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η AND X^0 MESON PRODUCTION BY NEUTRINOS
AND THE POSSIBLE EXISTENCE OF SECOND CLASS CURRENTS

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A B S T R A C T

The influence of possible second class currents on η and X^0 production by neutrinos is investigated. If these currents have the same strength as first class currents, they could give the major contribution to the cross-sections.

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1. INTRODUCTION

With the discovery of CP violation ¹⁾, interest has revived ²⁾ in the possible existence of second class hadronic currents ³⁾. These second class vector and axial currents have opposite G reflection character to the first class vector and axial currents and, if one assumes the same isospin properties, they have opposite CP conjugation properties to the first class ones. Thus, for instance, interference between first and second class currents may cause certain observable CP violating effects. This is one way to discover the existence of such currents.

Alternatively, if one considers matrix elements of the vector currents between states with definite G parity, then for first class currents the G parity of initial and final states should be the same, for second class ones opposite (in the case of axial currents it is just the reverse). Thus the existence of second class currents allows for transitions, which are forbidden ^{*)} for first class currents. An example of a pure second class vector process is. $\eta \rightarrow \pi^+ e^- + \bar{\nu}$, which involves the matrix element $\langle \pi^+ | J_\mu^V | \eta \rangle$. The branching ratio, however, is very small ^{4), 5)}. But it may be that the related process in peripheral neutrino induced reactions will become measurable in future neutrino experiments.

In this paper we investigate neutrino η and X^0 production through intervention of second class currents. Under various theoretical assumptions some competing processes, involving only first class currents are considered.

The reason for choosing the pseudoscalar η and X^0 mesons is related to the fact that only a vector current can be formed from two pseudoscalar mesons. In the diagram involving one-pion exchange (Fig. 1) the matrix element for η or X^0 production is of pure second type.

*) We neglect in this discussion the small electromagnetic corrections to weak interactions.

For the production of vector mesons ⁶⁾, one can always find a first class matrix element which would obscure the possible existence of the second class current. For instance, in ρ production through pion exchange, the vector current $\langle \rho | J_{\mu}^V | \pi \rangle$ is indeed of second class, but also an axial current $\langle \rho | J_{\mu}^A | \pi \rangle$ is present, which is of first class. If there is any chance of directly detecting second class currents in neutrino processes, then η and X^0 production are the best candidates, as one expects the first class contributions to be less important, being related to heavy particle exchanges.

It turns out indeed that for equal coupling constants second class contributions tend to be larger than first class ones, although both are rather small ($10^{-39} - 10^{-40} \text{ cm}^2$). Due to the arbitrariness in the form factors involved it is very hard to make definite statements on the differences in the various differential cross-sections between first and second class processes.

In Section 2 we consider the second class contributions to the processes $\nu + n \rightarrow \mu^- + p + \eta (X^0)$, in Section 3 the first class contributions. The results are the same for the reaction $\bar{\nu} + p \rightarrow \mu^+ + n + \eta (X^0)$, a specific property of neutrino induced peripheral processes.

2. SECOND CLASS PROCESSES

The most important diagram to be expected, if second class currents do indeed occur, is the one pion exchange diagram of Fig. 1.

- Figure 1 -

In the following we shall often use the five invariants which describe the process kinematically ⁷⁾ namely

$$\begin{aligned}
 W^2 &= -(k+p)^2 \\
 \omega^2 &= -(k'+q')^2 \\
 u^2 &= -(q'+p')^2 \\
 t^2 &= (k-k')^2 \\
 \Delta^2 &= (p-p')^2
 \end{aligned} \tag{1}$$

The various momenta are defined on Fig. 1.

For the upper vertex one can take quite generally the form

$$iG[F_1(t^2)(q+q')_\mu + F_2(t^2)(q-q')_\mu] \bar{U}_\ell(k') \gamma_\mu (1+\gamma_5) U_\nu(k) \tag{2}$$

For the lower vertex we take

$$i g_{\pi^-} F(\Delta^2) \bar{u}(p') \gamma_5 u(p) \quad (3)$$

with

$$g_{\pi^-}^2 / 4\pi = 28, \quad G^2 = 1.0068 \cdot 10^{-10} / m_p^4 \quad 8)$$

and

$$F(\Delta^2) = \frac{1}{1 + \Delta^2 / M^2} \quad (4)$$

Equation (4) gives a form factor of the Clementel - Villi type ⁹⁾ which is often used in weak interaction calculations. If one wants to take a phenomenological form factor from strong interactions, then such a form factor contains also the influence of another vertex (and again the propagator). One could try to take the square root of such a form factor instead of Eq. (4). Use of a form $\sqrt{(0.132 - m_\pi^2) / (0.132 + \Delta^2)}$ as suggested from Ref. ¹⁰⁾, decreases the cross-section roughly by a factor $\frac{1}{2}$ as compared to the form factor (4).

This gives the differential cross-section

$$d\sigma = \frac{1}{4} \frac{g_{\pi^-}^2}{(2\pi)^5} \delta(\kappa' + q' + p' - \kappa - p) G^2 \frac{F^2(\Delta^2)}{(-\kappa, p)} \frac{|cA|^2 \Delta^2}{(m_\pi^2 + \Delta^2)^2} \frac{d\vec{\kappa}'}{\kappa'_0} \frac{d\vec{q}'}{q'_0} \frac{d\vec{p}'}{p'_0} \quad (5)$$

$$\begin{aligned}
|\mathcal{A}|^2 &= \frac{|F_1(t^2)|^2}{2} \left[(t^2 + m_\mu^2)(-4\omega^2 + m_\mu^2) - 4(\omega^2 + \Delta^2)(-\omega^2 + m_\eta^2) \right] + \\
&\quad \frac{|F_2(t^2)|^2}{2} m_\mu^2(t^2 + m_\mu^2) + \operatorname{Re}(F_1(t^2)F_2^*(t^2)) m_\mu^2(2\omega^2 + 2\Delta^2 - t^2 - m_\mu^2)
\end{aligned}
\tag{6}$$

Thus the total cross-section becomes :

$$\sigma = \frac{1}{4(2\pi)^3} G^2 g_{\pi\pi}^2 \int d\Delta^2 \int d\omega^2 \int dt^2 \frac{F(\Delta^2)}{(m_\pi^2 + \Delta^2)^2} \frac{\Delta^2}{\omega^2 + \Delta^2} \frac{|\mathcal{A}|^2}{(W^2 - M^2)^2}$$

(7)

The integration region is :

$$\begin{aligned}
\Delta_{\min}^2 &= -2M^2 + \frac{1}{2W^2} (W^2 + M^2)(W^2 + M^2 - (m_\eta + m_\mu)^2) \\
&\quad + \frac{W^2 - M^2}{2W^2} \left[(W^2 + M^2 - (m_\eta + m_\mu)^2)^2 - 4W^2 M^2 \right]^{1/2}
\end{aligned}
\tag{8}$$

$$\omega_{\max}^2 = \frac{-\Delta^2(W^2 + M^2)}{2M^2} + \frac{W^2 - M^2}{2M^2} \left[\Delta^2(\Delta^2 + 4M^2) \right]^{1/2}$$

$$\omega_{\min}^2 = (m_\mu + m_\eta)^2
\tag{9}$$

6.

$$t_{\min}^2 = -m_\mu^2 + \frac{(\omega^2 + m_\mu^2 - m_\eta^2)(\omega^2 + \Delta^2)}{2\omega^2} \pm \frac{\omega^2 + \Delta^2}{2\omega^2} \left[(\omega^2 + m_\mu^2 - m_\eta^2)^2 - 4m_\mu^2 \omega^2 \right]^{1/2} \quad (10)$$

In our formulae, m_η stands for the mass of the η (548.9 MeV) or X^0 (958.6 MeV) mesons.

Let us now discuss briefly the assumptions we made about the second class current. We consider therefore

$$\langle \eta | J_\mu^V | \pi \rangle = F_1(t^2) (q + q')_\mu + F_2(t^2) (q - q')_\mu \quad (11)$$

for a real pion and assuming that the off-mass shell effect is small, we can apply our reasonings to the peripheral diagram, involving a virtual pion.

To this matrix element only the second class vector current contributes, so F_1 and F_2 are purely imaginary.

Concerning this second class current one could consider the following possibilities :

- i) The current is not conserved. Thus one has no relation between F_1 and F_2 . We omit the F_2 contribution, being proportional to the lepton mass squared, although this is somewhat less satisfactory for muons. For F_1 one takes a form factor similar to the electromagnetic one :

$$F_1(t^2) = \frac{1}{1 + t^2/N^2} F_1(0) \quad (12)$$

where for the electromagnetic pion form factor $N = 0.66$. If one thinks in terms of dispersion relations, the form factor would get contributions from spin one $\eta - \pi$ resonances ; however no such resonances are known as yet. Therefore it could be that N has to

be taken quite large. We calculate $\bar{G}$ for different N values, using the same strength for the second class current as for the pion current, i.e., $F_1(0) = i$. One should remark here that the question whether the second class current, if detected, is conserved or not might possibly be tested experimentally along lines suggested by Adler¹¹⁾.

ii) The current is conserved. One finds the relation

$$F_1(t^2)(-m_\pi^2 + m_\eta^2) + F_2(t^2)t^2 = 0 \quad (13)$$

Assuming a regular behaviour of $F_2(t^2)$ for $t^2 \rightarrow 0$, one deduces

$$F_1(0) = 0$$

and

$$F_2(t^2) = \frac{-m_\eta^2 + m_\pi^2}{t^2} F_1(t^2) \quad (14)$$

In the actual calculations we tried to take into account the off-mass shell effect by using

$$F_2(t^2) = \frac{-m_\eta^2 - \Delta^2}{t^2} F_1(t^2) \quad (15)$$

It turned out that the calculations are not sensitive to this refinement. For $F_1(t^2)$ we will use form factors which are damped at high t^2 . We take

$$F_A(t^2) = i \frac{t^2}{N^2} \exp -\frac{\sqrt{t^2}}{N} \quad (16-a)$$

$$F_B(t^2) = \frac{i t^2}{N^2 (1 + t^2/N^2)^2} \quad (16-b)$$

The exponential decrease is suggested by the behaviour of electromagnetic form factors at very high t^2 ¹²⁾. The mean square radius belonging to these form factors is $6/N^2$. The value $N \approx 0.6$ GeV would correspond to the mean square radius of the proton, which was for instance suggested as the strength for the hypothetical electromagnetic C violating decay $\eta \rightarrow \pi^0 + e^+ + e^-$ ¹³⁾.

- iii) The first and second class currents belong to the same SU(3) octet. In this case one can show that in exact SU(3) second class matrix elements are forbidden ¹⁴⁾. Only through symmetry breaking effects the matrix elements are different from zero. This could imply that weakening of the form factors F_A, F_B by some factor is needed. The magnitude of this factor is difficult to estimate at this stage. Feinberg ¹⁵⁾ has attempted to calculate the similar factor occurring in the above mentioned C violating decay $\eta \rightarrow \pi^0 + e^+ + e^-$ by assuming that the symmetry breaking is entirely due to $\eta - X^0$ mixing. If this is true, the factor multiplying Eq. (16) could be as small as $\approx 1/100$. However such a model is at this stage by no means compelling by any experimental or theoretical evidence. In any case, no such forbidding factor should occur for $\langle X^0 | J_\mu^V | \pi \rangle$, due to the fact that X^0 is mainly a unitary singlet and from this stem the special interest in X^0 production by neutrinos.

The general group theoretical structure of $F_1(t^2)$ in broken SU(3) symmetry for a second class octet current between meson octet states is

$$F_1(t^2) = a(t^2) \text{Tr}([M_1, M_2][\lambda_i, \lambda_j]) + b(t^2) [\text{Tr}(M_1 \lambda_i) \text{Tr}(M_2 \lambda_j) - \text{Tr}(M_1 \lambda_j) \text{Tr}(M_2 \lambda_i)] \quad (17)$$

when $\langle M_1 | J_\mu^i | M_2 \rangle$ is considered. For M_1, M_2 one takes the appropriate SU(3) λ matrices ¹⁶⁾. From this one sees that for strangeness conserving decays only $b(t^2)$ contributes while for the $K_{\ell 3}$ decays only $a(t^2)$. Hence the existence of second class contributions in leptonic η decay does not imply second class contributions in $K_{\ell 3}$ decay and vice versa.

The cross-sections are calculated under the above assumptions (i) and (ii) for different N values. The morale of the discussion concerning assumption (iii) is that for this case the results for η production (not however for X^0) calculated under assumption (ii) should be multiplied by an unknown factor which probably lies between 1 and 0.01. The results, which according to the different assumptions (Eqs. (12), (16-a), (16-b)) are denoted by σ_1 , σ_{2A} , σ_{2B} are listed in Table I for the η , in Table II for the X^0 production.

3. FIRST CLASS PROCESSES

As most important first class contributions to the process $\nu + n \rightarrow p + \mu^- + \eta (X^0)$ we take the peripheral graphs a and b in Fig. 2, in which a pion and a ρ is exchanged respectively. These processes involve the axial current $\langle \pi | J^A | 0 \rangle$ and the vector current $\langle \eta | J_\mu^V | \rho \rangle$ which are both of first class.

- Figure 2 -

One could argue that the production could also take place via a nucleon resonance, which decays in an $\eta (X^0)$ meson and a proton. Such a mechanism seems to be dominant for neutrino production of pions ¹⁷⁾. However the nucleon resonances needed for the $\eta (X^0)$ production have a substantially higher mass than the ones for π production and the partial decay into η and a fortiori for X^0 is small. Moreover, such contributions should be distinguishable experimentally from other mechanisms.

Graph a) gives the differential cross-section

$$d^5\sigma = \frac{2}{(2\pi)^3} g^2 m_\mu^2 \frac{[M^2 t^2 + (p \cdot q)^2]^{1/2}}{(-k \cdot p)} \frac{t^2 + m_\mu^2}{(t^2 + m_\pi^2)^2} d^2\sigma(\pi^+ n \rightarrow p \eta) \frac{d\vec{k}'}{k'_0} \quad (18)$$

where M is the nucleon mass and g is related to the $\pi^+ \rightarrow \mu^+ + \nu$ rate Γ ;

$$\Gamma = \frac{g^2}{4\pi} m_\pi m_\mu^2 \left[1 - \frac{m_\mu^2}{m_\pi^2} \right]^2 \quad (19)$$

Integration of (18) gives :

$$\sigma_a(W^2) = \frac{g^2 m_\mu^2}{2\pi^2} \frac{1}{(W^2 - M^2)^2} \int_{t_{\min}^2}^{t_{\max}^2} dt^2 \int_{u_{\min}^2}^{u_{\max}^2} du^2 \sigma(u^2) \cdot \frac{m_\mu^2 + t^2}{(t^2 + m_\pi^2)^2} \cdot \left[\frac{1}{4} (u^2 - M^2 + t^2)^2 + M^2 t^2 \right]^{1/2} \quad (20)$$

Here $\sigma(u^2)$ is the total cross-section for $\pi^+ n \rightarrow \eta p$ as a function of the c.m. $\eta - p$ energy u^2 . The integration limits are :

$$t_{\min}^2 = -m_\mu^2 + \frac{W^2 - M^2}{2W^2} \left\{ W^2 + m_\mu^2 - (M + m_\eta)^2 \pm \left[(W^2 + m_\mu^2 - (M + m_\eta)^2)^2 - 4W^2 m_\mu^2 \right]^{1/2} \right\} \quad (21)$$

$$u_{\min}^2 = (M + m_\eta)^2$$

$$u_{\max}^2 = \frac{(m_\mu^2 M^2 + W^2 t^2)(W^2 - M^2 - m_\mu^2 - t^2)}{(t^2 + m_\pi^2)(W^2 - M^2)} \quad (22)$$

Using experimental data ¹⁸⁾ for $\sigma(u^2)$, the cross-section is evaluated for a set of neutrino laboratory energies (see Table I). The cross-section is very small, of the order of 10^{-43} cm^2 , and is therefore of no particular relevance for our considerations. It is reasonable to expect the same for X^0 production, as the cross-section for X^0 production in pion-nucleon collisions is generally smaller than for η production ¹⁹⁾ at the same energy.

For the calculation of graph b) we use at the upper vertex

$$i \bar{u}_\ell(\kappa) \gamma_\alpha (1 + \gamma_5) u_\nu(\kappa') g_V(t^2) \varepsilon_{\alpha\rho\sigma\beta} q'_\rho q_\sigma \varepsilon_\beta \quad (23)$$

Here ε_β is the polarization vector of the ρ meson, $q = p - p'$ and $g_V(t^2)$, the form factor of the weak $\eta - \rho$ vector current is taken to be

$$g_V(t^2) = \frac{1}{1 + t^2/N^2} g_V(0) ; N = 0.66 \quad (24)$$

Thus, the same kind of form factor is used as for $\pi - \pi$ currents, or in other words the pion electromagnetic form factor. $g_V(0)$ is obtained by using the conserved vector current theory ²⁰⁾ from the $\rho \eta \gamma$ coupling, which in turn is estimated from $\omega \rightarrow \pi + \gamma$ decay via SU(3) and $\omega - \varphi$ mixing. With a $\rho \eta \gamma$ vertex given by

$$\frac{f_{\eta\rho\gamma}}{m_\pi} \varepsilon_{\mu\nu\sigma\tau} P_\mu^\rho P_\nu^\sigma \varepsilon_\sigma^\delta \varepsilon_\tau^\rho \quad (25)$$

one finds ²¹⁾

$$\frac{f_{\eta\rho\gamma}^2}{4\pi} = 0.06 \alpha \quad (26)$$

If there is $\eta - X^0$ mixing²²⁾ the value given in (26) changes to 0.03α or 0.09α ²³⁾, referring to a mixing angle of $10^\circ(-10^\circ)$ respectively. The results presented in Table I are calculated with the value given in Eq. (26). In order to calculate $f_{X^0 \rho \gamma}$, we have to rely completely on a symmetry model with $\eta - X^0$ mixing^{22),23)}. If X^0 is supposed to be a pure SU(3) singlet, and no higher symmetry is assumed, no relation to experimentally measured processes is available and unfortunately, the width of the observed $X^0 \rightarrow \pi^+ \pi^- \gamma$ decay has not yet been measured. Hence, we use the model of Refs.²²⁾ and ²³⁾ which gives for $X^0 \rightarrow \rho \gamma$ a calculated width of $0.25(0.14)$ MeV. This corresponds to a coupling

$$\frac{f_{X^0 \rho \gamma}^2}{4\pi} = 0.12\alpha \quad (0.066\alpha) \quad (27)$$

where $f_{X^0 \rho \gamma}$ is defined by a vertex analogous to Eq. (25), with η replaced by X^0 and the two numbers in Eq. (27) correspond to a mixing angle of $10^\circ(-10^\circ)$. From these electromagnetic couplings we get the weak coupling by the prescription $e \rightarrow \frac{G}{\sqrt{2}} \cdot \sqrt{2}$.

Thus, one finds

$$(\eta) g_v^2 = \frac{0.06}{m_{\pi^0}^2} G^2 = 3.3 G^2 (\text{GeV})^{-2} \quad (28)$$

for the η case (without mixing), whereas for the X^0 case one gets

$$(X^0) g_v^2 = 6.5 (3.6) G^2 (\text{GeV})^{-2} \quad (29)$$

corresponding to a mixing angle of $10^\circ(-10^\circ)$. The lower vertex of the diagram b is given by

$$i\sqrt{2} f_v(\omega^2) \varepsilon_{\mu\nu} \bar{u}(p') (\gamma_\mu + \lambda \sigma_{\mu\nu} (p_\nu - p'_\nu)) u(p) \quad (30)$$

for charged ρ exchange, where f_V is the ρNN coupling constant. For $f_V(\Delta^2)$ we tried $\frac{1}{1+\Delta^2/M^2} f_V(0)$ and $\exp(-2.5 \frac{\Delta^2}{2}) f_V(0)$, i.e., the same form factor as we used for the πNN vertex and the square root of the form factor which is suggested by Jackson and Pilkuhn²⁴⁾, respectively. For $f_V(0)$ we take the value obtained in Ref.²⁵⁾, from an analysis of ω production at threshold in pion-nucleon collisions, giving $\frac{f_V^2}{4\pi} = 0.4$. This value is fairly close to the universality prediction²⁶⁾ of $\frac{f_V^2(0)}{4\pi} = 0.6$, obtained by relating it to the $f_{\rho\pi\pi}$ coupling constant measured in ρ decay. For λ are taken the values $-1.06, -1.5, -1.97$ (GeV^{-1}), the first corresponding to the value of Ref.²⁵⁾, the last to the value suggested by the electromagnetic (ρ dominated) nucleon form factors.

So we get the matrix element

$$M = \bar{u}_\mu(k') \gamma_\alpha (1 + \gamma_5) u_\nu(k) g_\nu(t^2) \varepsilon_{\alpha\tau\sigma\beta} q'_\tau q_\sigma \cdot \quad (31)$$

$$\frac{\delta_{\beta\delta} + q_\beta q_\delta / m_\rho^2}{m_\rho^2 + \Delta^2} f_V(\Delta^2) \bar{u}_\tau(p') (\gamma_\delta + \lambda \sigma_{\delta\tau} q_\tau) u_\mu(p)$$

This leads to the differential cross-section

$$d\sigma = \frac{1}{2(2\pi)^5} \delta(p' + k' + q' - p - k) \frac{g_V^2(t^2) f_V^2(\Delta^2)}{(\Delta^2 + m_\rho^2)^2} \frac{|\mathcal{M}|^2}{(-k \cdot p)} \frac{d\bar{k}'}{k'_0} \frac{d\bar{q}'}{q'_0} \frac{d\bar{p}'}{p'_0} \quad (32)$$

where $g_V^2 f_V^2 |\mathcal{M}|^2 / (\Delta^2 + m_\rho^2)^2$ is $|\mathcal{M}|^2$ summed over final spins, averaged over initial spins, multiplied by the appropriate kinematical factors. $|\mathcal{M}|^2$ is given in the Appendix. For the total cross section one finds

$$\sigma_b = \frac{4}{(2\pi)^4} \int d\Delta^2 \int d\omega^2 \int dt^2 \int du^2 \frac{|R|^2}{(\Delta^2 + m_\rho^2)^2} \frac{W^2}{(W^2 - M^2)^2} \frac{g_v^2(t^2) f_v^2(\Delta^2)}{(au^4 + bu^2 + c)^{1/2}} \quad (33)$$

Here a, b, c , are given in the Appendix. The integration region is given by Eqs. (8) - (10) and

$$u_{\min}^2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \quad (34)$$

The results are presented in Tables I and II. The use of the exponential form factor decreases the cross-section by a factor $\frac{1}{2}$ at the lower to $1/3$ at the higher energies as compared to the $\frac{1}{1 + \Delta^2/M^2}$ form. So the use of a larger ρ NN coupling constant, as done in Ref. ²² (three times larger) combined with an exponential form factor gives about the same results as the smaller coupling constant of Ref. ²⁵, combined with the more conventional form factor. In the Tables we therefore only list the values of the cross-section obtained in the latter way. For X^0 production the largest coupling constant in Eq. (29) was used. So if the η - X^0 mixing angle is -10° the X^0 production through ρ exchange decreases by a factor 0.5. The same kind of calculation for η production was done by Henry and Veltman ⁶, however with different coupling constants and no ω - η mixing was taken into account.

We have also calculated the differential cross-sections for the ρ exchange and the π exchange (Section 2) in the various variables $t^2, \Delta^2, \omega^2, u^2$. This obviously involves changes in Eqs. (8) - (10), (34), when the orders of integration are changed. The possibility of distinguishing between first and second class currents through an analysis of differential cross-sections will be discussed in Section 4.

Finally we remark that the cross-section σ_b (ρ exchange) and the cross-sections of Section 2 (π exchange) add without interference due to the occurrence of a γ_5 matrix in the pion-nucleon vertex.

4. COMPARISON BETWEEN ρ AND π EXCHANGE

In order to disentangle the first and second class processes one would like to have a simple tool, e.g., a different behaviour of the differential cross-sections. It turns out however that $\frac{d\sigma}{dt^2}$, $\frac{d\sigma}{d\Delta^2}$, $\frac{d\sigma}{d\omega^2}$ are in such a way sensitive to the form factors that no marked differences between ρ and π exchange can be stated. As an illustration $\frac{d\sigma}{dt^2}/\sigma$ and $\frac{d\sigma}{d\Delta^2}/\sigma$ at $k_0 = 3$ GeV neutrino lab. energy, are plotted in Figs. 3 and 4 respectively for the following cases: ρ exchange for $\lambda = -1.97$ and the "normal" and exponential $f_V(\Delta^2)$. π exchange for $N = 0.66$, for hypothesis (i) - no C.V.C. - and (ii) with here the exponential form factor Eq. (16-a). The form factor of Eq. (16-b) gives a curve somewhere between the two plotted π exchange curves. Use of the other N and λ values does not change the picture essentially. Thus one is lead to consider the dependence of the matrix element of two variables. A known difference between ρ and π exchange in peripheral strong interaction processes with essentially two particles in the final state (which may be resonances) is the dependence of $\frac{d\sigma}{d\Delta^2}$ on W^2 (*). The π exchange process decreases due to W^4 factors in the denominator whereas ρ exchange has also W^4 factors in the numerator, which compensate the former ones. Because this property is due to the spin of the exchanged particle the same kind of factors also appear in our matrix elements, besides many others, involving the other invariants. These other factors, and the fact that the integration limits have a W^2 dependence obscure the above mentioned distinction between ρ and π exchange in our case of one weak vertex and a three particle final state. In the region of interest this is shown in Fig. 5, where $\frac{d\sigma}{d\Delta^2}$ is plotted as a function of W^2 for two different fixed Δ^2 values for both the ρ exchange (exponential $f_V(\Delta^2)$) and the π exchange (form factor of Eq. (16-a)).

(*) We like to thank Drs. J. and T. Høgaasen for a discussion on this point.

So to get rid of the difficulties in above mentioned tests, one should test the presence or absence of u^2 dependence in the matrix element. This leads to the well-known Treiman-Yang tests ²⁷⁾, where for this case one could test at fixed W^2, ω^2, Δ^2 the differential cross-section as a function of t^2 at different u^2 values. For π exchange it is independent of u^2 , for ρ exchange not.

In conclusion we may remark that in general cross-sections for η and X^0 production are expected to be small, although existence of second class currents of the same strength as first class currents could make the cross-sections an order of magnitude larger.

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k_0 Lab. in GeV										
	N or λ	1.0	1.5	2.0	2.5	3.0	4.0	5.0	7.0	10.0
σ_1	0.66	0.046	1.11	2.75	4.45	5.97	8.47	10.4	13.1	15.7
	1.00	0.058	1.54	4.08	6.91	9.59	14.3	18.2	24.0	30.0
	1.50	0.066	1.87	5.23	9.26	13.3	21.0	27.7	38.8	51.0
	2.00	0.068	2.02	5.83	10.6	15.6	25.4	34.6	50.5	69.3
	∞	0.073	2.26	6.90	13.2	20.4	36.4	53.6	89.6	145
σ_{2A}	0.66	$1.0 \cdot 10^{-3}$	0.138	0.694	1.73	3.12	6.58	10.5	18.3	28.6
	1.00	$0.30 \cdot 10^{-3}$	0.050	0.293	0.825	1.64	4.03	7.20	14.9	27.9
	1.50	$0.77 \cdot 10^{-4}$	0.015	0.100	0.307	0.656	1.82	3.56	8.57	18.8
	2.00	$0.28 \cdot 10^{-4}$	0.006	0.042	0.134	0.299	0.884	1.83	4.80	11.6
σ_{2B}	0.66	$1.0 \cdot 10^{-3}$	0.080	0.287	0.566	0.862	1.44	1.95	2.77	3.67
	1.00	$3.7 \cdot 10^{-4}$	0.046	0.209	0.480	0.813	1.56	2.32	3.73	5.46
	1.50	$9.9 \cdot 10^{-5}$	0.018	0.102	0.276	0.527	1.20	2.00	3.75	6.32
	2.00	$3.5 \cdot 10^{-5}$	0.0075	0.049	0.148	0.305	0.784	1.43	3.05	5.79
σ_a		$0.67 \cdot 10^{-3}$	$4.0 \cdot 10^{-3}$	$4.7 \cdot 10^{-3}$	$4.6 \cdot 10^{-3}$	$4.3 \cdot 10^{-3}$	$3.4 \cdot 10^{-3}$	$2.8 \cdot 10^{-3}$	$2.0 \cdot 10^{-3}$	$1.4 \cdot 10^{-3}$
σ_b	-1.06	$6.1 \cdot 10^{-4}$	0.026	0.089	0.176	0.278	0.509	0.754	1.25	1.96
	-1.50	$9.7 \cdot 10^{-4}$	0.041	0.137	0.271	0.430	0.787	1.17	1.95	3.07
	-1.97	$1.4 \cdot 10^{-3}$	0.061	0.203	0.402	0.637	1.17	1.75	2.93	4.64

- Table I -

Total cross-sections for γ production in units of 10^{-40} cm^2

k_0 Lab. in GeV		2.0	2.5	3.0	4.0	5.0	7.0	10.0
σ_1	N or λ							
	0.66	0.146	0.898	1.98	4.35	6.47	9.71	12.9
	1.00	0.207	1.35	3.11	7.22	11.2	17.7	24.6
	1.50	0.250	1.73	4.17	10.3	16.7	28.2	41.7
	2.00	0.269	1.92	4.77	12.3	20.6	36.4	56.3
	∞	0.299	2.26	5.91	16.8	30.7	62.7	115
σ_{2A}	0.66	$1.3 \cdot 10^{-2}$	0.205	0.738	2.81	5.78	12.7	22.9
	1.00	$4.5 \cdot 10^{-3}$	0.084	0.344	1.59	3.75	10.0	21.8
	1.50	$1.3 \cdot 10^{-3}$	0.028	0.126	0.671	1.77	5.58	14.4
	2.00	$5.0 \cdot 10^{-4}$	0.011	0.054	0.315	0.885	3.06	8.81
σ_{2B}	0.66	$8.7 \cdot 10^{-3}$	0.090	0.249	0.685	1.16	2.01	2.99
	1.00	$4.5 \cdot 10^{-3}$	0.063	0.208	0.697	1.32	2.64	4.40
	1.50	$1.6 \cdot 10^{-3}$	0.029	0.116	0.492	1.08	2.57	5.01
	2.00	$6.4 \cdot 10^{-4}$	0.014	0.061	0.302	0.739	2.04	4.52
σ_b	-1.06	0.015	0.094	0.220	0.551	0.938	1.77	3.03
	-1.50	0.025	0.149	0.347	0.869	1.48	2.80	4.79
	-1.97	0.037	0.225	0.522	1.31	2.22	4.22	7.26

- Table II -

Total cross-sections for $\bar{X}^0$ production in units
of 10^{-40} cm^2

A P P E N D I X

In the c.m. system, a , b and c are related to the azimuthal angle φ of $\vec{p}'$ ($\vec{k}$ is the z axis, $\vec{k}$, $\vec{k}'$ span the azimuthal plane ⁷⁾) in the following way

$$(2|\vec{k}||\vec{p}'| \sin \Theta_k \sin \Theta_{p'})^2 \sin^2 \varphi = \frac{a u^4 + b u^2 + c}{4 W^2 (W^2 - M^2)}$$

where

$$a = -4 W^4 (\Delta^2 + \omega^2)^2$$

$$b = -8 W^4 (t^2 + m_\mu^2) \left[(W^2 - M^2)(\Delta^2 - \omega^2) + \omega^2(\Delta^2 + \omega^2) \right] +$$

$$8 W^4 (\omega^2 + \Delta^2) \left[(W^2 - M^2)(m_\eta^2 - m_\mu^2) + \Delta^2(m_\mu^2 + W^2) + \omega^2(m_\mu^2 + M^2) \right]$$

$$c = -4 W^2 \left[(W^2 + M^2 - \omega^2)^2 - 4 M^2 W^2 \right] \left[m_\mu^2 (W^2 - M^2)^2 + W^2 (t^2 + m_\mu^2)^2 - \right.$$

$$\left. (t^2 + m_\mu^2)(W^2 - M^2)(W^2 + m_\mu^2) \right] - (W^2 - m_\mu^2)^2 \left[2 W^2 (\Delta^2 + 2 M^2) - \right.$$

$$\left. (W^2 + M^2)(W^2 + M^2 - \omega^2) \right]^2 - (W^2 - M^2)^2 \left[W^2 (\omega^2 - W^2 + m_\mu^2 + M^2 - 2 m_\eta^2) - \right.$$

$$\left. m_\mu^2 (\omega^2 - M^2) \right]^2 - 2 \left[2 W^2 (\Delta^2 + 2 M^2) - (W^2 + M^2 - \omega^2)(W^2 + M^2) \right] \cdot$$

$$\left[W^2 (\omega^2 - W^2 + m_\mu^2 + M^2 - 2 m_\eta^2) - m_\mu^2 (\omega^2 - M^2) \right] \left[2 W^2 (t^2 + m_\mu^2) - (W^2 - M^2)(W^2 + m_\mu^2) \right]$$

The quantity $|M|^2$ in Eq. (32) is defined as follows :

$$\begin{aligned}
 \frac{|M|^2}{2} = & \lambda^2 \Delta^2 \left\{ -(\kappa \cdot \kappa') \left[M^2 (q' \cdot q)^2 + M^2 m_\eta^2 \Delta^2 - \frac{1}{4} \Delta^4 m_\eta^2 \right] - \right. \\
 & \frac{1}{2} \left[2 (q' \cdot q) (p \cdot \kappa) - \Delta^2 (q' \cdot \kappa) \right]^2 + 2 m_\eta^2 \Delta^2 (p \cdot \kappa) \left[(q \cdot \kappa) - (p \cdot \kappa) \right] \left\{ \right. \\
 & - 2 M \lambda \Delta^2 \left\} - (\kappa \cdot \kappa') \left[m_\eta^2 \Delta^2 + (q' \cdot q)^2 \right] + \Delta^2 (q' \cdot \kappa)^2 - m_\eta^2 (q \cdot \kappa)^2 - \right. \\
 & 2 (q' \cdot q) (q \cdot \kappa) (q' \cdot \kappa) \left\{ + M^2 (\kappa \cdot \kappa') \left[m_\eta^2 \Delta^2 + (q' \cdot q)^2 \right] - 2 M^2 \Delta^2 (q' \cdot \kappa)^2 + \right. \\
 & 2 M^2 m_\eta^2 (q \cdot \kappa)^2 + 4 M^2 (q \cdot q') (q \cdot \kappa) (q' \cdot \kappa) - \frac{m_\eta^2 \Delta^4}{4} (\kappa' \cdot \kappa) - 2 \Delta^2 m_\eta^2 (p \cdot \kappa)^2 \\
 & - \frac{\Delta^2 m_\eta^2}{2} (q \cdot \kappa)^2 + 2 \Delta^2 m_\eta^2 (p \cdot \kappa) (q \cdot \kappa) - 2 (q' \cdot q)^2 (p \cdot \kappa)^2 - \frac{\Delta^2}{2} (q' \cdot q)^2 (\kappa \cdot \kappa') \\
 & - \Delta^2 (q' \cdot q) (q \cdot \kappa) (q' \cdot \kappa) + 2 \Delta^2 (q' \cdot q) (q' \cdot \kappa) (p \cdot \kappa) - \frac{1}{4} (\lambda^2 \Delta^2 + 1) (\varepsilon - u^2)^2. \\
 & \left[\Delta^2 (\kappa \cdot \kappa') + 2 (q \cdot \kappa)^2 \right] - (\lambda^2 \Delta^2 + 1) (\varepsilon - u^2) \left[2 \Delta^2 (p \cdot \kappa) (q' \cdot \kappa) - \right. \\
 & \left. \left. 2 (q \cdot q') (p \cdot \kappa) (q \cdot \kappa) - \Delta^2 (q \cdot \kappa) (q' \cdot \kappa) - \frac{\Delta^2}{2} (q \cdot q') (\kappa \cdot \kappa') \right] \right\} ,
 \end{aligned}$$

where

$$\varepsilon = M^2 + \Delta^2 - t^2$$

$$(q' \cdot q) = \frac{1}{2} (\Delta^2 - t^2 - m_\eta^2)$$

$$(\kappa \cdot \kappa') = -\frac{1}{2} (m_\eta^2 + t^2)$$

$$(q' \cdot \kappa) = \frac{1}{2} (-\omega^2 - \Delta^2 + t^2 + m_\eta^2)$$

$$(\kappa \cdot p) = \frac{1}{2} (M^2 - W^2)$$

$$(\kappa \cdot q) = -\frac{1}{2} (\omega^2 + \Delta^2)$$

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FIGURE CAPTIONS

Figure 3 $\frac{d\sigma}{dt^2} / \sigma$ as function of t^2 at 3 GeV neutrino lab. energy.

Figure 4 $\frac{d\sigma}{d\Delta^2} / \sigma$ as function of Δ^2 at 3 GeV neutrino lab. energy.

Figure 5 $\frac{d\sigma}{d\Delta^2}$ as function of W^2 at two fixed Δ^2 values. The
 scale is fixed by X , which takes the values 10^{-42}
 (10^{-40}) for $\rho(\pi)$ exchange at $\Delta^2 = 0.06$ and
 $10^{-43} (10^{-41})$ for $\rho(\pi)$ exchange at $\Delta^2 = 2.5$.

FIG. 3

FIG. 4

FIG.5