The Spectrum of the Hybrid Mesons with Heavy Quarks from the B.S. Equation

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Abstract

We construct the B.S. equation for the hybrid mesons under instantaneous approximation. The kernel is chosen as the sum of an one-gluon exchange potential and a linear confining potential. The equations are solved by numerical method, and the spectrum of hybrid mesons $b\bar{b}g$ and $c\bar{c}g$ are obtained.

Keywords: hybrid, spectrum, B.S. equation.
1. Introduction  Besides the conventional hadrons, quantum chromodynamics, the
theory of strong interaction, predicts that the states containing gluonic excitations,
namely glueballs and hybrid hadrons may exist. The confirmation of the existence
of such states would be a strong support for QCD to be the true theory of strong
interaction. Therefore, such states attract a lot of interest both from theories and
experiments. Several experimental candidates of glueballs and hybrid states have been
reported. For example, a candidate for the scalar glueball was reported by Crystal
Barrel Collaboration at LEAR [1], and possible evidence for a $1^{-+}$ light exotic hybrid
candidate was reported by VES [2]. As experiments begin to focus on the search for
 glueballs and hybrid states, it’s important to get more theoretical knowledge of such
states from theory. In this paper we will concentrate on the hybrid states with heavy
quarks.

Many methods have been applied in the investigation of hybrid in the literature,
which include the flux tube model [3], the bag model [4], the constituent gluon model
[5], and lattice Monte Carlo simulations [6]. In bag model, the mass of the lowest
hybrid mesons with light quarks is about 1.5GeV. The hybrid mesons with heavy
quarks are studied by taking into account the bag deformation, and it’s found that the
masses of the lightest hybrid are 3.9GeV for $c\bar{c}g$, and 10.5GeV for $b\bar{b}g$. In the flux tube
model, hybrid is interpreted as the phonon-type excitations of the string connecting
the quark and antiquark. It predicts that the masses of the lowest hybrid mesons are
1.8 – 2.0GeV for $u\bar{u}g$ or $d\bar{d}g$, 4.2 – 4.5GeV for $c\bar{c}g$, and 11.2GeV for $b\bar{b}g$.

The constituent gluon models for hybrid hadrons were introduced by Horn and
Madula [5]. In these models the constituent gluon plays the same role just as the
constituent quarks. It’s assumed that the constituent gluon can have a finite effective
mass $m_g$ and carries angular momentum $l_g$. As a result, it predicts non-exotic quantum
numbers for the ground states of hybrid. Exotic hybrid states including $1^{-+}$ and $0^{--}$
are predicted in the higher-lying multiplet.

In this paper, we study the hybrid mesons with heavy quark in the framework
of B.S. equation. This method is also based on the idea of constituent gluon. We
only concentrate on the hybrid with heavy quarks. In contrast to the light quark sector, where the complicated flavor mixing between $q\bar{q}$ and $q\bar{q}g$ bound states may be important, and the $q\bar{q}$ spectrum itself is not very clear, for the heavy quark system, the $Q\bar{Q}$ spectrum is well described by the the potential model, and the mixing between $Q\bar{Q}$ and $Q\bar{Q}g$ is small. This implies that it is possible to find pure hybrid states in heavy quark sector.

It’s also interesting to note that in the constituent gluon model for hybrid, when the mass of the quark and antiquark tends to infinity, spin-symmetry would show up. We will make a more detailed discussion of this point later.

The paper is arranged as follows. In section 2, we construct the B.S. equation for the hybrid states. In section 3, we construct the B.S. wave function of hybrid states. In section 4, the numerical result and a discussion are presented.

2. The B.S. equation of hybrid states

Let $\psi(x_1)$ and $\bar{\psi}(x_2)$ be the quark and antiquark fields at points $x_1$ and $x_2$, $A_\mu(x_3)$ the gluon field at point $x_3$. Then the B.S. wave function is defined as

$$\chi_\mu(x_1, x_2, x_3) = \langle 0| T\psi(x_1)\bar{\psi}(x_2)A_\mu(x_3)|M\rangle,$$  \hspace{1cm} (1)

where $|M\rangle$ is the hybrid state with mass $M$ and momentum $P$. The color indices has been suppressed. We define the relative and center of mass kinematic variables as

$$X = \frac{\eta_1}{2}(x_1 + x_2) + \eta_2 x_3,$$

$$x = x_1 - x_2, \quad x' = \frac{x_1 + x_2}{2} - x_3,$$  \hspace{1cm} (3)

where $\eta_1 = \frac{2m_Q}{2m_Q + m_g}$, $\eta_2 = \frac{m_g}{2m_Q + m_g}$.

With the translation invariance, we have

$$\chi_\mu(x_1, x_2, x_3) = e^{-iP_X} \chi_\mu(x, x').$$  \hspace{1cm} (4)

We can further define the B.S. wave function in momentum space by the Fourier
transformation

$$\chi_\mu(P, q, k) = \int d^4x d^4x' e^{-i(qx + kx')} \chi_\mu(x, x'),$$  \hspace{1cm} (5)$$

where $q$ is the relative momentum between the heavy quark and heavy antiquark, and $k$ is the relative momentum between the constituent gluon and the center of the two heavy quarks. With a standard method we can obtain the B.S. equation for the hybrid state.

$$\chi_\mu(P, q, k) = -\frac{1}{p_1 - m_Q} \frac{i}{p_3 - m_g} \int \frac{d^4q'}{(2\pi)^4} \frac{d^4k'}{(2\pi)^4} G_{\mu\nu}(P, q, q', k, k') \chi_\nu(P, q', k') \frac{1}{p_2 + m_Q}. \hspace{1cm} (6)$$

In obtaining the above equation, the propagators have been replaced by the free ones. $p_1$, $p_2$ and $p_3$ are the momentum of quark, antiquark and gluon respectively, and we have the following relations

$$p_1 = \eta_1 P + q + \frac{k}{2},$$
$$p_2 = \eta_1 P - q + \frac{k}{2},$$
$$p_3 = \eta_2 P - k. \hspace{1cm} (7)$$

The integral kernel $G_{\mu\nu}$ in the above equation is defined as the sum of all three-particle irreducible graphs. However, it’s impossible to calculate the kernel exactly from the first principle. In practice, some approximations and phenomenological assumptions have to be made in obtaining the kernel. As usual, we divide the kernel $G_{\mu\nu}$ into two parts: the long distance part $G_{\mu\nu}^{(l)}$ and the short distance $G_{\mu\nu}^{(s)}$. $G_{\mu\nu}^{(l)}$ can be obtained approximately by calculating the following three one-gluon-exchange diagrams shown in fig.1.

Besides equation (7), we also have

$$p_1' = \eta_1 P + q' + \frac{k'}{2},$$
$$p_2' = \eta_1 P - q' + \frac{k'}{2},$$
$$p_3' = \eta_2 P - k'. \hspace{1cm} (8)$$
Hybrid is color singlet. Each of the two-body system in the hybrid belongs to a definite representation of color SU(3). The $Qg$ system belongs to a $3$, the $\bar{Q}g$ system belongs to a $3^*$, and the $Q\bar{Q}$ system belongs to a $8$. The color factor for the $Qg$ and $\bar{Q}g$ systems is $-\frac{3}{2}$, while the color factor for $Q\bar{Q}$ system is $\frac{1}{6}$. This implies that the force between the constituent gluon and quark or antiquark is attractive, while the force between the color-octet quark–antiquark pair is repulsive and much week. Therefore, we have such a picture for the hybrid with heavy quarks: the constituent quark and antiquark are almost static with weak and repulsive force between them; the constituent gluon turns round the $Q\bar{Q}$ pair and bind them together to form a bound state. Because the force between the quark and antiquark is very weak, in our treatment we simply neglect it. Then for the short distance part of the kernel, we have (see Fig. 1)

$$G^{(s)}_{\mu\nu}(P, q, q', k, k') = (2\pi)^4\delta^4(p_1 - p'_1)\frac{3(4\pi\alpha_s)}{2l^2}\Gamma_{\mu\nu\rho}\gamma^\rho(p'_1 - m_Q)$$

$$+ (2\pi)^4\delta^4(p_2 - p'_2)\frac{3(4\pi\alpha_s)}{2l^2}\Gamma_{\mu\nu\rho}\gamma^\rho(p'_2 + m_Q),$$

(9)

where $l = k - k'$, and $\Gamma_{\mu\nu\rho}$ is the three-gluon-vertex,

$$\Gamma_{\mu\nu\rho} = (p_3 + p'_3)_{\rho}g_{\mu\nu} + (l - p_3)_{\nu}g_{\mu\rho} - (l + p'_3)_{\mu}g_{\nu\rho}.$$  

(10)

As for the long distance part of the kernel, we have to construct it phenomenologically. According to the experience with the ordinary mesons in the potential model, the long distance part of the kernel is mainly of scalar property. Therefore we only choose the first term of equation (10) as the spin dependence of the long distance part of the kernel, and we assume

$$G^{(l)}_{\mu\nu} = -g_{\mu\nu}(p_3 + p'_3)\cdot V \frac{8\pi\lambda}{M^4} \left( (2\pi)^4\delta^4(p_1 - p'_1)(y'_1 - m_Q) - (2\pi)^4\delta^4(p_2 - p'_2)(y'_2 + m_Q) \right).$$

(11)

where $V = \frac{P}{M}$, and $\frac{8\pi\lambda}{M^4}$ corresponds to a linear potential in the position space.

Substituting the kernel into equation (6), and complete the integration over $q'$, we
We see that now \( q \) is not a dynamical variable. By neglecting the \( q \) dependence of the propagators, this variable is integrated out. The propagator of the quark can be expressed as

\[
\frac{1}{p_1^2 - m_i^2} = \left( \frac{\Lambda^+(p_1)}{p_{10} - E_1} + \frac{\Lambda^-(p_1)}{p_{10} + E_1} \right) \gamma_0,
\]

(13)

\[
\frac{1}{p_2^2 + m_Q} = \gamma_0 \left( \frac{\Lambda^+(p_2)}{p_{20} + E_2} + \frac{\Lambda^-(p_2)}{p_{20} - E_2} \right),
\]

(14)

where \( E_i = \sqrt{m_i^2 + \mathbf{p}_i^2} \), and \( \Lambda^+(p) \) \( (\Lambda^-(p)) \) is positive (negative) energy projector defined as

\[
\Lambda^\pm(p_i) = \frac{E_i \pm \gamma_0 (\mathbf{p}_i + m_Q)}{2E_i}.
\]

(15)

For the constituent gluon propagator, we have

\[
\frac{1}{p_3^2 - m_g^2} = \frac{1}{2E_3} \left( \frac{1}{p_{30} - E_3} - \frac{1}{p_{30} + E_3} \right).
\]

(16)

As an approximation, we neglect the negative energy part of the propagator. Such approximation is reasonable especially for heavy particles. Then as usual, instantaneous approximation is made. Under this approximation we ignore the \( dk_0 \) dependence of the kernel. After making \( k_0 \) integration over the two sides of equation (12), we have

\[
2E_3(M - E_1 - E_2 - E_3)\phi_\mu(k)
\]

\[
= -\int \frac{d^3k'}{(2\pi)^3} \left( \frac{3(4\pi\alpha_s)^2}{2\mathbf{l}^2} \Gamma_{\mu\nu\rho} \gamma^\rho - g_{\mu\nu}(p_3 + p_3') \cdot V \frac{8\pi\lambda}{\mathbf{l}^4} \right) \phi'(k') \gamma_0 \Lambda^-\phi_\mu(k)
\]

\[
- \Lambda^+(p_1) \gamma_0 \int \frac{d^3k'}{(2\pi)^3} \left( \frac{3(4\pi\alpha_s)^2}{2\mathbf{l}^2} \Gamma_{\mu\nu\rho} \gamma^\rho + g_{\mu\nu}(p_3 + p_3') \cdot V \frac{8\pi\lambda}{\mathbf{l}^4} \right) \phi'(k'),
\]

(17)

where \( \phi_\mu(k) \) is the instantaneous three dimensional B.S. wave function.

\[
\phi_\mu(P, k) = \int dk_0 \chi_\mu(P, k)
\]

(18)
3. The B.S. wave function of the hybrid

In order to solve the B.S. equation, we should construct the B.S. wave function of the hybrid with the given quantum numbers. In the following we construct the wave function in heavy quark limit. The spin of the quark-antiquark system, $S_{Q\bar{Q}}$, can take two values, 0 and 1. When $S_{Q\bar{Q}} = 0$, we have the following hybrid states and its four dimensional wave functions.

The $1^{-+}$ hybrid:

$$
\chi_\mu(P,k) = (1 + V) \gamma_5 (F_1 e_\mu + F_2 k \cdot e k_{\perp \mu} + F_3 k \cdot e P_\mu).
$$

(19)

The $0^{--}$ hybrid

$$
\chi_\mu(P,k) = (1 + V) \gamma_5 (F_1 k_{\perp \mu} + F_2 P_\mu).
$$

(20)

The $1^{--}$ hybrid

$$
\chi_\mu(P,k) = F(1 + V) \gamma_5 \epsilon_{\mu \nu \rho \sigma} P^\nu k^\rho e^\sigma.
$$

(21)

The $2^{--}$ hybrid

$$
\chi_\mu(P,k) = (1 + V) \gamma_5 (F_1 \eta_{\mu \nu} k^\nu + F_2 \eta_{\lambda \nu} k^\lambda k_{\perp \mu} + F_3 \eta_{\lambda \nu} k^\lambda k^\nu P_\mu).
$$

(22)

When $S_{Q\bar{Q}} = 1$ we have another series of hybrid states and its four dimensional wave functions.

The $0^{++}$ hybrid

$$
\chi_\mu(P,k) = (1 + V)(F_1 \gamma_\mu + F_2 k_{\perp \mu} + F_3 k_{\perp \mu} P_\mu).
$$

(23)

The $1^{++}$ hybrid

$$
\chi_\mu(P,k) = (1 + V)(F_1 \epsilon_{\mu \nu \rho \sigma} \gamma^\nu e^\rho P^\sigma + F_2 \epsilon_{\mu \nu \rho \sigma} \gamma^\nu e^\rho k^\sigma + F_3 \epsilon_{\mu \nu \rho \sigma} e^\rho P^\sigma k^\nu k_{\perp \mu}
$$

$$
+ F_4 \epsilon_{\mu \nu \rho \sigma} \gamma^\nu P^\rho k^\sigma k_{\perp \mu} + F_5 \epsilon_{\mu \nu \rho \sigma} \gamma^\lambda e^\nu P^\rho k^\sigma P_\mu + F_6 \epsilon_{\mu \nu \rho \sigma} \gamma^\lambda e^\nu P^\rho k^\sigma k_{\perp \mu}).
$$

(24)

The $2^{++}$ hybrid

$$
\chi_\mu(P,k) = (1 + V)(F_1 \eta_{\mu \nu} \gamma^\nu + F_2 \eta_{\mu \nu} k_{\perp \mu} + F_3 \eta_{\mu \nu} \gamma^\rho k^\sigma P_\mu).
$$
$$ + F_4 \eta_{\rho \sigma} \gamma^\rho k^\sigma k_{\perp \mu} + F_5 \eta_{\rho \sigma} k^\rho k_{\perp \sigma} k_{\perp \mu} + F_6 \eta_{\rho \sigma} k^\rho k_{\perp \sigma} k_{\perp \mu} + F_7 \eta_{\rho \sigma} k^\rho k_{\perp \sigma} k_{\perp \mu} ) . \tag{25}$$

The $0^{++}$ hybrid

$$ \chi_\mu (P, k) = F(1 + V) \varepsilon_{\mu \nu, \rho \sigma} \gamma^\nu P^\rho k^\sigma . \tag{26}$$

The $1^{++}$ hybrid

$$ \chi_\mu (P, k) = (1 + V)(F_1 e_{\mu} k_{\perp \mu} + F_2 e \cdot k \gamma_{\perp \mu} + F_3 k_{\perp} e_{\mu} + F_4 e_{\mu} P_{\mu} + F_5 k_\perp k \cdot e_{\mu} + F_6 k_{\perp} k \cdot e P_{\mu} ) . \tag{27}$$

The $2^{++}$ hybrid

$$ \chi_\mu (P, k) = (1 + V)(F_1 \varepsilon_{\mu \nu, \rho \sigma} \eta_{\mu \lambda} P^\rho \gamma^\sigma k_{\lambda} + F_2 \varepsilon_{\mu \nu, \rho \sigma} \eta_{\mu \lambda} P^\rho \gamma^\sigma k_{\lambda} + F_3 \varepsilon_{\mu \nu, \rho \sigma} \eta_{\mu \lambda} \gamma^\rho k^\sigma k_{\perp \mu} + F_4 \varepsilon_{\mu \nu, \rho \sigma} \eta_{\mu \lambda} \gamma^\rho k^\sigma k_{\perp \mu} + F_5 \varepsilon_{\mu \nu, \rho \sigma} \eta_{\mu \lambda} P^\rho \gamma^\sigma k_{\lambda} + F_6 \varepsilon_{\mu \nu, \rho \sigma} \eta_{\mu \lambda} P^\rho \gamma^\sigma k_{\lambda} k_{\perp \mu} . \tag{28}$$

.....

In the above expressions $e_\mu$ is the polarization vector of the vector hybrid, and $\eta_{\mu \nu}$ the polarization tensor of the tensor hybrid. $q_{\perp \mu}$ is a transverse vector defined as $q_{\perp \mu} = q_\mu - q \cdot V V_\mu$, and $\gamma_{\perp \mu}$ is defined similarly. $F_i$'s are scalar functions of $q^2$, $P \cdot q$. For simplicity, the same $F_i$ in different hybrid states represent different functions.

If we substitute the wave function into the B.S. equation, the spin symmetry can be seen explicitly. For example, for $1^{++}$ state, the components of its wave function, $F_1$, $F_5$ and $F_6$ are decoupled from the total wave function $\chi_\mu$, and the equations about these components are exactly the same as those of $0^{++}$ hybrid. Similarly, for $2^{++}$ state, the components of the wave function, $F_1$, $F_3$ and $F_4$ are decoupled out from the total wave function, and the equations about these components are also the same as those of $0^{++}$ hybrid. The reason is that when the interaction between $Q \bar{Q}$ is week and negligible, the quark spin decouples from the light freedom in heavy quark limit. This result does not depend on the form of the kernel (10) and valid for more general case in which the interaction between $Q \bar{Q}$ is omitted. Therefore, if the constituent gluon model is true, we wish to see this symmetry at least in the low-lying $Q \bar{Q}g$ hybrid.
The numerical results

In the non-relativistic limit, in our numerical treatment we only choose the spatial part of the wave function. In the center of mass frame, the form of the wave functions are simplified significantly.

For $1^{+-}$ state

$$\phi_i(k) = (1 + \gamma_0)\gamma_5 f_1 e_i + f_2 e \cdot k k_i.$$ (29)

For $0^{--}$ state

$$\phi_i(k) = f_1 (1 + \gamma_0)\gamma_5 k_i.$$ (30)

For $1^{--}$ state

$$\phi_i(k) = f_1 (1 + \gamma_0)\gamma_5 e_{ijk} k_j e_k.$$ (31)

For $2^{--}$ state

$$\phi_i(k) = (1 + \gamma_0)\gamma_5 (f_1 \eta_{ij} k_j + f_2 \eta_{ijk} k_i k_i).$$ (32)

For $0^{++}$ state

$$\phi_i(k) = (1 + \gamma_0)(f_1 \gamma_i + f_2 \gamma \cdot k k_i).$$ (33)

For $1^{++}$ state

$$\phi_i(k) = (1 + \gamma_0)(f_1 \varepsilon_{ijk} \gamma_j e_k + f_2 \varepsilon_{ijk} k_j \gamma k k \cdot e + f_3 \varepsilon_{ijk} k_j \gamma k k \cdot \gamma + f_4 \varepsilon_{ijk} k_j \gamma k k \cdot \gamma k_i).$$ (34)

For $2^{++}$ state

$$\phi_i(k) = (1 + \gamma_0)(f_1 \eta_{ij} \gamma_j + f_2 \eta_{ijk} k_j \cdot \gamma + f_3 \eta_{ijk} k_j \gamma k g_i + f_4 \eta_{ijk} k_j \gamma k_i + f_5 \eta_{ijk} k_j k k \cdot \gamma k_i).$$ (35)

Substituting the above wave functions into the B.S. equation, we can calculate the spectrum of hybrid states. It’s easy to see that the states $1^{+-}$ and $(0, 1, 2)^{++}$ degenerate, and also the states $(0, 1, 2)^{--}$ and $(0, 1, 2)^{++}$ degenerate.

It’s easy to see that the long distance part of the kernel with the form of expression (11) is very singular at the point $\mathbf{l}^2 = 0$. Some form of regularization is necessary. The method is to make the following replacement

$$G_{\mu \nu}^{(l)} \rightarrow -g_{\mu \nu}(p_3 + p_3') \cdot V \left( (2\pi)^4 \delta^4(p_1 - p_1') (y_1 - m_Q) - (2\pi)^4 \delta^4(p_2 - p_2') (y_2 + m_Q) \right) \frac{8\pi \lambda}{(p^2 + u^2)^2}$$
\[ + \delta^3(1) \int d^3k \left\{ g_{\mu\nu}(p_3 + p'_3) \cdot V \frac{8\pi \lambda}{(l^2 + u^2)^2} \left( (2\pi)^4 \delta^4(p_1 - p'_1)(g'_{1} - m_Q) \
- (2\pi)^4 \delta^4(p_2 - p'_2)(g'_{2} + m_Q) \right) \right\} \]  \tag{36}

where \( u \) is a small quantity. In actual calculation, we let \( u \to 0 \). In this way, the infrared divergence is subtracted out.

In our calculation, the strong coupling constant \( \alpha_s \) is chosen as a running one.

\[ \alpha_s = \frac{12\pi}{27} \frac{1}{ln(\frac{a + l^2}{\Lambda^2_{QCD}})} \]  \tag{37}

where the parameter \( a \) is introduced to avoid the infrared divergence. We choose \( a = 4.0 \) which implies that \( \alpha_s \) tends to its largest value 1 when \( l^2 \to 0 \).

The other parameters including the string tension \( \lambda \), and the quark mass, \( m_c \) and \( m_b \), are determined by fitting the \( Q\bar{Q} \) spectrum, and we have: \( \lambda = 0.2(\text{GeV})^2 \), \( m_c = 1.47\text{GeV} \) and \( m_b = 4.80\text{GeV} \).

The mass of the constituent gluon has been studied in a lattice calculation [7], and in non-perturbative QCD based on gauge invariant Lagrangian [8]. These studies yield \( m_g = (500 \sim 800)\text{MeV} \). In our study, we adopt \( m_g = 600\text{MeV} \).

All the parameters are determined as above, we can solve the B.S. equation numerically. The results are list in the table 1.

As mentioned above, the choice \( m_g = 600\text{MeV} \) is reasonable. However, we can’t say it’s exact. We calculate the hybrid mass by changing \( m_g \) in the the range of several hundred \( \text{MeV} \). Fig. 2 shows the dependence of the mass of \( c\bar{c}g \) on \( m_g \), and fig.3 shows the dependence of the mass of \( b\bar{b}g \) on \( m_g \). We can see that the hybrid mass is not too sensitive to the gluon mass. The mass of lightest \( c\bar{c}g \) changes from 3.65\( \text{GeV} \) to 3.80\( \text{GeV} \) when \( m_g \) changes from 300\( \text{MeV} \) to 800\( \text{MeV} \). In the same range of \( m_g \), the mass of the lightest \( b\bar{b}g \) changes from 10.26\( \text{GeV} \) to 10.39\( \text{GeV} \).

There is another kind of uncertainty due to the neglecting of the interaction between the quark and anti-quark. Recall that for the ground state of charmonium or bottumonium in quark model, the interaction energy between the two quarks is less
than one hundred MeV, and here the strength of the interaction between the color-octet $Q\bar{Q}$ is only one eighth of that between the singlet $Q\bar{Q}$. Therefore, we expect this uncertainty is small and can be neglected reasonably.

5. Conclusion and discussion

We have studied the hybrid states with heavy quarks in the framework of B.S. equation. In the heavy quark limit, we show clearly the existence of the spin symmetry which results in the degeneracy of the states of $(0,1,2)^{++}$ and also the degeneracy of $(0,1,2)^{-+}$. The states $1^{+-}$ and $(0,1,2)^{++}$ are also degenerate, and they are the ground state of hybrid, while the degenerate states $(0,1,2)^{--}$ and $(0,1,2)^{++}$ belong to the first excited states. With a reasonable choice of $m_g(600MeV)$, we find that the ground state of $c\bar{c}g$ hybrid is near the threshold of $D\bar{D}$ meons, while the ground state of $b\bar{b}g$ hybrid is below the threshold of $B\bar{B}$ mesons. The energy gap between the first excited states and the ground states are 0.52GeV for $c\bar{c}g$, and 0.48GeV for $b\bar{b}g$, and such values are quite insensitive to the mass of the constituent gluon.

In the experiment of $e^+e^-$ collision, several resonant states are found near 4.0GeV, such as $\psi(4040)$, $\psi(4160)$ and $\psi(4415)$. Within the conventional picture of charmonium, $\psi(4040)$ is assigned as $3S$ state, and $\psi(4415)$ is assigned as $4S$ (or possibly $5S$) state. But as has been argued by F. Close [9], $\psi(4160)$ perhaps is a $1^{--}$ $c\bar{c}g$ hybrid state. In our model, the mass of the $1^{--} c\bar{c}g$ hybrid is 4.24GeV, so our result is roughly in agreement with this interpretation.
References


Figuer Caption

Fig.1: The diagrams of B.S. kernel in the lowest order.

Fig.2: The $m_g$ dependence of the mass of the ground and first excited $c\bar{c}g$ hybrid.

Fig.3: The $m_g$ dependence of the mass of the ground and first excited $b\bar{b}g$ hybrid.
Table Caption

Table 1: The mass of the hybrid mesons (MeV).