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ON CURRENT ALGEBRA AND CVC IN PION BETA DECAY

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Callan and Treiman ¹⁾, using the notions of current algebra ²⁾ and pion dominated divergence of axial current ³⁾ (PDDAC), have obtained the relation

$$|f_{\pi}| = |g_A M / \mu g_r| |f_+|$$

The quantity f_{π} is the effective coupling constant for $\pi \rightarrow \mu + \nu$, g_A the axial beta decay coupling, M the nucleon mass, μ the pion mass, g_r the pion-nucleon coupling, and f_+ the principal form factor (taken to be constant) for $\pi \rightarrow \pi + e + \nu$. Substituting the familiar Goldberger-Treiman relation ⁴⁾

$$f_{\pi} = g_A M / \sqrt{2} \mu g_r$$

the new relation becomes

$$|f_+| = \sqrt{2}$$

This is just the value implied by the conventional CVC hypothesis ⁵⁾, now generally accepted ⁶⁾. The view might then be taken that nothing new has been achieved. However in our opinion the possibility of obtaining the CVC result without the CVC hypothesis is not of negligible interest. We try here to exhibit very explicitly what is involved.

Consider the quantities

$$R_{\mu}(p^0, p^+) = \langle 0 | \int dx e^{-ip^+x} T(\partial_{\nu} A_{\nu}^0(x), W_{\mu}(0)) | \pi^+ \rangle = a_1 p_{\mu}^+ + a_2 p_{\mu}^0$$

$$S_{\mu}(p^0, p^+) = \langle \pi^0 | \int dx e^{ip^+x} T(\partial_{\nu} A_{\nu}^+(x), W_{\mu}(0)) | 0 \rangle = b_1 p_{\mu}^+ + b_2 p_{\mu}^0$$

in which A_ν^+ is the axial strangeness conserving weak interaction current, A_ν^0 is the neutral member of the isovector to which A_ν^+ belongs, and W_μ is for the moment any other local current. The a 's are functions of $(p^0)^2$ and q^2 , and the b 's of $(p^+)^2$ and q^2 , where in each case $q = p^+ - p^0$. These quantities will have pion pole terms from diagrams

from which

$$a_1 = -i\alpha f_1^W(q^2) (m_\pi^2 + p^{02})^{-1} + \text{---}$$

$$a_2 = -i\alpha f_2^W(q^2) (m_\pi^2 + p^{02})^{-1} + \text{---}$$

$$b_1 = -i\alpha f_1^W(q^2) (m_\pi^2 + p^{+2})^{-1} + \text{---}$$

$$b_2 = -i\alpha f_2^W(q^2) (m_\pi^2 + p^{+2})^{-1} + \text{---}$$

where the form factors f are defined by

$$\langle \pi^0 | W_\mu | \pi^+ \rangle = f_1^W(q^2) p_\mu^+ + f_2^W(q^2) p_\mu^0$$

and

$$\alpha = \langle 0 | \partial_\nu A_\nu^0 | \pi^0 \rangle = \langle \pi^+ | \partial_\nu A_\nu^+ | 0 \rangle$$

Note that $\alpha \neq 0$, because $\pi \rightarrow \mu + \nu$ occurs.

The fundamental assumption (PDDAC) is that the one-pion pole is dominant in R_μ at $p^0 = 0$ and in S_μ at $p^+ = 0$:

$$R_\mu(0, p^+) = -i\alpha f_1^W(-m_\pi^2) p_\mu^+$$

$$S_\mu(p^0, 0) = -i\alpha f_2^W(-m_\pi^2) p_\mu^0$$

Then we need to suppose the following formal consequence of partial integration :

$$\begin{aligned} \left\{ \langle \beta | \int d^4x e^{-ipx} T(\partial_\nu A_\nu(x), W_\mu(0)) | \alpha \rangle \right\}_{p=0} \\ = \langle \beta | \left[\int d^3\vec{x} A_0(\vec{x}, 0), W_\mu(0) \right] | \alpha \rangle \end{aligned}$$

If the commutators vanish we have

$$R_\mu(0, p^+) = S_\mu(p^0, 0) = 0$$

whence

$$f_1^W(-m_\pi^2) = f_2^W(-m_\pi^2) = 0$$

So : for currents which commute at a given time with the space integrals of the axial currents, the form factors are zero at $q^2 = -m_\pi^2$..

To obtain the Callan-Treiman result suppose that the weak vector current V_μ has the same relevant commutators as the isospin current I_μ with $\int A_0 d\vec{x}$. Then $W_\mu = I_\mu - V_\mu$ has zero commutators, and it follows that at $q^2 = -m_\pi^2$ the form factors of I_μ and V_μ are equal ⁷⁾.

As a model of the situation consider a world with elementary nucleons and elementary pions, and let V and A be the purely nucleonic forms

$$V_\mu = \bar{\Psi} i\gamma_\mu \tau \Psi \quad A_\mu = \bar{\Psi} i\gamma_\mu \gamma_5 \tau \Psi$$

Then I_μ would differ from V_μ by terms, call them I_μ^π , involving the elementary pion fields. Moreover, I_μ^π would commute at given time with A_ν . It would follow that for zero momentum transfer

$$\langle \pi^0 | I_\mu^\pi | \pi^+ \rangle \approx 0$$

At first sight this is quite remarkable, as is the appearance of g_r^{-1} in the amplitude for pion decay. Both results indicate that in such a model PDDAC implies strong coupling.

We are indebted to L. Van Hove and J. Prentki for useful remarks.

REFERENCES AND FOOTNOTES

- 1) G.C. Callan and S.B. Treiman - Phys.Rev.Letters 16, 153 (1966).
- 2) M. Gell-Mann - Physics 1, 63 (1964).
- 3) M. Gell-Mann and M. Lévy - Nuovo Cimento 16, 705 (1960).
- 4) M. Goldberger and S.B. Treiman - Phys.Rev. 110, 1178, 1478 (1958).
- 5) R.P. Feynman and M. Gell-Mann - Phys.Rev. 109, 193 (1958) ;
S.S. Gerstein and J.B. Zeldovitch - Soviet Phys. - JETP 2, 576 (1955).
- 6) In the current whose matrix elements define f_π , g_A , f_+ , and f_- , the universal coupling constant $G/\sqrt{2}$ (known from muon decay), and the factor $\cos \theta$ of Cabibbo, are omitted. They must then be included, of course, in the formulae for decay amplitudes.
- 7) One may wish to verify that the algebraic argument applied to I gives indeed the CVC result. From the above reasoning and the usual commutators one finds

$$-i\alpha f_1^I(-m_\pi^2)p_\mu^+ = m_\pi^2 \langle 0 | [d\vec{x} A_0^0, I_\mu^-] | \pi^+ \rangle = -m_\pi^2 \langle 0 | \sqrt{2} A_\mu^- | \pi^+ \rangle$$

$$-i\alpha f_2^I(-m_\pi^2)p_\mu^0 = m_\pi^2 \langle \pi^0 | [d\vec{x} A_0^+, I_\mu^-] | 0 \rangle = -m_\pi^2 \langle \pi^0 | \sqrt{2} A_\mu^0 | 0 \rangle$$

Evaluation of the right hand sides in terms of α yields

$$-ip_\mu^+ \sqrt{2}\alpha \quad \text{and} \quad -ip_\mu^0 \sqrt{2}\alpha$$

6.

respectively, whence

$$f_1^I(-m_\pi^2) = f_2^I(-m_\pi^2) = \sqrt{2}$$

On the other hand CVC gives

$$f_1^I(q^2) = f_2^I(q^2) \quad f_+^I(0) = \frac{1}{2}(f_1^I(0) + f_2^I(0)) = \sqrt{2}$$

The difference of arguments, $q^2 = 0$ or $q^2 = -m_\pi^2$, cannot be regarded as significant in the context of PDDAC.