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ANALYSIS OF CP VIOLATION IN POSSIBLE EXPERIMENTS ON K^0 DECAY

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A B S T R A C T

The phenomenological analysis of CP violation in $K^0 \rightarrow 2 \pi$ is reviewed in order to relate the information obtainable to various classes of Hamiltonian models. In particular, it is shown that two additional experiments beyond the rate of $K_L^0 \rightarrow \pi^+ + \pi^-$ are necessary to determine one new CP violating parameter, which is proportional to the $\Delta I = \frac{1}{2}$ violating amplitude. While a non-zero value of this parameter would rule out several classes of models, a zero value would rule out none. A test of CPT invariance from $K^0 \rightarrow 2 \pi$ decays is presented. The analysis is extended to the charge asymmetry in the leptonic decays of K_L^0 .

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I. INTRODUCTION

The decay of the long-lived K^0 meson into two pions ¹⁾ indicates that CP invariance is violated in the fundamental Hamiltonian of elementary particles. Theoretical discussions of this problem have been in the form of a description of K^0 decay processes in terms of phenomenological parameters ²⁾ or in specific attempts to formulate Hamiltonian models of CP violation ³⁾. The present paper consists of an extension and a discussion of the phenomenological analysis of Wu and Yang ²⁾ and an attempt to relate this to Hamiltonian models.

The major conclusion of the paper of Wu and Yang is that the "source" of the CP violation in the process $K^0 \rightarrow 2\pi$ must be found primarily either in a CP violation proportional to the $I = 2$ decay channel amplitude or in a single real parameter describing the CP violation in the dispersive part of the K^0 self-energy matrix or a combination thereof. As a consequence, not emphasized in their paper, all experiments on the process $K^0 \rightarrow 2\pi$ beyond the original experiment of Christenson et al. ¹⁾ are likely to determine only one additional piece of information about CP violation; namely, how the CP violation is divided between these two "sources". Section II of this paper is devoted to a review of this result and Section III to a discussion of its implications with respect to testing proposed Hamiltonians.

In Section IV the question of the relation between the magnitude of the CP violation as measured by phenomenological parameters and the magnitude of the CP violation in the Hamiltonian is discussed. The analysis is extended in Section V to observations on the charge asymmetry in the leptonic decays of the long-lived K^0 mesons. A basic assumption in the considerations of Sections II to V is CPT invariance. In Section VI we review our results from the view-point of defining experimental tests of CPT invariance. The major conclusions are summarized in Section VII.

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II. ANALYSIS OF $K^0 \rightarrow 2\pi$ EXPERIMENTS

There are nine phenomenological parameters needed in the analysis of possible observations on $K^0 \rightarrow 2\pi$ decays provided we assume CPT invariance. First, there are five real parameters from the mass matrix $\Gamma + iM$, which is discussed in Appendix A :

$$\begin{array}{ll}
 m_S - m_L & = \text{mass difference of } K_S \text{ and } K_L \\
 \gamma_S & = \text{width of } K_S \\
 \gamma_L & = \text{width of } K_L
 \end{array}
 \left. \vphantom{\begin{array}{l} m_S - m_L \\ \gamma_S \\ \gamma_L \end{array}} \right\} \begin{array}{l} \text{three weak} \\ \text{CP conserving} \\ \text{parameters} \end{array}$$

$$r = 1 - \epsilon = \text{complex admixture coefficient} \left. \vphantom{r} \right\} \begin{array}{l} \text{two weak} \\ \text{CP violating} \\ \text{parameters} \end{array}$$

where K_S and K_L are the short-lived and long-lived K^0 mesons related to K and $\bar{K}$ by

$$\begin{aligned}
 |K_S\rangle &= (|K\rangle + r |\bar{K}\rangle) / (1 + |r|^2)^{\frac{1}{2}} \\
 |K_L\rangle &= (|K\rangle - r |\bar{K}\rangle) / (1 + |r|^2)^{\frac{1}{2}}
 \end{aligned}$$

Next there are three real parameters describing the decay amplitudes

$$\begin{array}{ll}
 A_0 & = \text{real amplitude for } K \rightarrow 2\pi \text{ in} \\
 & \quad I = 0 \text{ standing wave state} \\
 A_2 & = \text{complex amplitude for } K \rightarrow 2\pi \text{ in} \\
 & \quad I = 2 \text{ standing wave state}
 \end{array}
 \left. \vphantom{\begin{array}{l} A_0 \\ A_2 \end{array}} \right\} \begin{array}{l} \text{two weak CP} \\ \text{conserving and} \\ \text{one weak CP} \\ \text{violating parameters.} \end{array}$$

The phase convention by which A_0 is chosen to be real and the relation between the physical decay amplitudes and A_0 and A_2 is discussed in Appendix B. This relation introduces one additional parameter

$$\delta_2 - \delta_0 = \left. \begin{array}{l} \text{difference between } \pi\pi \text{ scattering} \\ J = 0 \text{ phase shifts with isotopic} \\ \text{spins } I = 2 \text{ and } I = 0 \end{array} \right\} \begin{array}{l} \text{one} \\ \text{strong} \\ \text{parameter.} \end{array}$$

The CP violation in the decay amplitude is expressed by the relative phase of A_0 and A_2 and so, with A_0 real, is proportional to $\text{Im } A_2$.

Of the six CP conserving parameters four are known with adequate precision for the discussion of CP violating experiments

$$\begin{aligned} \gamma_S &\approx 2 A_0^2 = 1.1 \times 10^{10} \text{ sec}^{-1} \\ \gamma_L &= 18 \pm 2 \times 10^6 \text{ sec}^{-1} \\ |\text{Re } A_2|^2 &\leq [1 / \cos(\delta_2 - \delta_0)] \times 10^6 \text{ sec}^{-1} \end{aligned}$$

Of the remaining two, $(m_S - m_L)$ can be determined from experiments that do not involve CP violation ⁴⁾ whereas $(\delta_2 - \delta_0)$ most likely cannot.

A major conclusion from the analysis of Wu and Yang ²⁾ is that, if $|\epsilon|$ is of the order 4×10^{-3} or greater, the phase δ of ϵ is approximately determined. If it is assumed that $\Delta Q = -\Delta S$ decays are effectively forbidden and that there is at most a small CP violation in the decay $K^0 \rightarrow 3\pi$, then to a high degree of accuracy

$$\tan \delta = \tan \delta_s = -2 \left(\frac{m_S - m_L}{\gamma_S - \gamma_L} \right) \quad (1)$$

where $\epsilon = \epsilon_0 \exp(i\delta)$. This equation has previously been discussed under the more restrictive assumption of a superweak interaction ³⁾.

Since only the tangent of δ is determined it is convenient to restrict δ between -90° and $+90^\circ$ leaving ϵ_0 as a real number which may be either positive or negative. Equation (1) is assumed to be approximately valid throughout this analysis. It is to be noted that even fairly large amounts of CP violation in the $K^0 \rightarrow 3\pi$ decays will not seriously modify the value of δ . Considering an extreme possibility ⁵⁾ we find $\delta = \delta_s \pm \Delta$ where values of Δ , which depend on ϵ_0 and $(m_S - m_L)$, are shown in the Table. On the other hand if we included the large CP violation in $\Delta Q = -\Delta S$ amplitudes that is allowed by present experiments ⁶⁾ a considerably greater range of values of δ is allowed. These questions are discussed in more detail in Appendix A. Further experiments on CP violation in K^0 leptonic decays and 3π decays are of importance not only in their own right but in order to decrease this uncertainty in the analysis of $K^0 \rightarrow 2\pi$ decays.

It follows that experiments on CP violation in $K^0 \rightarrow 2\pi$ have as their goal the determination of three real parameters: $(\delta_2 - \delta_0)$, ϵ_0 and $\text{Im } A_2$. Of these, $(\delta_2 - \delta_0)$ has nothing directly to do with weak interactions but, as noted in Ref. ²⁾, is useful in determining $\text{Re } A_2$ from the measured branching ratio for $K_S^0 \rightarrow \pi^0 + \pi^0$. The quantity ϵ_0 measures the ratio of the CP violating part of the self-energy matrix to the CP conserving part [cf. Eq. (A.7)]. Unfortunately theoretical models are notoriously unreliable in calculating self-energies so that only the order of magnitude of ϵ_0 is really of theoretical interest. In all probability this is already known from the original experiment of Christenson et al. ¹⁾. Therefore, further experiments beyond the original one primarily yield one new piece of information about CP violation, namely the value of $\text{Im } A_2$.

To relate these three parameters to experiment it is convenient to introduce the complex amplitude ratios

$$\eta_{+-} = |\eta_{+-}| e^{i\chi} = \frac{A(K_L^0 \rightarrow \pi^+ \pi^-)}{A(K_S^0 \rightarrow \pi^+ \pi^-)} = \frac{1}{2} (\epsilon + \epsilon') \quad (2a)$$

$$\eta_{00} = |\eta_{00}| e^{i\psi} = \frac{A(K_L^0 \rightarrow \pi^0 + \pi^0)}{A(K_S^0 \rightarrow \pi^0 + \pi^0)} = \frac{1}{2} (\epsilon - 2\epsilon') \quad (2b)$$

$$\epsilon' = \sqrt{2} i \exp[i(\delta_2 - \delta_0)] \operatorname{Im} A_2 / A_0 \quad (3)$$

where the last equality in Eqs. (2) is based on the approximation, $|\epsilon| \ll 1$, $|\epsilon'| \ll 1$, justified in Appendix B. The three unknown parameters may be determined from three experiments :

- 1) The measurement ¹⁾ of the rate $K_L^0 \rightarrow \pi^+ + \pi^-$, which determines $|\eta_{+-}|$ and from which it can be deduced

$$|\epsilon + \epsilon'| = (4 \pm 0.5) \times 10^{-3} \quad (4)$$

- 2) Experiments on the interference between the decays $K_L \rightarrow \pi^+ + \pi^-$ and $K_S \rightarrow \pi^+ + \pi^-$ observed in beams prepared in various ways including regeneration when calculated to first order in CP violation ⁷⁾ depend only on η_{+-} and so may be used to determine one additional parameter, the phase χ .

- 3) The measurement of the rate $K_L^0 \rightarrow \pi^0 + \pi^0$, which determines $|\eta_{00}|$.

The graphical solution determining ϵ_0 , $(\delta_2 - \delta_0)$, and $|\epsilon'| = \sqrt{2} |\operatorname{Im} A_2 / A_0|$ from the experimental quantities $|\eta_{+-}|$, $|\eta_{00}|$, and χ is shown in the Figure. In general there are two solutions which can be distinguished only by some other knowledge about $\delta_2 - \delta_0$, by a rough measurement of the phase ψ , or possibly by a measurement of the charge asymmetry in leptonic decays discussed in

Section V. The uncertainty in the solutions arising from the range of values of δ allowed by the Table most likely will not be larger than the experimental uncertainty. In order that a solution exist at all, the data must satisfy the inequality

$$|\eta_{00}| \geq 2 |\eta_{+-} \sin(\delta_s - \chi)| \quad (5a)$$

where the equality corresponds to the case that the two solutions coincide. Since the major goal is the determination of $|\epsilon'|$ the following comments on the solution shown in the Figure are of particular interest.

a) The solution for $|\epsilon'|$ depends on χ and δ_s only in the combination $(\chi - \delta_s)$. Since δ_s depends on $(m_S - m_L)$, which is not known accurately, it is desirable to measure $(\chi - \delta_s)$ directly, which can be done by performing an interference experiment at the end of a long regenerator⁸⁾.

b) While the three experiments are not sufficient to determine $|\epsilon'|$ uniquely it is possible that two of them may set a lower limit on $|\epsilon'|$ by the inequalities

$$|\epsilon'| \geq |\eta_{+-} \sin(\chi - \delta_s)| \quad (5b)$$

$$|\epsilon'| \geq \frac{2}{3} (|\eta_{+-}| - |\eta_{00}|) \quad (5c)$$

c) If the experiments yield $|\eta_{+-}| \approx |\eta_{00}|$ and $\sin(\chi - \delta_s) \approx 0$ then in addition to the solution $|\epsilon'| = 0$ there exists a second solution

$$|\epsilon'| = \frac{2}{3} (|\eta_{+-}| + |\eta_{00}|) = \frac{4}{3} |\eta_{+-}| \approx 2.7 \times 10^{-3}$$

This solution would require $\delta_2 - \delta_0 = \delta_s - \frac{\pi}{2} + n\pi$, where n is an integer, and it is conceivable that other information could rule out this value of $(\delta_2 - \delta_0)$. However, the value of $(\delta_2 - \delta_0)$ is very sensitive to small changes in $(\chi - \delta_s)$ so that it is unlikely that this condition will prove very useful in practice.

III. CONSEQUENCES OF HAMILTONIAN MODELS

We wish to classify proposed Hamiltonian models of CP violation from the point of view of what they say about the experimental results on $K^0 \rightarrow 2\pi$; in particular, as to the magnitude of $\text{Im } \Lambda_2$. We may write the interaction Hamiltonian of elementary particles (omitting purely leptonic terms) as

$$H = H_{00} + \sum c_{qs} H_{qs} + \text{h.c.} \quad (6)$$

where the notation H_{qs} implies that the strongly-interacting particles satisfy $\Delta Q = q$, $\Delta S = s$. Thus H_{00} includes the usual strong (plus electromagnetic) interaction, H_{10} the β decay interaction, H_{01} the non-leptonic $\Delta S = 1$ decay interaction, and H_{11} the $\Delta Q = \Delta S = 1$ leptonic decay interaction. The summation covers $q \geq 0$ and omits $q = 0$, $s = 0$. In the standard current-current form used to describe weak interactions $c_{10} = G_1$, $c_{11} = c_{01} = G \tan \theta$, and all other c_{qs} are zero.

The various Hamiltonian models may be divided ³⁾ into three general classes according to the strength of the coupling of the CP violating term: (a) the coupling is somewhat weaker than the strong interaction coupling, (b) the coupling is somewhat weaker than the usual weak coupling, and (c) the coupling is much weaker than the usual weak coupling. For our purposes we consider a further subdivision and describe these classes in terms of Eq. (6):

- 1A. The CP violation occurs in H_{00} and is characterized by $\Delta I = 0$; thus, the CP violation appears as a feature of the strong interactions ⁹⁾.
- 1B. The CP violation occurs in H_{00} and is characterized by violation of charge independence. One possibility would be that the CP violation is a feature of the electromagnetic interaction ¹⁰⁾.

- 2A. The CP violation occurs in H_{01} , but the parity-violating part of H_{01} is CP invariant ¹¹⁾.
- 2B. CP violation occurs in the parity-violating part of H_{01} and perhaps also in the parity-conserving part ¹²⁾.
- 2C. There exist CP violating $\Delta Q = -\Delta S$ interactions; that is, $c_{1,-1}$ is non-zero and complex ¹⁴⁾.
3. There exists a superweak interaction of the form $c_{02} H_{02}$ (containing a parity-conserving part) with either c_{02} complex or H_{02} intrinsically CP violating ¹⁵⁾.

Models of type 2A and 2B may differ in whether CP violation also occurs in H_{10} and H_{11} , but this does not concern us here. Models in which CP violation occurs only as a phase difference between the P conserving and P violating parts of H_{01} may be classed either 2A or 2B and are discussed in more detail in Section IV.

These models have the following consequences for the $\text{Im } A_2$.

a) Models in class 2A, 2C, and 3 yield a zero value for $\text{Im } A_2$ neglecting higher-order weak interaction effects. This follows since in these cases the CP violating interaction makes no lowest order contribution to the amplitude $K \rightarrow 2\pi$, which is caused by the parity-violating part of H_{01} . For model 3 CP violation results from a first-order contribution to M , whereas in models 2A and 2C it is due to the second-order contributions to $\Gamma + iM$ of the transitions (primarily virtual) to 3π and leptonic states, respectively.

b) For model 1A we expect

$$\frac{\text{Im } A_2}{A_0} \approx \frac{\Delta I = \frac{1}{2} \text{ amplitude}}{\Delta I = \frac{1}{2} \text{ amplitude}} \times \frac{\text{CP violating amplitude}}{\text{CP conserving amplitude}}$$

$$\approx 5 \times 10^{-2} \times 4 \times 10^{-3} = 2 \times 10^{-4}$$

(7)

where we have used the rate $K^+ \rightarrow \pi^+ + \pi^0$ as a measure of the $\Delta I \neq \frac{1}{2}$ amplitude and the measured value given by Eq. (4) as a measure of CP violations. The point is that in this model CP violation and $\Delta I = \frac{1}{2}$ rule violation are completely independent. It is possible, of course, that the measure of CP violation is much larger than 4×10^{-3} and that the small value of ϵ_0 is accidental; this question is discussed in Section IV.

- c) If it is believed that the $\Delta I = \frac{1}{2}$ rule holds because of strong-interaction enhancement¹⁶⁾ and not because of the special structure of the weak interaction then we would expect Eq. (7) to hold for models 1B and 2B as well.
- d) If the $\Delta I = \frac{1}{2}$ rule is a basic feature of the CP invariant part of the weak interaction then for models of class 1B and some models of class 2B one would expect a violation of the $\Delta I = \frac{1}{2}$ rule to be directly associated with the CP violation¹⁷⁾. In this case Eq. (7) would not hold and larger values of $\text{Im } A_2$ would result¹⁸⁾.

It follows that a non-zero value of $\text{Im } A_2/A_0$ would definitely rule out models of class 2A, 2C, and 3, while a value of $\text{Im } A_2/A_0$ of the order of 2×10^{-3} or greater would argue against models of type 1A. On the other hand a value of $\text{Im } A_2/A_0$ equal to zero within an experimental error of the order of 10^{-4} would not clearly rule out any of the classes of models of CP violation discussed above.

Different models may be distinguishable even in this case, of course, by their predictions concerning CP violation in other processes. Thus models of class 1 have as their most characteristic prediction CP and C violation in strong and electromagnetic processes. Models of class 2A and 2B will in general predict CP violation in non-leptonic hyperon decays. If a single dominant decay channel can be assumed, the ratio of $K_S^0 \rightarrow 3\pi$ to $K_L^0 \rightarrow 3\pi$ is a measure of CP violation. In

models of class 2A and 2B this ratio may be significantly larger than ϵ_0^2 , the value it has if there is no CP violation in the $K \rightarrow 3\pi$ decay amplitudes but only in the mass matrix. On the other hand the ratio will be ϵ_0^2 for models in which the CP violation occurs only as a relative phase between P conserving and P violating parts of H_{01} or in those models of class 2B in which the P conserving part of H_{01} is CP invariant. Models of class 2C predict characteristic CP violations in the time dependence of K^0 leptonic decays.

However, as discussed in Section IV, it is probable that the CP violation is effectively small so that experimentally if it turns out that $\text{Im } A_2/A_0 \simeq 0$, it may prove hard to distinguish any other models from model 3, in which the only effective CP violation is in the mass matrix of the K^0 .

IV. MAGNITUDE OF CP VIOLATION

In this Section we wish to consider the significance for Hamiltonian models of the probable result that $|\epsilon|$ is of the order of 4×10^{-3} . The possibility that $|\epsilon|$ is considerably larger than this but that ϵ and ϵ' in Eq. (4) cancel one another is shown to be highly unlikely in Appendix B for our assumed phase convention. It must be noted that if a considerably different phase convention had been chosen, both $|\epsilon|$ and $|\epsilon'|$ would have been considerably larger [see Eqs. (B.6)] and a cancellation would take place. Thus it might appear that it is only our phase convention that makes the CP violating parameters small; that is, there might be a large CP violation in the $\Delta I = \frac{1}{2}$ transition but this does not show up because of our choice that A_0 be real. This question is closely related to the Weinberg argument¹⁹⁾ that the $\Delta I = \frac{1}{2}$ rule automatically makes CP violation in the process $K^0 \rightarrow 2\pi$ small. We wish to present arguments that, in fact, unless the smallness of $|\epsilon|$ is in some sense accidental, we can conclude that CP violation in the Hamiltonian is similarly small. The arguments are amplifications of remarks by Sachs and Treiman^{14),20)}.

Our discussion is limited to models of class 2A and 2B, in which CP violation occurs in the non-leptonic $\Delta S = 1$ Hamiltonian H_{01} , or of class 1, in which there is an effective CP violating H_{01} after the perturbation of the CP violating part of H_{00} is taken into account²¹⁾. In order that there be a large CP violation in H_{01} and only a small value of $|\epsilon|$, which essentially means a small value of M_1 (see Appendix A), it is necessary to make two assumptions, the simultaneous validity of which does not seem to follow naturally from any model.

1) It is necessary to neglect the contribution to the matrix M of virtual transitions to states other than 2π ; in particular, states of one or three mesons. While 3π states are known to contribute very little to the real decays of K^0 they would be expected to be considerably less depressed for virtual transitions for which the phase space is not so limited. If one eliminates 3π states on the philosophy of considering the simplest intermediate states; one expects a dominant contribution from the π^0 and η^0 poles. In fact, a number of calculations of the CP conserving part of M , which determines $(m_S - m_L)$, have been made solely on the basis of these pole contributions²²⁾.

2) It is necessary to assume that the phase of the virtual transitions $K^0 \rightarrow 2\pi$ is independent of the energy of the 2π state. Explicitly, if we write

$$\langle s | c_{01} H_{01} | K^0 \rangle = f(s) \exp(i\alpha(s)) \quad (8)$$

where $|s\rangle$ is the $I = 0$ 2π standing-wave state with total energy s , and f by definition is real, the assumption is that α , which would be zero if CP invariance held, is independent of s . This seems unlikely since it is to be expected that the CP violating part of H_{01} in general will have a different dynamical structure from the CP conserving one.

We now consider possible explanations for assumption 2). One is that the independence of α with respect to s follows from the fact that both the virtual and real transitions to 2π states are completely dominated by one resonant state. If $m_S < m_L$, as indicated by one experiment²³⁾, the most likely candidate would be the " ϵ meson"²⁴⁾ with mass $M_\epsilon \simeq 730$ MeV and $\Gamma_\epsilon \simeq 90$ MeV. In this case one finds a value for $|m_S - m_L|/\gamma_s$ about ten times larger than indicated by experiment²⁵⁾. A second possible explanation is to assume that $c_{01} H_{01}$ has the form

$$c_{01} H_{01} = g_c H_c + g_v e^{i\alpha} H_v \quad (9)$$

where H_c is parity conserving, H_v is parity violating, and g_c and g_v are real. The notation is meant to imply that if $\alpha = 0$, $c_{01} H_{01}$ is CP invariant. The independence of $\alpha(s)$ in Eq. (8) on s is obvious with this assumed form. In this case it is possible by a phase transformation $g'_v = g_v e^{i\alpha}$, $g'_c = g_c e^{i\alpha}$ to transform the theory to one of class 2A in which there is no CP violation in the decay amplitude $K^0 \rightarrow 2\pi$. We still in general will have trouble with assumption 1) since the one- and three-meson contributions to the mass matrix will be out of phase with the 2π contributions. This will not be true for the special case $\alpha = \pi/2$, since the mass matrix in second-order perturbation theory involves the square of the interaction and $(e^{i\pi/2})^2$ is real. Indeed this special case corresponds to a P violating, CP violating, but C conserving non-leptonic $\Delta S = 1$ Hamiltonian, in which case there can be no decay $K_L^0 \rightarrow 2\pi$ by C invariance. One knows that this is not even approximately the true situation since it would require the s and p wave amplitudes of $\Lambda \rightarrow N + \pi$ to be 90° out of phase (before inserting the final state phase shifts) in contradiction to present experiments²⁶⁾. Also such a C conserving theory would have the consequence that the decay $K_S^0 \rightarrow 3\pi$ would be much more probable than $K_L^0 \rightarrow 3\pi$ since the most probable 3π configuration is even under C.

We conclude therefore that a Hamiltonian with a large CP violation is consistent with the small CP violation observed in $K^0 \rightarrow 2\pi$ decay only if the smallness is in some sense accidental. The most natural conclusion is that, if there is an effective CP violation in H_{01} , the CP violating part of H_{01} is of the order of 10^{-2} to 10^{-3} times the CP conserving part. The best candidate for a theory that might allow an order-of-magnitude larger CP violation than this is a model of class 2A in which the CP violating part of H_{01} is parity conserving. One need then only assume that for dynamical reasons or due to accidental cancellations the single meson and 3π virtual states make a much smaller contribution to the matrix M than do the 2π states. A simple model of this type within the framework of a current-current interaction has been discussed by Alles¹¹⁾.

V. LEPTONIC DECAYS OF THE K_L^0

Detailed discussions of possible CP violation in the leptonic decays of K^0 mesons have been given by Sachs and co-workers^{20),27)}. The relevant decay amplitudes are

$$\begin{aligned} A(K \rightarrow \pi^- + \ell^+ + \nu) &= f \\ A(\bar{K} \rightarrow \pi^+ + \ell^- + \bar{\nu}) &= f^* \\ A(\bar{K} \rightarrow \pi^- + \ell^+ + \nu) &= g \\ A(K \rightarrow \pi^+ + \ell^- + \bar{\nu}) &= g^* \end{aligned}$$

Here ℓ stands for e or μ , the amplitudes g have $\Delta Q = -\Delta S$, and the CPT theorem has been applied. f and g are functions of the kinematical variables of the final state and also of the helicity of ℓ^+ . It is understood for f^* and g^* that ℓ^- has the opposite helicity from ℓ^+ , but we will really only be interested in sums over final spins so this will be of no concern. It therefore is sufficient for our purpose to consider only a single amplitude f and a single g . Setting

$$g = X f e^{i\varphi} \quad (10)$$

φ is a measure of CP violation and X is a measure of $\Delta Q = -\Delta S$ amplitudes. The charge asymmetry in K_L^0 decays is given by

$$\begin{aligned} R &= \frac{R(K_L^0 \rightarrow \pi^+ + \ell^- + \bar{\nu})}{R(K_L^0 \rightarrow \pi^- + \ell^+ + \nu)} = \left| \frac{g^* - r f^*}{f - r g} \right|^2 \\ &= 1 + 2\epsilon_0 \cos \delta \left(\frac{1 - X^2}{1 - 2X \cos \varphi + X^2} \right) \end{aligned} \quad (11)$$

provided only terms to first order in ϵ_0 are retained.

If it is assumed that $\Delta Q = -\Delta S$ leptonic decays do not occur

$$R = 1 + 2 \epsilon_0 \cos \delta_s \quad (12)$$

It then is possible by an approximate measurement of R to distinguish between the two possible solutions for ϵ_0 discussed in Section II. For example, if it turns out that there exists a solution with $|\text{Im } A_2/A_0| = |\epsilon'| = 0$, the two possibilities for R are $R = 1 \pm 8 \times 10^{-3} \cos \delta_s$ corresponding to $|\epsilon'| = 0$ and $R = 1 \pm (8/3) \times 10^{-3} \cos \delta_s$ corresponding to $|\epsilon'| \simeq 2.7 \times 10^{-3}$. The sign of the charge asymmetry $(R-1)$ in this example is completely determined for both solutions by the sign of ϵ_0 , which is negative if $(\chi - \delta_s) \simeq \pi$ as indicated by preliminary experiments⁸⁾. Of course, present experimental limits on $\Delta Q = -\Delta S$ amplitudes⁶⁾ are not stringent enough to ensure that Eq. (12) is an adequate approximation to Eq. (11).

VI. TESTS OF CPT INVARIANCE

Since our equations are all based on CPT invariance we may review our results from the point of view of testing this assumption. It follows from Eqs. (1) and (2) that

$$\frac{\text{Im}(2\eta_{+-} + \eta_{00})}{\text{Re}(2\eta_{+-} + \eta_{00})} = \tan \delta = \tan \delta_s = -2 \frac{m_S - m_L}{\gamma_S - \gamma_L} \quad (13)$$

This may be solved for the phase ψ of η_{00} (most simply by applying the law of sines to triangle OAC in the Figure)

$$\sin(\psi - \delta_s) = -2 \sin(\chi - \delta_s) |\eta_{+-}| / |\eta_{00}| \quad (14)$$

Thus an interference experiment on the decay $K \rightarrow \pi^0 + \pi^0$ provides a possible check on CPT invariance. As noted previously such an interference experiment could also distinguish between the two solutions in the Figure, which correspond to the two values of ψ consistent with Eq. (14). A less striking test of CPT invariance is given by the inequality (5a), which is seen from Eq. (14) to correspond to $|\sin(\psi - \delta_s)| \leq 1$. Assuming $\Delta Q = -\Delta S$ decays are ruled out, Eq. (12) could provide another test of CPT; in particular, it is quite likely that both solutions of the Figure will predict the same sign of the charge asymmetry.

Naturally before abandoning CPT one would seriously question other assumptions used in deriving the tests. The tests here depend first on the approximations $|\epsilon| \ll 1$, $|\epsilon'| \ll 1$ used in Section II, which are justified in Appendix B. Secondly they depend on the approximation $\delta \simeq \delta_s$. The absolute limits on the possible value of $\Delta = \delta - \delta_s$ are not very small on the basis of present data; however, they can be made much smaller by further experiments on CP violation in $K^0 \rightarrow 3\pi$ and in K^0 leptonic decays with $\Delta Q = -\Delta S$. The above tests of CPT are significant only in so far as these further experiments are also done.

It is of interest to see how a violation of CPT invariance might change these results. For this purpose we consider the possibility of CPT violation in the mass matrix but not in the decay amplitudes. In this case the eigenstates are ²⁸⁾

$$|K_S\rangle = (1 + \frac{1}{2} \epsilon_1) |K\rangle + (1 - \frac{1}{2} \epsilon_1) |\bar{K}\rangle \quad (15a)$$

$$|K_L\rangle = (1 + \frac{1}{2} \epsilon_2) |K\rangle - (1 - \frac{1}{2} \epsilon_2) |\bar{K}\rangle \quad (15b)$$

where we will assume $|\epsilon_1| \ll 1$, $|\epsilon_2| \ll 1$, and where if CPT invariance were assumed $\epsilon_1 = \epsilon_2 = \epsilon$. In place of Eqs. (2) we now have

$$\eta_{+-} = \frac{1}{2} (\epsilon_2 + \epsilon') \quad (16a)$$

$$\eta_{00} = \frac{1}{2} (\epsilon_2 - 2\epsilon') \quad (16b)$$

As discussed in Section II and Appendix A, with the phase convention we have chosen, most of the CP violation in $\Gamma + iM$ must be attributed to the matrix M . Neglecting all CP violation in Γ , we find ²⁸⁾, if $\epsilon_j = \epsilon_{0j} e^{i\lambda_j}$,

$$\epsilon_{01} = \epsilon_{02}$$

$$\frac{1}{2} (\lambda_1 + \lambda_2) = \delta_s$$

The CPT violation is then measured by one parameter

$$\eta = \frac{1}{2} (\lambda_1 - \lambda_2)$$

The previous results in Sections II and V are now to be modified by replacing δ_s by $\delta_s - \eta$. In particular we have in place of Eqs. (13) and (11)

$$\frac{\text{Im} (2 \eta_{+-} + \eta_{00})}{\text{Re} (2 \eta_{+-} + \eta_{00})} = \tan (\delta_s - \eta) \quad (17a)$$

$$R = 1 + 2 \epsilon_0 \cos (\delta_s - \eta) \left(\frac{1 - X^2}{1 - 2 X \cos \varphi + X^2} \right) \quad (17b)$$

Thus the tests of CPT invariance may be interpreted directly as measuring the parameter η .

One possible set of results may serve to illustrate this discussion. It might turn out that $|\eta_{+-}| = |\eta_{00}|$ but that the phase χ of η_{+-} is significantly different from δ_s . The solutions in Section II would then indicate that $\text{Im } A_2$ was significantly different from zero and that the equality of $|\eta_{+-}|$ and $|\eta_{00}|$ was an accident depending on the phase $(\delta_2 - \delta_0)$. In this case the phase ψ of η_{00} would be different from the phase χ of η_{+-} . However, an alternative interpretation would be a CPT violating superweak interaction in which case the equality $|\eta_{+-}| = |\eta_{00}|$ would not be accidental and the phases would be related by $\psi = \chi = \delta_s - \eta$.

VII. SUMMARY

The major conclusions are summarized here.

1. Assuming CPT invariance and the $\Delta Q = \Delta S$ rule for leptonic decays, we conclude that all experiments on $K^0 \rightarrow 2\pi$ determine primarily two parameters that measure CP violation. One of these, ϵ_0 , measures the CP violation in the dispersive part of the self-energy matrix; the other, $\text{Im } A_2$, is proportional to the violation of the $\Delta I = \frac{1}{2}$ rule.
2. The order-of-magnitude of ϵ_0 is probably known from the original experiment of Christenson et al.¹⁾. From this value the most reasonable conclusion is that the CP violating part of the effective weak interaction Hamiltonian is at least about two orders-of-magnitude smaller than the CP conserving part.
3. By combining the results of two additional experiments, an interference experiment on $K^0 \rightarrow \pi^+ + \pi^-$ and a measurement of the rate $K_L^0 \rightarrow \pi^0 + \pi^0$, it is possible to limit $\text{Im } A_2$ to two possible values. One possible method of resolving the ambiguity would be a relatively crude measurement of the small charge asymmetry in the leptonic decay $K_L^0 \rightarrow \pi^\pm + \ell^\mp + \nu$. This method depends crucially on the $\Delta Q = \Delta S$ assumption. Another conceivable method would be an interference experiment on the decay $K_L^0 \rightarrow \pi^0 + \pi^0$; such an experiment could serve also as a test of CPT invariance.
4. A non-zero value for the parameter $\text{Im } A_2$ would rule out several theoretical models; in particular, the model of a superweak interaction¹⁵⁾. A large value for this parameter would argue against but not rule out other models; in particular, the model of CP violation in charge independent strong interactions⁹⁾. A zero value for this parameter within a reasonable experimental error would rule out none of the general classes of Hamiltonian models that have been proposed.

5. If it is assumed that the $\Delta I = \frac{1}{2}$ rule holds for CP violating decay amplitudes (this means $\text{Im } A_2 \simeq 0$) and that $\Delta Q = -\Delta S$ leptonic decays do not occur, then all possible experimental results on CP violation in $K^0 \rightarrow 2\pi$ decays and on the charge asymmetry of K_L^0 leptonic decays are determined to a good degree of accuracy by the original experimental result of Christenson et al.¹⁾ except for one algebraic sign. This sign (the sign of ϵ_0) determines whether the interference effect in $K^0 \rightarrow \pi^+ + \pi^-$ is constructive or destructive and also the sign of the charge asymmetry.

These results follow from Eqs. (1) - (4) and (12) if

$$\epsilon' = \text{Im } A_2 = 0.$$

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A P P E N D I X A

Mass matrix

Assuming CPT invariance the "mass matrix" of the K^0 has six real parameters ²⁹⁾

$$\Gamma + iM = \begin{pmatrix} A & p^2 \\ q^2 & A \end{pmatrix} \quad (A.1)$$

where we use the $K - \bar{K}$ representation and A , q^2 , and p^2 are complex. The eigenvectors of this matrix are $|K_S\rangle$ and $|K_L\rangle$ given in Section II where

$$r = 1 - \epsilon = q/p \quad (A.2)$$

$$|\bar{K}\rangle = CP |K\rangle \quad (A.3)$$

The eigenvalues are given by

$$\begin{aligned} A + qp &= \gamma_S + 2im_S \\ A - qp &= \gamma_L - 2im_L \end{aligned} \quad (A.4)$$

We note that in addition to the five physical quantities listed in Section II there is a sixth parameter, $m_S + m_L$, the mean K^0 self-energy.

The quantities p^2 and q^2 may be written as follows ²⁾

$$\begin{aligned} p^2 &= A_0^2 + A_2^2 + x + iy + iM_r - M_i \\ q^2 &= A_0^2 + A_2^{*2} + x - iy + iM_r + M_i \end{aligned} \quad (\text{A.5})$$

where the first four terms come from the Hermitian matrix Γ and represent the "off-diagonal" transition probability to real final states :

A_0^2 , transitions to $I = 0$ final $\pi\pi$ states ;

A_2^2 , transitions to $I = 2$ final $\pi\pi$ states ;

x , CP conserving contribution from transitions to other states (3π and leptonic) ;

y , CP violating contribution from transitions to other states (3π and leptonic).

The last two terms in Eq. (A.5) represent the off-diagonal part of the self-energy matrix M , associated in second-order perturbation theory with transitions to virtual intermediate states ; M_r represents the CP conserving part and M_i the CP violating part. Since $|\epsilon|$ is assumed to be much less than unity (see Appendix B) we can write

$$\begin{aligned} \epsilon &= \epsilon_0 e^{i\delta} = \frac{p-q}{p} = \frac{p^2 - q^2}{p(p+q)} \approx \frac{p^2 - q^2}{2pq} \\ &= \frac{-2M_i + 2iy + 4i \operatorname{Re} A_2 \operatorname{Im} A_2}{(\gamma_S - \gamma_L) + 2i(m_S - m_L)} \end{aligned} \quad (\text{A.6})$$

Neglecting the last term in the numerator, which we discuss later, we have

$$\epsilon_0 = -\frac{2M_i}{|M_i|} \left[\frac{M_i^2 + \gamma^2}{(\gamma_S - \gamma_L)^2 + 4(m_S - m_L)^2} \right]^{\frac{1}{2}} \quad (\text{A.7})$$

$$\begin{aligned} \delta &= \delta_S + \Delta \\ \tan \Delta &= -\gamma/M_i \end{aligned} \quad (\text{A.8})$$

where δ_S is given by Eq. (1).

If we assume there are no $\Delta Q = -\Delta S$ leptonic decays (or for present purposes that any such decays are CP conserving) the only contribution to γ comes from 3π decays. If only a single final 3π configuration with $I = 1$ is considered the decay amplitudes are given by

$$\begin{aligned} A(K \rightarrow 3\pi) &= A_1 e^{i\varphi_1} \\ A(\bar{K} \rightarrow 3\pi) &= A_1 e^{-i\varphi_1} \end{aligned}$$

where A_1 is real. Considering the possibility of large CP violations we may set $r \simeq 1$ in which case

$$\begin{aligned} \Gamma(K_L \rightarrow 3\pi) &= 2 A_1^2 \cos^2 \varphi_1 \\ \Gamma(K_S \rightarrow 3\pi) &= 2 A_1^2 \sin^2 \varphi_1 \\ \gamma &= -2 A_1^2 \sin \varphi_1 \cos \varphi_1 \\ &= -\Gamma(K_L \rightarrow 3\pi) \tan \varphi_1 \end{aligned} \quad (\text{A.9})$$

To obtain a reasonable limit on Φ_1 we employ the arguments of Gaillard ⁵⁾ that the agreement between the rate of $R(K_L \rightarrow 3\pi)$ and the $\Delta I = \frac{1}{2}$ rule is not fortuitous yielding $|\Phi_1| \lesssim 30^\circ$. From Eq. (A.9) we then have

$$|y| \lesssim 4 \times 10^6 \text{ sec}^{-1} \quad (\text{A.10})$$

Experiments such as those reported by Anderson et al. ³⁰⁾, if done to greater precision, could confirm such a limit. The values of Δ given in the Table correspond to the value of $|y|$ given by Eq. (A.10) and are obtained by use of Eqs. (A.7) and (A.8). Note that with this limit on y the sign and magnitude of ϵ_0 (assuming it is of the order of 4×10^{-3}) is almost completely determined by M_1 from Eq. (A.7).

In the case of a theory of the type proposed by Sachs ¹⁴⁾ (class 2C in Section III) in which there is no CP violation in non-leptonic processes but a maximum violation in $\Delta Q = -\Delta S$ leptonic decays

$$|y| = R(K_L^0 \rightarrow \text{leptons})(2X / 1 + X^2) \quad (\text{A.11})$$

where X equals the ratio of $\Delta Q = -\Delta S$ to $\Delta Q = +\Delta S$ decay amplitudes (here suitably averaged over all leptonic decays) defined by Eq. (10). If the limit $X \leq 0.4$ indicated by the experiments of Aubert et al. ⁶⁾ is accepted, $|y| \leq 8 \times 10^6 \text{ sec}^{-1}$, so that values of Δ approximately twice as large as indicated in the Table are allowed.

We return now to the last term in the numerator of Eq. (A.6); this may be estimated by assuming

1) $|\text{Re } A_2| \simeq 0.05 A_0$ using as a measure of the $\Delta I = \frac{1}{2}$ violation the rate $K^+ \rightarrow \pi^+ + \pi^0$; and

2) $|\epsilon'| \lesssim 4 \times 10^{-3}$ (based on Eq. (4) and the discussion in Appendix B) which yields $|\text{Im } A_2| \lesssim 3 \times 10^{-3} A_0$ from Eq. (3).

Then we find

$$|4 \text{Re } A_2 \text{ Im } A_2| \lesssim 6 \times 10^{-4} A_0^2 = 6 \times 10^{-4} (\gamma_s/2)$$

$$|4 \text{Re } A_2 \text{ Im } A_2| \lesssim 3.5 \times 10^6 \text{ sec}^{-1} \quad (\text{A.12})$$

From Eqs. (A.6), (A.10), and (A.12) it is seen that the maximum contribution of $4 \text{Re } A_2 \text{ Im } A_2$ to the phase δ is less than half the possible contribution from $|y|$, which is given by the values of Δ in the Table. Since these values are fairly small, it seems reasonable to neglect $4 \text{Re } A_2 \text{ Im } A_2$ in a first approximation. However, it is clearly possible to carry out a process of successive approximations if this is warranted since the analysis of Section II allows for the determination of both $\text{Re } A_2$ and $\text{Im } A_2$. In particular, from Eq. (3) together with Eq. (18) of Ref. ²⁾

$$4 \text{Re } A_2 \text{ Im } A_2 = \frac{1}{2} \frac{|\epsilon'|^2}{\text{Re } \epsilon'} \left[R(K_S \rightarrow \pi^+ \pi^-) - 2 R(K_S \rightarrow \pi^0 \pi^0) \right]$$

Thus, except for the compounding of experimental uncertainties, no approximation is really required as far as this term is concerned.

A P P E N D I X B

Phase convention

There exists an ambiguity in the relative phase of $|K\rangle$ and $|\bar{K}\rangle$, which are related by Eq. (A.3). This exists because of the possibility of the phase transformation

$$\begin{aligned} |K\rangle_\alpha &= e^{-i\alpha S} |K\rangle = e^{-i\alpha} |K\rangle \\ |\bar{K}\rangle_\alpha &= e^{-i\alpha S} |\bar{K}\rangle = e^{+i\alpha} |\bar{K}\rangle \end{aligned} \tag{B.1}$$

which has no effect on the usual strong or electromagnetic interactions because of the conservation of strangeness S . Thus the weak interactions must be used in some way to define the relative phase, but it is not clear how to do this since the weak interaction processes violate CP. It is possible to express all physical results in terms of relative phases, but in practice it is much more convenient to establish an arbitrary phase convention which we have done by setting A_0 to be real.

In this Appendix we first show why it is to be expected with this phase convention that neither $|\epsilon|$ nor $|\epsilon'|$ is much greater than the observed quantity 4×10^{-3} ; that is, a large cancellation between ϵ and ϵ' in Eq. (4) would be purely accidental. The argument is based on two points.

- 1) The relative phase of ϵ and ϵ' is determined by the relative phase of $(\delta_2 - \delta_0)$ and δ ; these are unrelated in any model since $(\delta_2 - \delta_0)$ depends on the strong interactions whereas δ is given by Eq. (1) in terms of the width and mass difference of the K^0 mesons.

2) The magnitude of $|\epsilon'|$ depends linearly on the $I = 2$ amplitude whereas $I = 2$ amplitudes make only a quadratic and thus very small contribution to the mass matrix $\Gamma + iM$, which determines ϵ . Thus it is purely accidental if $|\epsilon|$ and $|\epsilon'|$ are approximately equal in magnitude so as to approximately cancel.

Nevertheless the possibility of an accidental cancellation needs to be kept in mind, since the validity of Eqs. (2a) and (2b) depend upon the approximation $|\epsilon| \ll 1$ and $|\epsilon'| \ll 1$. In particular, it is important to ask whether the tests of CPT invariance given in Section VI might be vitiated by the break-down of this approximation. Thus we now attempt to find a rigorous justification.

We first review the derivation of Eqs. (2). It follows from CPT, the definition of the isotopic spin amplitudes, and the phase convention that

$$A(K \rightarrow \pi^+ \pi^-) \equiv c = \sqrt{2/3} A_0 e^{i\delta_0} + \sqrt{1/3} A_2 e^{i\delta_2} \quad (\text{B.2a})$$

$$A(\bar{K} \rightarrow \pi^+ \pi^-) \equiv \bar{c} = \sqrt{2/3} A_0 e^{i\delta_0} + \sqrt{1/3} A_2^* e^{i\delta_2} \quad (\text{B.2b})$$

$$A(K \rightarrow \pi^0 \pi^0) \equiv d = \sqrt{1/3} A_0 e^{i\delta_0} - \sqrt{2/3} A_2 e^{i\delta_2} \quad (\text{B.2c})$$

$$A(\bar{K} \rightarrow \pi^0 \pi^0) \equiv \bar{d} = \sqrt{1/3} A_0 e^{i\delta_0} - \sqrt{2/3} A_2^* e^{i\delta_2} \quad (\text{B.2d})$$

As measures of CP violation we have ϵ' and ϵ'' defined by

$$\frac{\bar{c}}{c} \equiv (1 - \epsilon') = \frac{1 + \sqrt{\frac{1}{2}} F A_2^* / A_0}{1 + \sqrt{\frac{1}{2}} F A_2 / A_0} \quad (\text{B.3a})$$

$$\frac{\bar{d}}{d} \equiv (1 - 2\epsilon'') = \frac{1 - \sqrt{2} F A_2^*/A_0}{1 - \sqrt{2} F A_2/A_0} \quad (\text{B.3b})$$

where

$$F = \exp[i(\delta_2 - \delta_0)]$$

Believing that $\text{Re}(A_2/A_0)$ is small compared to unity from such evidence as the branching ratio for $K_S^0 \rightarrow 2\pi^0$ and assuming that $|\epsilon'|$ (and $|\epsilon''|$) is small, which requires $\text{Im}(A_2/A_0) \ll 1$, we can to a good approximation expand the denominators of Eqs. (B.3) giving

$$\epsilon'' \simeq \epsilon' \simeq \sqrt{2} i F \text{Im} A_2 / A_0 \quad (\text{B.4})$$

The decay amplitude ratios η_{+-} and η_{00} are given without approximation by

$$\eta_{+-} = \frac{c - r\bar{c}}{c + r\bar{c}} = \frac{\epsilon + \epsilon' - \epsilon\epsilon'}{2 - \epsilon - \epsilon' + \epsilon\epsilon'} \quad (\text{B.5a})$$

$$\eta_{00} = \frac{d - r\bar{d}}{d + r\bar{d}} = \frac{\epsilon - 2\epsilon'' + 2\epsilon\epsilon''}{2 - \epsilon + 2\epsilon'' - 2\epsilon\epsilon''} \quad (\text{B.5b})$$

The approximations $|\epsilon| \ll 1$, $|\epsilon'| \ll 1$ and $\epsilon'' = \epsilon'$ then yield Eq. (2).

We now note that there exists some phase convention for which $|\epsilon| \ll 1$. This follows from the absolute limits on the value of $|r|$ that can be obtained from Eqs. (35) and (36) of Lee, Oehme and Yang²⁹⁾ by using the known values of γ_S and γ_L . The limit depends only weakly on $(m_S - m_L)$; using $|m_S - m_L| = 0.5 \gamma_S$ we find

$$0.945 \leq |r| \leq 1.055$$

Thus if we choose a phase convention α such that r_α is real, it follows that $|\epsilon_\alpha| \leq 0.055$ and from Eq. (4) $|\epsilon'_\alpha| \leq 0.06$. (Note that Eqs. (B.5) do not depend on any phase convention and that Eq. (4) follows from Eq. (B.5a) provided $|\epsilon| \ll 1$.)

It is conceivable that this phase convention is very different from the one we have used, corresponding to a large value of α . Using Eq. (B.1) we can relate the parameters for the two phase conventions

$$r = (1 - \epsilon) = r_\alpha e^{2i\alpha} = (1 - \epsilon_\alpha) e^{2i\alpha} \quad (\text{B.6a})$$

$$\frac{\bar{c}}{c} = (1 - \epsilon') = \left(\frac{\bar{c}}{c}\right)_\alpha e^{-2i\alpha} = (1 - \epsilon'_\alpha) e^{-2i\alpha} \quad (\text{B.6b})$$

Since $|\epsilon'_\alpha| \ll 1$ we have approximately

$$1 - \epsilon' \simeq e^{-2i\alpha}$$

From Eq. (B.3a) this requires $F \simeq 1$ and

$$\text{Im } A_2 / A_0 = \sqrt{2} \tan \alpha$$

It then follows from Eq. (B.3b) that

$$\frac{\bar{d}}{d} \equiv 1 - 2\epsilon'' = e^{2i\lambda} \quad (\text{B.7})$$

$$\tan \lambda = 2 \tan \alpha$$

Now using Eq. (B.5b) together with Eq. (B.6a) neglecting ϵ_α we have

$$\eta_{00} = [1 - e^{2i(\alpha+\lambda)}] / [1 + e^{2i(\alpha+\lambda)}]$$

This will be much larger than $|\eta_{+-}|$, which has been presumed to be given by the known experimental value, unless $2(\lambda+\alpha) \approx 2\pi$. Given Eq. (B.7) the only solution other than $\alpha \approx 0$ is $\alpha \approx \pi/2$. This corresponds to $r \approx -1$ and thus to exchanging the definition of $|K_S\rangle$ and $|K_L\rangle$. Thus we conclude that if $|\eta_{00}|$ is found to have the same order-of-magnitude as $|\eta_{+-}|$ then $|\epsilon|$ and $|\epsilon'|$ are small enough to justify the approximations made.

Finally, it is of interest to define CP violating parameters that do not depend on the phase convention; these may be chosen as β and $|r|^2$:

$$e^{2i\beta} \equiv \frac{A_2/A_0}{A_2^*/A_0^*} = \frac{\text{Re}(A_2/A_0) + i \text{Im}(A_2/A_0)}{\text{Re}(A_2/A_0) - i \text{Im}(A_2/A_0)} \quad (\text{B.8a})$$

$$|r|^2 = 1 - 2 \text{Re } \epsilon + |\epsilon|^2 = 1 - 2\epsilon_0 \cos \delta + \epsilon_0^2 \quad (\text{B.8b})$$

As discussed in Section II all experiments on CP violation in $K^0 \rightarrow 2\pi$ determine two CP violating parameters, ϵ_0 and $\text{Im } A_2$. It is seen from Eqs. (B.8) that each of these in turn determines a phase independent parameter provided $\text{Re } A_2$ is known and Eq. (1) is used to determine δ .

T A B L E

Values of $\Delta = \delta - \delta_s$ for an extreme case
corresponding to a large CP violation in $K^0 \rightarrow 3\pi$ decay

$\frac{m_S - m_L}{\gamma_S - \gamma_L}$	$ \delta_s $	$ \epsilon_0 \times 10^3$	$ \Delta $
0.5	45°	2	15°
		4	7.5°
		6	5°
0.75	56.3°	2	12°
		4	6°
		6	4°
1.0	63.4°	2	9.5°
		4	5°
		6	3°

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 in place of ϵ_1 , ϵ_2 , and ϵ used in Ref. ³⁾. It should
 also be noted that in the "simple phenomenological model", that
 is, the superweak interaction, discussed in Ref. ³⁾, the CPT
 violating part of H_F must satisfy the selection rule $\Delta S = 0$
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FIGURE CAPTION

Construction of solution for phenomenological parameters from experimental data. Given δ_s from Eq. (1) and χ , $|\eta_{+-}|$, and $|\eta_{00}|$ from experiment, draw a circular arc of length $\frac{1}{2}|\eta_{00}|$ with centre at A (end of the vector η_{+-}) intersecting the line OB at C. (a) and (b) show the two possible solutions. Extend the line AC back through A a distance $|\eta_{00}|$ to D. It follows from Eqs. (2) that OC is equal $\frac{3}{4}\epsilon$ and OD is equal to $\frac{3}{2}\epsilon'$. The parameters desired are $\epsilon_0 = \frac{4}{3} \times (OC)$ (positive if C is towards B), $|\text{Im } A_2/A_0| = \sqrt{2}/3 \times (OD)$, and $\delta_2 - \delta_0$ shown on the figure. Since the sign of $\text{Im } A_2/A_0$ is not known, $(\delta_2 - \delta_0)$ is only determined modulo π .