:

$$M_B^A \big|_{\alpha=1, \beta=1} = M_\theta^a + \frac{1}{\sqrt{6}} \delta_\theta^a M$$

$$M_B^A \big|_{\alpha=2, \beta=2} = M_{\bar{\theta}}^{\bar{a}} - \frac{1}{\sqrt{6}} \delta_{\bar{\theta}}^{\bar{a}} M \quad (7)$$

$$M_B^A \big|_{\alpha=1, \beta=2} = M_{\bar{\theta}}^a$$

$$M_B^A \big|_{\alpha=2, \beta=1} = M_{\theta}^{\bar{a}}$$

where

$$\left. \begin{aligned} M_\theta^a &= \frac{1}{\sqrt{2}} P_\theta^a + V_\theta^a (\chi_8)_1^1 \\ M_{\bar{\theta}}^{\bar{a}} &= \frac{1}{\sqrt{2}} P_{\bar{\theta}}^{\bar{a}} + V_{\bar{\theta}}^{\bar{a}} (\chi_8)_2^2 \quad (\chi_8)_2^2 = -(\chi_8)_1^1 \\ M_{\bar{\theta}}^a &= V_\theta^a (\chi_8)_2^1 + \frac{1}{\sqrt{3}} \delta_\theta^a (\chi_1)_2^1 \\ M_{\theta}^{\bar{a}} &= V_{\bar{\theta}}^{\bar{a}} (\chi_8)_1^2 + \frac{1}{\sqrt{3}} \delta_{\bar{\theta}}^{\bar{a}} (\chi_1)_1^2 \\ M &= \sqrt{2} (\chi_1)_1^1 = -\sqrt{2} (\chi_1)_2^2 \end{aligned} \right\} \quad (8)$$

4.

The representation 56 of U_6 contains 10 $\frac{3}{2}^+$ particles and 8 $\frac{1}{2}^+$ baryons:

$$B_{ABC} = D_{abc}(\chi_{10})_{\alpha\beta\gamma} + \frac{1}{3\sqrt{2}} \left\{ N_a^d \epsilon_{abc} (\chi_8)_\alpha \epsilon_{\beta\gamma} + N_b^d \epsilon_{dca} (\chi_8)_\beta \epsilon_{\gamma\alpha} + N_c^d \epsilon_{dab} (\chi_8)_\gamma \epsilon_{\alpha\beta} \right\} \quad (9)$$

Under $U_3 \times U_3$, 56 splits into $(10,1) \oplus (1,10) \oplus (6,3) \oplus (3,6)$:

$$\left. \begin{aligned} B_{abc} &= D_{abc}(\chi_{10})_{111} \\ B_{\bar{a}\bar{b}\bar{c}} &= D_{abc}(\chi_{10})_{222} \\ B_{a\bar{b}\bar{c}} &= \sqrt{3} D_{abc}(\chi_{10})_{112} + \frac{1}{\sqrt{6}} (N_a^d \epsilon_{dbc} + N_b^d \epsilon_{dac}) (\chi_8)_1 \\ B_{a\bar{b}\bar{c}} &= \sqrt{3} D_{abc}(\chi_{10})_{122} + \frac{1}{\sqrt{6}} (N_b^d \epsilon_{dac} + N_c^d \epsilon_{dab}) (\chi_8)_2 \end{aligned} \right\} \quad (10)$$

Higher representation could be analyzed in a similar way.

III. BARYON-MESON SCATTERING IN THE FORWARD AND BACKWARD DIRECTION

Using (8) and (10), it is easy to write down the most general expression which contains baryon-pseudoscalar-meson scattering. The contribution with opposite baryon helicity may be obtained by a parity transformation. One gets:

$$\begin{aligned}
& A_1 \bar{B}^{abc} B_{abc} \bar{M}_e^d M_a^e + A_2 \bar{B}^{abc} B_{abc} \bar{M}_e^d M_d^e \\
& + A_3 \bar{B}^{abc} B_{dec} \bar{M}_a^d M_b^e \\
& + A_4 \bar{B}^{abc} B_{deb} \bar{M}_e^d M_a^e + A_5 \bar{B}^{abc} B_{deb} M_e^d \bar{M}_a^e \\
& + A_6 \bar{B}^{abc} B_{aed} \bar{M}_e^d M_c^e + A_7 \bar{B}^{abc} B_{aed} M_e^d \bar{M}_c^e \\
& + A_8 \bar{B}^{abc} B_{ade} \bar{M}_b^d M_c^e + A_9 \bar{B}^{abc} B_{ade} M_b^d \bar{M}_c^e .
\end{aligned} \quad (11)$$

The different terms of (11) are not necessarily independent. Eq. (11) describes also various processes where the decuplet with helicity $\pm \frac{1}{2}$ and the vector mesons with helicity 0 may enter. It should be stressed that processes which involve decuplet baryon with helicity $\pm \frac{3}{2}$ and vector mesons with helicity ± 1 are not related to the previous ones. This diminishes the predicting power of the collinear $U_3 \times U_3$ group, as compared with $(U_6)_W$.

IV. THE JOHNSON-TREIMAN AND OTHER RELATIONS

When (8) and (10) are applied to (11), one gets for baryon-meson scattering expressions of the form $\text{tr } \bar{B}B \bar{P}P$ $\text{tr } \bar{B}\bar{P}$ $\text{tr } BP$ and various permutations of those. We now show that the term

$$\text{tr } \bar{B}\bar{P} \text{tr } BP - \text{tr } \bar{B}P \text{tr } B\bar{P} \quad (12)$$

is never present.

The first three terms of Eq. (11) are symmetric in the interchange of P and $\bar{P}$. The next four terms contain the product $P\bar{P}$ or $\bar{P}P$.

6.

Direct computation shows that the last two terms do not contribute either.

Using standard SU_3 calculations¹³⁾, the absence of terms (12) implies for the amplitudes of elastic scattering in the forward direction:

$$(K^+p) - (K^-p) = (\pi^+p) - (\pi^-p) + (K^0p) - (\bar{K}^0p) \quad (13)$$

Using the optical theorem, Eq. (13) is also true for total cross-sections.

This relation was also found by W. Rühl⁸⁾, using the inhomogeneous $SL(6, C)$ group. It is very well satisfied experimentally¹⁴⁾ from 6 GeV/c to 18 GeV/c, kinetic energy of the incident meson.

The absence of Eq. (12) also implies¹³⁾ for the differential cross-sections in the forward and backward direction:

$$\frac{1}{4} \sigma(K^-p \rightarrow K^+ \Xi^-) = \sigma(K^-p \rightarrow K^0 \Xi^0) = \sigma(\pi^-p \rightarrow K^+ \Xi^-) \quad (14)$$

Using SU_3 invariance:

$$\sigma(K^-p \rightarrow K^0 \Xi^0) = \sigma(K^-p \rightarrow \pi^+ \Sigma^-) \quad (15)$$

Eq. (14) was derived in ref. 7 under much more restrictive hypotheses.

The relation $1/4 \sigma(K^-p \rightarrow K^+ \Xi^-) = \sigma(K^-p \rightarrow \pi^+ \Sigma^-)$ does not seem to be very well satisfied experimentally¹⁵⁾. The same is true for Eq. (15), which requires only SU_3 . However, the best test would be provided by the relation

$$\sigma(K^-p \rightarrow K^+\Xi^-) = 4 \sigma(K^-p \rightarrow K^0\Xi^0)$$

where the kinematical factors are the same for both processes. A rough estimate of the experimental^{15),16)} results at 1.5 GeV/c and at 3.0 GeV/c gives

1.5 GeV/c		3.0 GeV/c	
0°	180°	0°	180°
> 4	1.5	16	1.4

whereas the value of the experimental ratio for $\theta = 180^\circ$ is consistent with the predicted value 1, within experimental errors, there is a large discrepancy for $\theta = 0^\circ$. If these experimental values are confirmed¹⁷⁾, it is surprising that the Johnson-Treiman relation holds so well.

A P P E N D I X

We rederive the value for the ratio of magnetic form-factors of the proton and the neutron⁵⁾. In the Breit system, the two baryons which interact with the photon have opposite momenta. Assume that in this system, the charge operator transforms like (8,1) and (1,8), and the magnetic moment operator like (3, $\bar{3}$) and ($\bar{3}$,3)¹⁸⁾. In the latter case, when $U_3 \times U_3$ is reduced to SU_3 , only the octet part should be retained.

The magnetic part of the vertex may therefore be written:

$$\mu(q^2) = G_M(q^2) \bar{B}^{abc}(\vec{q}) B_a \bar{c} d(-\vec{q}) Q_c^d \quad (A1)$$

where Q is the SU_3 charge matrix.

The contribution of (A1) to the nucleus is given by Eq. (10):

$$\begin{aligned} \mu(q^2) &= G_M(q^2) (5 \operatorname{tr} \bar{N} Q N + \operatorname{tr} \bar{N} N Q) \\ &= G_M(q^2) [3 \operatorname{tr} \bar{N} (Q N + N Q) + 2 \operatorname{tr} \bar{N} (Q N - N Q)] \end{aligned} \quad (A2)$$

This gives

$$\frac{\mu_p(q^2)}{\mu_n(q^2)} = -\frac{3}{2} \quad (A3)$$

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