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THE RELATION BETWEEN W MESON PRODUCTION
AND CHARGED LEPTON PAIR CREATION BY HADRON-HADRON COLLISIONS

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Recently, the W^+ meson production due to proton-nucleus collision has been studied at Argonne ¹⁾ and Brookhaven ²⁾ by detecting high energy muons, supposedly originated from W meson decays, at (large) angles. [The W meson is as usual supposed to mediate weak interactions.] These experimental groups have tried to derive a lower limit on the W mass based on muon counts.

It is the purpose of this short paper to show that these experiments are not suited to learn about W mesons. Rather, these experiments only give us some idea on the electromagnetic structures of hadrons in an extremely complex situation.

In what follows we shall discuss hadron-hadron collisions producing charged leptons either by virtual photons or by leptonic decays of W mesons. We do not consider effects of charged leptons coming from decays of hadrons ($\pi\ell_2$, $K\ell_2$, etc.). We also ignore the leptonic decays of W 's of $\geq$ three-body type, $W \rightarrow \ell + \bar{\nu}_\ell + \text{mesons}$, since we are interested in very energetic charged leptons, and the two-body leptonic decays $W\ell_2$ ($W \rightarrow \ell + \bar{\nu}_\ell$) give leptons of the highest energy. Moreover, the theoretical arguments support the conclusion that $W\ell_2$ must have the largest (practically 100%) branching ratio among all leptonic decays $W\ell_n$ ($n \geq 2$) of $W^\pm$ mesons.

Let us compare ³⁾ two intimately related reactions initiated by two hadrons :

$$\begin{array}{l} a + A \rightarrow W + B \\ \quad \quad \quad \searrow \\ \quad \quad \quad \ell + \bar{\nu}_\ell \quad (\bar{\ell} + \nu_\ell) \end{array} \quad (1.a)$$

and

$$\begin{array}{l} a' + A' \rightarrow \text{"}\gamma\text{"} + B' \\ \quad \quad \quad \searrow \\ \quad \quad \quad \ell^+ + \ell^- \end{array} \quad (1.b)$$

where (a, a') and (A, A') belong to the same isomultiplets, the final products B and B' consisting of a set of hadrons, $B = (b_1 + b_2 + \dots)$, $B' = (b'_1 + b'_2 + \dots)$: b_j and b'_j belong to a same isomultiplet, but B and B' need not have a same total isospin. " γ " is a virtual photon which "decays" into a charged lepton pair ($\ell = \mu^-$ or e^-).

Our discussions, given below, consist of two parts. Firstly, we compare the productions of W and " γ " in the same direction and with similar energies, as well as invariant masses - (I). Next, we compare " γ " emissions with large invariant masses at small emission angles and those with small invariant masses at large emission angles - (II). Then it is concluded that, under the experimental conditions, muons from " γ " dominate over muons of " W " origins.

(I) We shall consider these twin reactions with exactly the same (kinematical) situation, namely the incident particles a and a' must have the same energy (and polarization), W and " γ " are emitted into the same direction with the same invariant mass $-q^2$ (the same four-momentum), and so on.

The ratio of differential cross-sections for (1.a) and (1.b) with respect to q^2 (and any other variables which we shall not write down explicitly for simplicity) can be written as :

$$\frac{d\sigma_W/dq^2}{d\sigma_{em}/dq^2} = \frac{\left| g^2 (\bar{u}_\ell \gamma_\alpha (1 + \gamma_5) v_{\bar{\nu}}) \langle B | J_\alpha^W | a A \rangle / \{ (m_W - \frac{i}{2}\Gamma)^2 + q^2 \} \right|^2}{\left| e^2 (\bar{u}_\ell \gamma_\alpha v_{\bar{\ell}}) \langle B' | J_\alpha^{em} | b' A' \rangle / q^2 \right|^2} \quad (2)$$

where $-q^2$ (> 0) is the invariant mass squared for the lepton pair ($\ell \bar{\nu}_\ell$) for (1.a) or ($\bar{\ell} \ell$) for (1.b), $J_\alpha^W = J_\alpha^V + J_\alpha^A$ and J_α^{em} are the weak current and the electromagnetic currents, respectively, which appear in the weak and electromagnetic interactions :

$$g W_\alpha (J_\alpha^W + j_\alpha^l) + h.c.,$$

and

$$e J_\alpha^{em} A_\alpha$$

We notice that

$$g^2 = \frac{G_\mu m_W^2}{\sqrt{2}}, \quad \frac{e^2}{4\pi} = \frac{1}{137}, \quad (3)$$

$$G_\mu = \mu\text{-decay constant} \simeq 10^{-5}/m_p^2$$

m_W and m_p being W meson and proton masses, while Γ is the total width (inverse of the mean life) of the $W^\pm$ meson.

It should be emphasized that the matrix elements $\langle B | J_\alpha^V | aA \rangle$ for the weak vector part and the isovector part of the electromagnetic one $\langle B' | J_\alpha^{em} | a'A' \rangle$ are identically equal (apart from a trivial Clebsch-Gordan coefficient and a factor depending on the Cabibbo angle) according to the conserved weak vector current hypothesis (CVC). The matrix elements $\langle B | J_\alpha^W | aA \rangle$ and $\langle B' | J_\alpha^{em} | a'A' \rangle$ are functions of q^2 and other relevant variables for the reactions (1.a) and (1.b).

For application to a real experimental situation, it is more appropriate to consider the ratio :

$$\frac{\langle d\sigma_W \rangle_{\Delta q^2}}{\langle d\sigma_{em} \rangle_{\Delta q^2}} \equiv \left\{ \int_{-m_W^2 - \frac{1}{2}\Delta q^2}^{-m_W^2 + \frac{1}{2}\Delta q^2} \frac{d\sigma_W}{dq^2} dq^2 \right\} / \left(\frac{d\sigma_{em}}{dq^2} \Delta q^2 \right)$$

$$\simeq \left(\frac{1}{\Delta q^2} \int_{-\infty}^{\infty} \frac{d\sigma_W}{dq^2} dq^2 \right) / \left(\frac{d\sigma_{em}}{dq^2} \right),$$

4.

where Δq^2 is the experimental resolution for q^2 [$\Delta q^2 \gg \Gamma^2$ in any case, since most likely $\Gamma \lesssim 1$ MeV for $m_W \lesssim 10 m_p$]. The last quantity is (approximately) given by

$$\left(\frac{\langle d\sigma_W \rangle_{\Delta q^2}}{\langle d\sigma_{em} \rangle_{\Delta q^2}} \right)_{q^2 = -m_W^2} = \pi \left(\frac{g^2/4\pi}{e^2/4\pi} \right)^2 \frac{m_W^2}{\Delta q^2} \frac{m_W}{\Gamma} X,$$

where

$$X = \left| \frac{(\bar{u}_\ell \gamma_\alpha (1 + \gamma_5) v_\nu) \langle B | J_\alpha^W | a A \rangle}{(\bar{u}_\ell \gamma_\alpha v_\ell) \langle B' | J_\alpha^{em} | a' A' \rangle} \right|_{q^2 = -m_W^2}^2$$

Introducing the leptonic decay branching ratio $R_{\ell 2} = \Gamma_{\ell 2} / \Gamma$ of W decays, $\Gamma_{\ell 2} = (g^2 m_W) / (6\pi)$, and making use of the values in (3), we find

$$\left(\frac{\langle d\sigma_W \rangle_{\Delta q^2}}{\langle d\sigma_{em} \rangle_{\Delta q^2}} \right)_{q^2 = -m_W^2} \simeq \frac{3}{8\sqrt{2}} (137)^2 \times 10^{-5} \left(\frac{m_W}{m_p} \right)^2 \frac{m_W^2}{\Delta q^2} R_{\ell 2} X \quad (4)$$

$$\simeq 5.0 \times 10^{-2} \left(\frac{m_W}{m_p} \right)^2 \frac{m_W^2}{\Delta q^2} R_{\ell 2} X$$

If m_W is not larger than a few GeV, $R_{\ell 2}$ is roughly estimated ⁴⁾ as $(m_p/2m_W)$. The ratio X of nuclear matrix elements squared is of the order of unity, provided :

- i) there is no particular enhancement of the weak matrix element for the axial vector current and
- ii) there is no striking cancellation between the isoscalar and isovector part of J_α^{em} in their matrix elements.

Excluding these two abnormal (and unlikely) cases, one may conclude that

$$\left(\frac{\langle d\sigma_W \rangle_{\Delta q^2}}{\langle d\sigma_{em} \rangle_{\Delta q^2}} \right)_{q^2 = -m_W^2} < \left(\frac{m_W}{10 m_p} \right) \left(\frac{m_W^2}{\Delta q^2} \right) \quad (5)$$

We are now in a position to criticize the experiments ^{1),2)}. If one does detect two charged leptons in (1.b) with a good resolution Δq^2 (a formidable demand!), one can test the existence of W mesons.

On the other hand, if one only detects energetic muons at large angle, one immediately loses information on q^2 and the direction of the parent $W^\pm$ as well as " γ "; and hence $\Delta q^2 \approx m_W^2$. The ratio of muons from W and " γ " will certainly become small:

$$\frac{(\mu' \text{'s from } W)}{(\mu' \text{'s from } "\gamma")} < \frac{m_W}{10 m_p} \quad (6)$$

for the same energy and angular interval of muons. It is clear that the experiment of the type of Refs. 1) and 2) would be sensible only when $m_W > 10 m_p$.

It is unfortunate that available proton beams produced by large accelerators are not energetic enough for this type of experiment.

The result ($m_W > 10 m_p$) has a very simple reason. It corresponds essentially to the situation where the effective W couplings are at least comparable to (or stronger than) the effective electromagnetic couplings: $g^2 \gtrsim e^2$. [As will become clear from (II), the ratio (4) does not correspond to the total W production versus the total lepton pair production via virtual photons, the latter ratio being smaller than what (4) would imply. If we use (4) $\gtrsim 1$ literally, we find, say, $g^2 \gtrsim (e^2/4\pi)/\pi = \alpha/\pi$.]

The argument presented just now can be worked out based on a particular model. Suppose there exists a heavy vector meson V which couples to hadrons via strong interactions with coupling constant f_V and decays by strong interactions. Occasionally V will decay into $\ell^+ + \ell^-$ with the branching ratio $R_{\ell 2}^{(V)}$ of the order of 10^{-4} (cf., $\omega^0 \rightarrow e^+ + e^-$). Next, we shall compare two reactions :

The ratio of these two rates is

$$\sim \frac{g^2}{f_V^2} \frac{R_{\ell 2}}{R_{\ell 2}^{(V)}}$$

Making use of the previous results (3), $R_{\ell 2} \sim m_p / 2m_W$, $R_{\ell 2}^{(V)} \sim 10^{-4}$ and also a reasonable guess :

$$f_V^2 / 4\pi \sim 1$$

will lead to

$$\frac{g^2}{f_V^2} \frac{R_{\ell 2}}{R_{\ell 2}^{(V)}} \sim \frac{10^{-1}}{2\sqrt{2} 4\pi} \cdot \frac{m_W}{m_p} \sim \frac{m_W}{300 m_p}$$

provided W and V have similar masses. This result is in agreement with our previous finding (4). This is not at all surprising : since if such a vector meson V exists, this meson must contribute to electromagnetic form factors of all hadrons. Hence this V model represents merely a possible example of the previous discussion, which has a more general validity.

(II) We can further state a better upper limit on muons of W origin than the above (6). Suppose that virtual photons " γ " are emitted into the direction toward the muon detector with large energies (as large as W meson energies) but with small q^2 , $-q^2 \ll m_W^2$. For example, take $-q^2$ of the order of m_μ^2 , m_μ being muon mass. We can estimate the ratio

$$\frac{(\mu's \text{ from } \gamma \text{ with } q^2 \simeq -m_W^2 \text{ but smaller emission angles})}{(\mu's \text{ from } \gamma \text{ with } q^2 \simeq -m_\mu^2 \text{ but larger emission angles})} \quad (7)$$

as follows : this is given by a product of four factors

- i) ratio of $(1/q^2)^2 \sim (m_\mu/m_W)^4$
- ii) ratio of the spread $\Delta q^2 \sim (m_W/m_\mu)^2$
- iii) ratio of the phase space $\sim (m_W/m_\mu)$, and
- iv) ratio of "electromagnetic form factors" for " γ " emission. (8)

We notice that emission of low q^2 photons of high energies at larger angles will be suppressed because of the larger transverse momenta involved, while large $-q^2$ photon emission should be suppressed by form factors for large $-q^2$ as usual. We may take the last quantity (8) to be at most ~ 1 (although it is likely to be appreciably smaller than 1). Then the ratio of muons specified there at detectors would be at most $(m_\mu/m_W) \lesssim 1/20$ for $m_W \gtrsim 2$ GeV. Notice again that we are only considering a special type of background muons.

These considerations strongly indicate that the muons, if observed in the experiments ^{1),2)}, are essentially of "photon" origin.

The muons of "photon" origin were discussed by Chilton et al. ⁵⁾ based on a particular model, which they have used to estimate W production cross-sections. But their conclusion that "background muons" are probably negligible is very puzzling.

(III) Finally, we would like to add a comment on the effect of thick targets made of high Z material (U or W). When energetic protons hit these targets we would expect noticeable π^0 production towards the muon detector. π^0 's decay into 2γ , which produces occasionally muon pairs in the same thick target. Muons produced in this way will preserve approximately the direction of (grand) parent π^0 (and γ). A number of such muons can be estimated using $\pi^\pm$ production spectra⁶⁾ by high energy protons, and amount to the order of 10^{-32} cm²/sr (GeV/c) per nucleon for muons of several GeV/c under the experimental conditions of Ref. 1).

How should we search for W mesons? To this end, first we must have a good deal of information about the "electromagnetic" reactions (1.b) [besides many other problems]. Then we can go on to ask experimentally whether the reaction (1.a) really takes place or not. Whenever one sees "excess muons" (or electrons), which one would like to attribute to the reaction (1.a), one should make the comparison expressed in Eq. (4), which furnishes the important internal consistency test for the experimental finding.

An essential part of this paper was written before the details of Ref. 2) were available to the author.

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