The halo white dwarf population

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Halo white dwarfs can provide important information about the properties and evolution of the galactic halo. In this paper we compute, assuming a standard IMF and updated models of white dwarf cooling, the expected luminosity function, both in luminosity and in visual magnitude, for different star formation rates. We show that a deep enough survey (limiting magnitude $\sim 20$) could provide important information about the halo age and the duration of the formation stage. We also show that the number of white dwarfs produced using the recently proposed biased IMFs cannot represent a large fraction of the halo dark matter if they are constrained by the presently observed luminosity function. Furthermore, we show that a robust determination of the bright portion of the luminosity function can provide strong constraints on the allowable IMF shapes.

*Subject headings*: stars: white dwarfs – stars: luminosity function – Galaxy: stellar content
1. Introduction

One way to understand the structure and evolution of the Galaxy is to study the properties of one of its fossil stars: white dwarfs. Their luminosity function has been extensively studied since it provides important information about the properties of the Galaxy and new deep surveys will open the possibility to observe the white dwarf population beyond the cutoff reported by Liebert, Dahn & Monet (1988) and Oswalt et al. (1996) as well as to discriminate, on the basis of their kinematically properties, those that belong to the halo. If the halo was formed sometime before the disk as a burst of short duration (Eggen et al. 1962), it would be possible to obtain information about the time elapsed between the formation of both structures (Mochkovitch et al. 1990). If mergers of protogalactic fragments have played an important role in the formation of the galactic halo (Searle & Zinn 1978), their signature should be apparent in the white dwarf luminosity function.

The observed properties of halo white dwarfs are very scarce. These properties can be summarized as follows: a) Liebert, Dahn & Monet (1989) provided a very preliminary luminosity function using six white dwarfs, which were identified as halo members because of their high tangential velocities. b) Flynn, Gould & Bahcall (1996) have found that the number of stellar objects in the Hubble Deep Field (HDF) with $V - I > 1.8$ is smaller than 3, while Méndez et al. (1996) have identified 6 objects with $0 < V - I < 1.2$ that could be white dwarfs (although they recommend to reject them as white dwarf candidates because of their colors). Both values can be considered as reliable upper limits to the number of white dwarfs in the HDF. c) A recent analysis of the microlensing events towards the Large Magellanic Cloud suggests that they are produced by halo objects with an average mass $\sim 0.5 M_\odot$ (Alcock et al. 1997) and the total contribution of such objects to the total halo mass could be as high as 40%. Obviously, white dwarfs are one of the most natural candidates to explain such observations. If this interpretation of the microlensing data...
turns out to be correct, the tight constraints imposed by galactic properties should demand the use of biased non-standard initial mass functions (Adams & Laughlin 1996; Fields, Mathews & Schramm 1996; Chabrier, Ségretain & Méra 1996).

These IMFs are characterized by a pronounced shortfall below \( \sim 1 \, M_\odot \) and above \( \sim 7 - 10 \, M_\odot \) in order to avoid the overproduction of red dwarfs in the first case, and to avoid problems with the luminosity of galactic haloes at high redshift (Charlot & Silk 1995) or to overproduce heavy elements by the explosion of massive stars in the second case. The problem is that the IMF determines the contribution of each kind of star to galactic evolution and, therefore, any ad-hoc change introduced to solve a problem, risks to introduce a desadjustement in other apparently well settled fields. In the case of the IMFs quoted here, the problem is twofold. First, the quantity of astrated mass that is returned on average to the interstellar medium through stellar winds and planetary nebula ejection is very high, it lies in the range of 40 to 80\%. If white dwarfs contribute substantially to the mass budget of the halo, the total energy necessary to eject this unwanted mass to the intergalactic medium is \( \sim 10^{60} \) erg, assuming a typical halo radius of 50 kpc and a mass of \( 10^{12} \, M_\odot \). Since the number of massive stars has already been suppressed to avoid an excess of supernovae, an important part of this material would remain locked into the galaxy (Isern et al. 1997a). Furthermore, since intermediate mass stars are the main producers of carbon and nitrogen, it would be hard to account for the \([C,N/O]\) ratios observed in Population II stars if an important part of this matter is invested in the formation of these stars (Gibson & Mould 1997). Second, even if these problems were solved, the huge increase of white dwarfs would increase the number of Type Ia supernovae unless new ad-hoc and completely unjustified hypothesis about the properties of binary stars are adopted. The resulting overproduction of Fe and the excessively high rate of supernova explosions have recently led (Canal, Isern & Ruiz-Lapuente 1997) to the conclusion that, in all cases, the contribution of white dwarfs to the halo mass should be well below 5–10 \%. At this point, it is worthwhile
to point out that both the estimated mass of the objects causing the microlensing events
and the total mass of the population responsible of such events are still preliminary (Mao

In view of the interest of the luminosity function of halo white dwarfs and since there is
not any recent study of such stars using standard hypothesis, it is worthwhile to construct
an updated series of standard models of halo white dwarf populations for comparison
purposes.

2. The luminosity function

The luminosity function is defined as the number of white dwarfs per unit volume and
per unit of bolometric magnitude, $M_{\text{bol}}$:

$$n(M_{\text{bol}}, T) = \int_{M_i}^{M_s} \Phi(M) \Psi(T - t_{\text{cool}} - t_{\text{MS}}) \tau_{\text{cool}} dM$$

(1)

where $M$ is the mass of the parent star (for convenience all white dwarfs are labelled with
the mass of their main sequence progenitors), $\tau_{\text{cool}} = dt/dM_{\text{bol}}$ is the characteristic cooling
time, $M_s$ and $M_i$ are the maximum and the minimum masses of the main sequence stars
able to produce a white dwarf of magnitude $M_{\text{bol}}$ at the time $T$, $t_{\text{cool}}$ is the time necessary
to cool down to this magnitude, $t_{\text{MS}}$ is the main sequence lifetime and $T$ is the age of the
population under study (disk, halo, . . . ). Of course, for evaluating equation (1) a relationship
between the mass of the white dwarf and the mass of its progenitor must be provided.
It is also necessary to provide a relationship between the mass of the progenitor and its
main sequence lifetime. We have used those of Wood (1992) instead of those of Iben &
Laughlin (1989) as we usually did in previous papers in order to compare with Adams &
Laughlin (1996). In order to properly compare with the observations, it is desirable to bin
this function in intervals of magnitude $\Delta M_{\text{bol}}$, usually of one or half magnitudes, in the following way:

$$\langle n(M_{\text{bol}}, T) \rangle_{\Delta M_{\text{bol}}} = \frac{1}{\Delta M_{\text{bol}}} \int_{M_{\text{bol}}-0.5\Delta M_{\text{bol}}}^{M_{\text{bol}}+0.5\Delta M_{\text{bol}}} n(M_{\text{bol}}, T) \, dM_{\text{bol}}$$

(2)

### 2.1. The cooling sequences

The values taken by $t_{\text{cool}}$ and $\tau_{\text{cool}}$ depend on the adopted evolutionary models of white dwarfs. Since there have been some misunderstandings about the cooling process, it is worthwhile to summarize here the most relevant points. After integrating over the entire star and assuming that the release of nuclear energy is negligible, the energy balance can be written as (Isern et al. 1997b):

$$L + L_\nu = - \int_0^{M_{\text{WD}}} C_v \frac{dT}{dt} \, dm - \int_0^{M_{\text{WD}}} T \left( \frac{\partial P}{\partial T} \right)_{V,X_0} \frac{dV}{dt} \, dm + (l_s + e_g) \dot{m}_c$$

(3)

where $L$ and $L_\nu$ are the photon and the neutrino luminosities respectively, $\dot{m}_c$ is the rate at which the crystallization front moves outwards, and the rest of the symbols have their usual meaning. Neutrinos are dominant for luminosities larger than $10^{-1} L_\odot$. Nevertheless, since the phase dominated by the neutrino cooling is very short and the luminosity function of very bright halo white dwarfs is still unknown we start our calculations at $10^{-1} L_\odot$.

All the four terms in the right hand side of this equation depend on the detailed chemical composition of the white dwarf. We have adopted the chemical profiles of Salaris et al. (1997a,b) for C–O white dwarfs (white dwarf masses in the range of 0.5–1 $M_\odot$ and progenitors in the mass range 0.7–8 $M_\odot$), which take into account the presence of high quantities of oxygen in the central regions due to the high rates of the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction, and from García-Berro et al. (1997) for O–Ne white dwarfs (white dwarf masses $\gtrsim 1 M_\odot$)
and progenitors in the mass range 8–11 $M_\odot$).

The first term of the right hand side of equation (3) represents the well known contribution of the heat capacity of the star to the total luminosity (Mestel 1952). It strongly decreases when the bulk of the star enters into the Debye regime (Lamb & Van Horn 1975). The second term takes into account the net contribution of compression to the luminosity. It is in general small since the major part of the compressional work is invested into increasing the Fermi energy of electrons (Lamb & Van Horn 1975, Shaviv & Kovetz 1976). The largest contribution to this term comes from the outer, partially degenerate layers. In fact, when the white dwarf enters into the Debye regime, this term can provide in some cases about 80% of the total luminosity preventing in this way the sudden disappearance of the star (D’Antona and Mazzitelli 1989). Of course, the relevance and the details of this contribution strongly depend on the characteristics of the envelope. It is clear, therefore, that extrapolating the behavior of the coldest white dwarfs just assuming that their unique source of energy is the heat capacity may not necessarily be the best procedure.

The third term in the right hand side represents the energy release associated to solidification. The term $l_s$ corresponds to release of the latent heat ($\sim kT_s$/nucleus, where $T_s$ is the solidification temperature), and the term $e_g$ corresponds to the gravitational energy release associated to the chemical differentiation induced by the freezing process (Mochkovitch 1983, Isern et al. 1997b). It is important to notice here that the C–O models of Salaris et al. (1997a,b) predict very high oxygen abundances in the central region. This fact minimizes the effect of the chemical differentiation as compared with the models assuming half carbon and half oxygen homogeneously distributed along the star. It is also interesting to notice that this effect is almost completely negligible in the case of O–Ne white dwarfs (García-Berro et al. 1997).
The main difference between C–O and O–Ne models is that the last ones cool down more quickly. For instance, two white dwarfs of 1 $M_\odot$, one made of C–O and the other one of O–Ne take 11.3 Gyr and 8.1 Gyr respectively to reach a luminosity of $10^{-5} L_\odot$. This is due to the smaller heat capacity and to the negligible influence of chemical differentiation settling in O–Ne mixtures — see García-Berro et al. (1997) for a detailed discussion.

We would like to stress the importance of the outer layers of the white dwarf in the cooling process. They not only can provide a major contribution to the total luminosity during the late stages, as previously stated, but also control the power radiated by the star. The difficulties come from the fact that matter is far from ideal conditions and that the convective envelope reaches the partially degenerate layers. Although our models take into account reasonably well the energy released by the compression of the outer layers, the rate at which the energy is radiated away remains quite uncertain.

### 2.2. Computational procedure

The white dwarf luminosity function averaged over bins of width $\Delta M_{\text{bol}}$ as given in equation (2) can also be directly computed in the following way. Assume a population that was formed according an arbitrary star formation rate $\Psi(t)$. After a time $T$ since the origin of the Galaxy, the number of white dwarfs that have a bolometric magnitude in the interval $M_{\text{bol}} \pm 0.5 \Delta M_{\text{bol}}$ is:

$$N(M_{\text{bol}}, T) = \int_t \int_M \Phi(M) \Psi(t) dM dt$$

where the integral is constrained to the domain that satisfies the condition:
\begin{align}
T - t_{\text{cool}}(M, M_{\text{bol}} - 0.5\Delta M_{\text{bol}}) &\leq t + t_{\text{MS}}(M) \leq T - t_{\text{cool}}(M, M_{\text{bol}} + 0.5\Delta M_{\text{bol}}) \\
\end{align}

Dividing this result by \( \Delta M_{\text{bol}} \) we obtain equation (2).

This expression can be easily computed using standard methods and, since it does not make use of the characteristic cooling time (which demands the use of numerical derivatives to evaluate it) it easily allows to obtain the luminosity function in visual magnitudes or in any other photometric band.

Figure 1 displays the luminosity function of a burst of constant star formation rate of arbitrary strength and a duration of 0.1 Gyr that started 13 Gyr ago obtained using both methods. The upper solid line has been obtained using equation (1), the lower lines have been obtained by applying the binning procedure of equation (2) to equation (1), solid line, and equations (4) and (5), dotted line. The last two lines have arbitrarily been shifted downwards by a fixed amount to allow comparison. The differences can be considered as negligible. It is worthwhile to note here how the sudden rise produced by crystallization at \( \log(L/L_\odot) \approx -3.7 \) in the luminosity function (upper curve) is smeared out when bins are taken. Note as well that the observational luminosity functions are actually derived using such binning. In all cases calculations were stopped at \( \log(L/L_\odot) = -5 \) to save computing time.

3. Results and discussion

3.1. Standard initial mass function

Figure 2a displays the luminosity functions of halo and disk white dwarfs computed with a standard initial mass function (Salpeter 1961). The observational data for both the disk
and the halo have been taken from Liebert et al. (1988, 1989). The theoretical luminosity functions have been normalized to the points \( \log(L/L_\odot) \approx -3.5 \) and \( \log(L/L_\odot) \approx -2.9 \) for the halo and the disk respectively due to their smaller error bars. The luminosity function of the disk was obtained assuming an age of the disk of 9.3 Gyr and a constant star formation rate per unit volume for the disk, and those of the halo assuming a burst that lasted 0.1 Gyr and started at \( t_{\text{halo}} = 10, 12, 14, 16 \) and 18 Gyr respectively. Due to their higher cooling rate, O–Ne white dwarfs produce a long tail in the disk luminosity function and a bump (only shown in the cases \( t_{\text{halo}} = 10 \) and 12 Gyr) in the halo luminosity function. It is important to realize here that the faintest white dwarf known, ESO 439–26, which has a mass \( M = 1.1 - 1.2 \, M_\odot \) (Ruiz et al. 1995) and a luminosity \( \log(L/L_\odot) \approx -5 \) is clearly an O–Ne white dwarf and cannot be a halo white dwarf unless the halo stopped its star formation activity less than 8 Gyr ago, since the time necessary for O–Ne white dwarfs to reach this luminosity is at maximum 8 Gyr. In order to make easier the comparison with observations, we display in Figure 2b the same luminosity function in visual magnitudes. The photometric corrections were obtained from the atmospheric tables of Bergeron et al. (1995). Beyond \( M_V \gtrsim 17 \) these corrections were obtained by extrapolating those tables. It is interesting to notice that the distance between the peaks of the halo luminosity functions has increased due to the fact that more and more energy is radiated in the infrared as white dwarfs cool down. Therefore, the detection of such peaks should allow the determination of the age of the galactic halo. It is also convenient to remark here that the disk white dwarf luminosity function of figure 2b was obtained with the age and normalization factor used for figure 2a.

If the halo formed from the merging of protogalactic fragments, the time scale for halo formation should be larger than 0.1 Gyr and therefore, the white dwarf luminosity function should be different from those of Figure 2. To show that we have computed the luminosity function for bursts that, starting at 12 Gyr, lasted 0.1, 1 and 3 Gyr. The last
one was inspired by the age distribution of the globular cluster sample of Salaris & Weiss (1997). We see from Figure 3 that because of the relative lack of sensitivity to the age and shape of the star formation rate of the hot portion of the luminosity function, the different curves merge when we normalize them to a fixed observational bin. As a consequence, it is necessary to have precise information about the white dwarf population in the region $M_V \gtrsim 16$ before being able to reach any conclusion.

The shape of the Hertzsprung–Russell diagram of halo white dwarfs also provides useful information about the halo population. Figure 4 displays the color–magnitude diagram for each one of the bursts of figure 2 using a simulated Monte Carlo sample of 2,000 stars. They can be interpreted as the isochrones, including the lifetime in the main sequence, of this halo population. The diagram displays a characteristic Z–shape produced by the combination of the different cooling times of white dwarfs and main sequence lifetimes of their progenitors. This feature moves downwards with the age and ultimately disappears. Therefore, its detection could provide an indication of the halo age.

The duration of the process of formation of the halo is also reflected in the color–magnitude diagram. If the halo took a relatively large time to form, the Z–feature would cover a large region of the color–magnitude diagram and its width could be used as a duration indicator if enough white dwarfs with good photometric data were available. Figure 5 displays the color–magnitude diagram for a burst of constant star formation rate that was 12 Gyr old and lasted 3 Gyr.

Another useful quantity is the discovery function. This function gives the number of white dwarfs per interval of magnitude which can be detected in the whole sky by a survey limited to a given apparent magnitude $m_\lambda$ in a photometric band centered at $\lambda$ (Mochkovitch et al. 1990). If we limit ourselves to nearby halo white dwarfs, this volume can be considered spherical and the discovery function, $\Delta_H(M_\lambda)$, is readily obtained from
the luminosity function

\[ \Delta_H(M_\lambda) = \frac{4\pi}{3} d^3(M_\lambda) n_H(M_\lambda) \]  

(6)

where \( d(M_\lambda) \) is the distance at which a white dwarf of absolute magnitude \( M_\lambda \) has an apparent magnitude \( m_\lambda \):

\[ d(M_\lambda) = 10^{1+0.2(m_\lambda-M_\lambda)} \]  

(7)

where \( d \) is in parsecs. Since white dwarfs with luminosities \( \log(L/L_\odot) \sim -5 \) have effective temperatures of \( \sim 3000 \) K and radiate most of their energy in the red or infrared, we have computed the discovery function for both the \( V \) and \( I \) bands assuming in both cases a limiting magnitude \( M_{V,I} \simeq 20 \). For reasonable ages of the halo (\( t_{\text{halo}} \sim 12-16 \) Gyr), the discovery function in both the \( V \) and \( I \) band yield \( \sim 500 \) stars/magnitude for bright objects and have a steady decrease with the magnitude. This decrease is more pronounced for the \( V \) band (Figure 6). In any case, the total number of stars that we would expect to find in a survey of such characteristics is about 1,500 stars. This implies that the average number of white dwarfs that we expect to find in a typical Schmidt plate of \( 6^\circ \times 6^\circ \) is about 1.5. One third of them should be brighter than magnitude 12.

At this point it is interesting to examine, just as an exercise, the impact of the recently discovered white dwarf WD 0346+246 (Hambly, Smartt & Hodgkin 1997). The first analysis indicates that this white dwarf is placed at a distance \( d \sim 40 \) pc and it has a tangential velocity \( v_T \sim 250 \) km/s, which indicates that it probably belongs to the halo. Its absolute visual magnitude is estimated to be in the range \( 16.2 \lesssim M_V \lesssim 16.8 \). Therefore, if we assume that it is the only star of these characteristics within this distance, the luminosity function would take the value of \( \sim 1.5 \times 10^{-5} \) mag\(^{-1}\) pc\(^{-3}\) at \( M_V \approx 16.5 \), which is in perfect
agreement with the results plotted in Figure 2b. Note also that the standard halo models could accommodate a density larger by a factor $\sim 3$ to that quoted here just assuming that the luminosity function has a peak in this region. In this case, the halo should be as young as $\sim 11$ Gyr. If the density finally turned out to be larger we could start to think about non-conventional hypothesis. It is also interesting to notice that the expected number of white dwarfs per plate with $16 \leq M_I \leq 17$ is in the range of $10^{-1}$ to $3 \times 10^{-2}$.

We have also computed the total number of white dwarfs per stereoradian in the direction of the Hubble Deep Field assuming a spheroidal distribution of stars of the kind $\rho(r) = \rho_0 a^2 / (a^2 + r^2)$, with $a = 2.5$ kpc, a distance of the Sun to the galactic center of 8.5 kpc, and a limiting apparent visual magnitude $V = 26.3$. The total number of halo white dwarfs in this photometric band goes from a minimum of 315,000 stars/str for the 10 Gyr burst to 321,000 stars/str for the 16 Gyr one, while the number of white dwarfs redder than $V - I = 1.8$ ranges from a minimum of 233 stars/str for the first case to 2,000 stars/sr for the second case. The number of stars in the window $0 < V - I < 1.2$ takes values in the range 195,000 to 198,000 stars/str. This behavior can be easily understood if we note that, due to the normalization procedure, the bright portion of the luminosity function is nearly coincident in all cases and that counts limited to a given apparent magnitude are dominated by the brightest stars. On the contrary, if we limit ourselves to the very red ones, which are also the dimmest ones, we are eliminating the brightest white dwarfs and, therefore, the number of them in the pencil becomes sensitive to the age. Unfortunately, the HDF pencil is so narrow ($\Delta \Omega = 4.4$ arc min$^2 = 3.723 \times 10^{-7}$ str) that it is impossible to extract from it any valuable information of the properties of the different bursts.

The local density of halo white dwarfs obtained from our luminosity functions ranges, depending on the adopted age of the halo, from $5.8 \times 10^{-5}$ to $1.1 \times 10^{-4}$ white dwarfs per pc$^3$ (that is from 0.24 to 0.46 white dwarfs in a sphere of 10 pc of radius around the Sun).
If we assume that the characteristic mass of halo white dwarfs is $\sim 0.6 \, M_\odot$, this density represents at most the 0.6% of the local dark halo density, $0.01 \, M_\odot/pc^3$ (Gilmore 1997). Finally, we have to mention that if we chose to normalize to the total density of discovered white dwarfs as in Mochkovitch et al. (1990), all the curves will move downwards by approximately a factor 0.3 dex.

### 3.2. Biased initial mass functions

In order to account for the MACHO results in terms of an halo white dwarf population, Adams & Laughlin (1996), and Chabrier et al. (1996) introduced ad-hoc non-standard initial mass functions that fall very quickly below $\sim 1 \, M_\odot$ and above $\sim 7 \, M_\odot$. These functions avoid the overproduction of red dwarfs, the overproduction of heavy elements by the explosion of massive stars (Ryu, Olive & Silk 1990) and the luminosity excess of the haloes of galaxies at large redshift (Charlot & Silk 1995) and, since the formation of very massive and very small stars has been inhibited, the proposed IMFs allow to increase the number of white dwarfs per unit of astrated mass.

Table 1 displays the mass density in the form of white dwarfs for bursts of star formation that started at different ages and lasted 0.1 Gyr using the IMFs proposed by Adams & Laughlin (1996), with $m_c = 2.3$ and $\sigma = 0.44$ — AL case — and the IMFs proposed by Chabrier et al. (1996) — CSM1 and CSM2 cases. The main differences between our calculations and those of Adams & Laughlin (1996) are that we have computed the luminosity function without neglecting the time spent in the main sequence, we take into account the full effects of crystallization and we normalize to the best known bin of the observed halo luminosity function instead of trying to reproduce a given density of white dwarfs in the halo. The main differences with the Chabrier et al. (1996) calculations rely on the normalization procedure and on the fact that we use realistic carbon–oxygen profiles.
instead of assuming that C–O white dwarfs are made of an homogeneous mixture of half carbon and half oxygen. Besides that, we use average binned functions.

Concerning the AL case, the maximum densities that can be reached are smaller than the 10% of the dark halo for any reasonable age of the Galaxy. The same happens with the CSM1 case. Only in the CSM2 case white dwarfs can represent a noticeable fraction of the halo dark matter. In fact, in the case of an age $\sim 16$ Gyr the halo would be saturated with white dwarfs. The differences with Chabrier et al (1996) are essentially due to the different normalization procedure used here. Therefore, it is clear that a robust determination of the bright portion of the halo white dwarf luminosity function would introduce strong constraints on the allowed shapes of the IMFs.

Figure 7 displays the luminosity functions obtained with the aforementioned IMFs for bursts that started 12 (left panel) and 14 Gyr (right panel) ago (dotted lines), which we believe are realistic values for the age of the halo, normalized to the brightest and more reliable observational bin. The luminosity function corresponding to the standard case is also displayed for comparison (thick solid line). As expected, the position of the peak does not change but its height increases since the number of main sequence stars below $\sim 1 M_\odot$ is severely depleted. This behavior is due to two different effects: i) The non-standard IMFs have been built to efficiently produce white dwarfs (0.18, 0.39, 0.53 and 0.44 white dwarfs per unit of astrated mass for the standard, AL, CSM1 and CSM2 cases respectively and for a burst 14 Gyr old for instance). ii) The time that a white dwarf needs to cool down to the luminosity of the normalization bin, $\log(L/L_\odot) = -3.5$, is $\sim 1.8$ Gyr and only main sequence stars with masses smaller than $1 M_\odot$ are able to produce a white dwarf with such a high luminosity if the halo is taken to be older than 12 Gyr. Since the new IMFs have been taylored to reduce the number of stars below $\sim 1 M_\odot$, it is necessary to shift the luminosity function to very high values to fit the normalization criterion. For instance, the
values that the different IMFs take at $M = 0.98 \, M_\odot$, the mass of the main sequence star that produces a white dwarf with the aforementioned luminosity, are $\Phi_S = 0.23$, $\Phi_{AL} = 0.06$, $\Phi_{CSM1} = 0.2$, and $\Phi_{CSM2} = 0.01$.

Figure 7 also shows that all the luminosity functions, except the one obtained from the standard IMF, are well above the detection limit of Liebert et al. (1988) (shown as triangles). This is due to the normalization condition adopted here. If we had normalized the luminosity function to obtain a density of $1.35 \times 10^{-5}$ white dwarfs per cubic parsec brighter than $\log(L/L_\odot) = -4.35$ as in Mochkovitch et al. (1990), all the luminosity functions (long dashed in Figure 7) would have been shifted downwards and only those corresponding to the CSM2 case would have remained above the detection limit. Note, however, that except for unrealistic ages of the galactic halo, these IMFs are not only unable to provide an important contribution to the halo (see Table 1 below), but also to fit the observed bright portion of the luminosity function of halo white dwarfs. For instance, no one of the CSM2 cases appearing in Figure 2 of Chabrier et al. (1996) fits the brightest bin. Therefore a robust determination of the bright portion of the luminosity function could introduce severe constraints to the different allowable IMFs.

As we have already mentioned in section 2, one of the major uncertainties is the transparency of the envelopes when white dwarfs are very cool. If they turn out to be more opaque than the models used here, the cooling would be slowed down and the height of the peaks would be consequently increased. On the contrary, if the atmospheres turn out to be more transparent, white dwarfs could be able to reach, for a given age, smaller luminosities and the peaks of Figure 7 would therefore be reduced. There is however one limitation: the properties of the envelopes for white dwarfs brighter than $\log(L/L_\odot) \approx -4$ are reasonably well known. Consequently, we have checked this behavior by arbitrarily increasing the transparency below $\log(L/L_\odot) \approx -4$. Adopting the appropriate factor, it is possible to
reduce the height of the peaks of figure 7 below the detection limits. Nevertheless, since the properties of the luminosity function at the normalization point have not changed, the contributions of white dwarfs to the halo mass budget remain the same as those quoted in Table 1. We have also checked if a change in the initial–final mass relationship (in the sense of favouring the formation of massive white dwarfs) could reduce the number of bright white dwarfs, but we have only obtained a slight change in the morphology of the peak since the bright part of the luminosity function is dominated by long lived main sequence stars.

4. Conclusions

We have computed the luminosity function of halo white dwarfs for different photometric bands assuming a standard IMF and several star formation rates. We have shown that a detailed knowledge of this function can provide critical information about the halo properties, in particular its age and duration of the process of formation. The discovery functions computed in this way show that the luminosity function can only be obtained if deep enough, $M \gtrsim 20$, surveys in the $I$ or $R$ bands are performed.

We have also examined the constraints introduced by the Huble Deep Field and we have found that it is too narrow to be useful in this issue. The differences between our results and those of Kawaler (1996) are probably due to the fact that we do not neglect the lifetime in the main sequence since this assumption is not true for bright dwarfs, which are dominant in the star counts below a given magnitude.

Finally, we have shown that, even using biased IMFs, it is impossible to appreciably fill the dark halo with white dwarfs if the luminosity functions are normalized to the observational bin with the smallest error bar. Besides the lack of any physical reason able to justify the radical changes introduced in biased IMFs and the secondary effects mentioned
in the Introduction, it is necessary to assume that white dwarfs become very transparent when they cool down below \( \log(L/L_\odot) \approx -4 \) (or that for some reason halo white dwarfs cool down more quickly than disk white dwarfs, or that they suffered some kind of selection effect) in order to have escaped detection during previous surveys.

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REFERENCES


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Table 1: Local density of halo white dwarfs ($M_\odot$/pc$^3$) for different IMFs and ages of the halo

<table>
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<th>IMF</th>
<th>12 Gyr</th>
<th>14 Gyr</th>
<th>16 Gyr</th>
</tr>
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<td>$5.1 \times 10^{-5}$</td>
<td>$5.6 \times 10^{-5}$</td>
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<td>$3.4 \times 10^{-4}$</td>
<td>$5.3 \times 10^{-4}$</td>
<td>$7.6 \times 10^{-4}$</td>
</tr>
<tr>
<td>CSM1</td>
<td>$1.2 \times 10^{-4}$</td>
<td>$2.1 \times 10^{-4}$</td>
<td>$3.5 \times 10^{-4}$</td>
</tr>
<tr>
<td>CSM2</td>
<td>$1.6 \times 10^{-3}$</td>
<td>$4.4 \times 10^{-3}$</td>
<td>$1.2 \times 10^{-2}$</td>
</tr>
</tbody>
</table>
Figure 1: Luminosity functions corresponding to a burst with a constant star formation rate that started at $T = 13$ Gyr and lasted $\Delta t = 0.1$ Gyr (upper solid line). The luminosity functions obtained with the two methods, after binning in intervals of 1 magnitude, are displayed below (they have been arbitrarily shifted for clarity). The differences between the usual method, solid line, and the direct method, dotted line, are very small.
Figure 2: Luminosity functions of halo white dwarfs assuming bursts of ages 10, 12, 14, 16 and 18 Gyr that lasted for 0.1 Gyr as a function of the luminosity (panel a), and visual magnitude (panel b). In both cases, the luminosity function of disk white dwarfs (dashed line) for $t_{\text{disk}} = 9.3$ Gyr is also plotted for comparison purposes. The observational data were obtained from Liebert et al. (1988, 1989).
Figure 3: Luminosity functions for three bursts that started at $T = 12$ Gyr and lasted 0.1, 1 and 3 Gyr.
Figure 4: Color–magnitude diagrams for the same bursts of figure 2.
Figure 5: Color–magnitude diagrams for a burst of age 12 Gyr that lasted 3 Gyr.
Figure 6: Discovery function of halo white dwarfs in the $I$ band (upper panel) and in the $V$ band (lower panel) for bursts that started at 12, 14 and 16 Gyr and lasted 0.1 Gyr. In all cases, the limiting magnitude is $M_{V,I} = 20$. 
Figure 7: Comparison between the luminosity functions of halo white dwarfs of ages 12 and 14 Gyr and different IMFs (see text for details).