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COHERENT BREAK UP OF THE DEUTERON
AND THE PHASE OF HIGH ENERGY AMPLITUDES

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Lately the problem of coherent scattering of high energy particles from bound states has attracted considerable interest. In particular, two experiments of coherent elastic scattering of high-energy protons on deuterium ¹⁾ at 2 GeV incident energy and on ⁴He ²⁾ at 1.7 GeV/c incident momentum have been performed, measuring the elastic angular distribution to rather high values of the momentum transfer [of the order of $1(\text{GeV}/c)^2$]. The most striking feature of those experiments is the sudden change in the angular distribution from a very fast decrease to a much flatter behaviour. The value t_0 of the transition point is $-t_0 \simeq 0.35$ for pd scattering and 0.25 for pHe scattering [in $(\text{GeV}/c)^2$]. Moreover the pHe angular distribution shows a dip around $t = t_0$ ^{*}), while the situation is unsettled in pd scattering, lacking the experimental data.

All these features (the strong t dependence at small t , the dip with a subsequent second maximum and the flattening of the large t scattering), are well explained by the multiple scattering theory proposed by Glauber ⁴⁾, and then discussed in further details in other papers ⁵⁾. The actual applications are due to Coleman and Franco ⁶⁾ (pd scattering) and to Czyz and Lesniak ⁷⁾ and Bassel and Wilkin ⁸⁾ (pHe scattering). Denoting by x any high energy incident hadron, the Glauber theory expresses the x nucleus amplitude as the sum of the single scattering amplitudes (usually referred as the impulse approximation), in which x interacts only with one of the nucleons in the target, plus the multiple scattering amplitudes, in which x suffers two successive collisions from two different nucleons (double scattering) or in general n successive collisions (n -ple scattering). The single scattering term is overwhelming in the forward direction, and goes down with t with a law which is governed by the form factor of the nucleus, i.e., by its "radius"; the double scattering term, which is small at $t = 0$ (the Glauber shadow correction to the total cross-section is in general only a few percent) goes down with t with a much smaller slope

^{*}) This dip has not to be confused with the dip observed in the elastic electron-He scattering ³⁾, which shows up at a value of $-t \simeq 0.4 (\text{GeV}/c)^2$.

(for pd scattering its slope is a half of the slope of the corresponding nucleon-nucleon diffraction peak). Double scattering therefore overwhelms single scattering at large t (multiple scattering terms start to be important at still larger t) ; however, there exists an interval of t , around a value $t = t_0$, in which the single and double scattering amplitudes are of the same order of magnitude, and therefore they interfere appreciably. The cancellation is complete at $t = t_0$ if the x nucleon amplitudes are, for all values of t , purely imaginary ; in this situation one would therefore have a zero in the angular distribution, resulting in a very pronounced dip. This dip fills up very rapidly if the real parts of the x nucleon scattering amplitudes are different from zero, and reasonable fits to the existing experimental data have been obtained ⁶⁾⁻⁸⁾ to the existing experimental data by taking t independent values of $\alpha_{p,n}(t) = (\text{Re } T_{pp,pn})/(\text{Im } T_{pp,pn})$, equals to the values of α at $t = 0$.

One has to stress, however, that the x nucleus cross-section on the dip depends upon all the values of $\alpha(t)$ for $t \leq t_0$, and particularly on the value of $\alpha(t_0)$ ^{*)} ; in fact, it has been shown ⁹⁾ that a better fit to the pd angular distribution in the shoulder region after the dip can be obtained by assuming a reasonable variation of $\alpha(t)$ with t . The precise measurement of the elastic x nucleus angular distribution in the region around the dip would therefore be a tool in order to have information about the t dependence of the real parts of the x proton and x neutron scattering amplitudes. Experiments of this kind with incident π mesons are planned at CERN ¹⁰⁾. In general, if one wants to have reliable information about the elementary particle physics, one has to avoid as much as possible the complication due to the nuclear physics problem, and therefore is preferable to limit the analysis to nuclei with only few nucleons, among which the deuteron is the simplest and the best known one.

The elastic $x-d$ scattering has, however, the drawback that the position of the dip, namely the value of t_0 , cannot be changed. In fact

*) If one takes for $\alpha(t)$ the linear expansion $\alpha(t) = \alpha(0) + \alpha_1 t$, then the cross-section at the dip depends on the values of $\alpha(t_0)$ and $\alpha(t_{0/2})$.

its value depends essentially only upon the internal structure of the deuteron, and is almost energy-independent, since variations of the energy of the total cross-section and of the widths of the elastic x nucleon scattering affect very little (only logarithmically) the value of t_0 . By using elastic scattering one could vary t_0 only by changing the target nucleus, at the price of introducing different nuclear physics complications. In any case, only a small finite number of values of t_0 will be obtained.

We would like to point out here that the interference between single and double scattering amplitudes is not limited to the elastic scattering, in which the target nucleus remains in the fundamental state, but will occur also in phenomena in which the target breaks up into non-relativistic components, inasmuch all the independent variables in the final configuration are measured; then by varying the values of some kinematical variables in the final state, one will correspondingly change the position of the (eventual) dip in a continuous way. The basis of the argument lies in the observation that the Glauber multiple scattering expansion is valid not only for the scattering amplitude, but for the scattering operator, the conditions for its validity being the high energy of the incident particle (so high that its wavelength is small as compared to the target "dimensions"), and the small momentum transferred to the scattered x particle. The expansion is therefore valid for any matrix element of the scattering operator, and in particular for the break-up. One can then, by a judicious choice of the kinematical variables, enhance the contribution of the double scattering with respect to that of the single scattering, pushing down in that way the value of t_0 . As for the elastic scattering, the destructive interference between single and double scattering is a consequence of the fact that, at least at small angles, all the high energy amplitudes are mostly imaginary.

For simplicity, we shall discuss here only the case of the deuteron break-up

$$x + d \rightarrow x' + p + n \quad (1)$$

neglecting all the complications connected with the spin variables.

For the kinematical description of the final state, we choose the following four independent variables (the total energy being fixed) in the lab system of the deuteron (see Fig. 1a)

$$(\vec{p}_x - \vec{p}_{x'})^2 = \vec{\Delta}^2; \quad k_l; \quad k_t^2; \quad \vec{k}_t \cdot \vec{\Delta} \quad (2)$$

where k_l and $\vec{k}_t$ are the projections of $\vec{k} = (\vec{k}_p - \vec{k}_n)/2$ along the beam direction and in the orthogonal plane.;

In the limit in which $p_0^2 \rightarrow \infty$ but $-t, k_p^2, k_n^2 \ll m^2$ (m = nucleon mass), the energy momentum conservation gives the constraints

$$\begin{aligned} t &\equiv (E_x - E_{x'})^2 - \Delta^2 \approx -\Delta^2; \quad \vec{k}_p + \vec{k}_n = -\vec{\Delta}; \quad \vec{\Delta} \cdot \vec{p}_x = 0 \\ k_p^2 + k_n^2 &= 2k^2 + \frac{\Delta^2}{2}; \quad k_p^2 - k_n^2 = -\vec{k} \cdot \vec{\Delta} = -k_t \cdot \vec{\Delta} \\ k_l &= k_{pl} = -k_{nl} \end{aligned} \quad (3)$$

A straightforward application of the Glauber method to reaction (1), corresponding to the diagrams of Fig. 1b and Fig. 1c, gives the result *

$$\begin{aligned} \frac{d^4\sigma}{dt d_3k} &= \frac{1}{8\pi^2 p_x^2} \cdot \left| f_{xn}(t) \psi(\vec{k}_p) + f_{xp}(t) \psi(\vec{k}_n) + \right. \\ &\quad \left. + \frac{i}{2\pi p_x} \int d_2\vec{q} \psi(\vec{q} - \vec{k}) f_{xp}\left(-(\vec{q} - \frac{\vec{\Delta}}{2})\right) f_{xn}\left(\vec{q} + \frac{\vec{\Delta}}{2}\right) \right|^2 \end{aligned} \quad (4)$$

where $\vec{q}$ is a two-dimensional vector, orthogonal to $\vec{p}_x$, and the x nucleon scattering amplitudes are normalized in such a way that $(d\sigma/d\Omega) = |f(t)|^2$. Only their dependence upon the momentum transfer is explicitly indicated. $\psi(\vec{k})$ is the Fourier transform of the deuteron wave function.

We can study now the t dependence of this differential distribution, keeping fixed all the remaining three variables. In order to enhance the double scattering contribution, we can observe that the maximal contribution to a double scattering with a net momentum transfer t arises from two

*) In this formula we have neglected the effect of the final state interaction between the proton and the neutron, which will be discussed below.

individual scattering each with partial momentum transfer $t/4$; in other words, the best configuration for a double scattering is a symmetric one, in which $E_p = E_n$, and therefore $\vec{k}_t \cdot \vec{\Delta} = 0$.

In order to give an estimate of the value of t_0 , we can use for f_{xn} and f_{xp} the forms $f_{xp} = f_p(0) \exp(b_s t/2)$, $f_{xn} = f_n(0) \exp(b_s t/2)$, which give good fits to the high energy elastic differential cross-section at small t , assuming for simplicity the phase of the scattering amplitude to be t independent.; moreover, in order to simplify as much as possible the calculation, we shall assume the Pomeranchuk limits

$$f_{xp} = f_{xn} = i \frac{p_x \sigma_T}{4\pi} \exp[b_s t/2] \quad (5)$$

All these detailed assumptions do not change in an essential way the value of t_0 . In order to compute analytically all the expressions, we can moreover assume for the deuteron a Gaussian wave function, with the value of b_f as given by Verde ¹¹⁾

$$\psi(p^2) = N_g \exp[-b_f p^2] ; \quad b_f = 66 \left(\frac{GeV}{c} \right)^{-2} \\ N_g = \left[\frac{(2b_f)^3}{\pi} \right]^{1/4} \quad (6)$$

In this way we do not expect the exact value of t_0 to be a reliable one, but we can compare it with the corresponding value of the elastic scattering, and study its dependence upon the variables k_e^2 , k_t^2 and $\vec{k}_t \cdot \vec{\Delta}$. We find in this way

$$-t_0 = \frac{4}{b_s + b_f} \ln \left[\frac{16\pi}{\sigma} (b_s + b_f) \cosh(b_f \vec{k}_t \cdot \vec{\Delta}) \right] - \frac{4 b_f^2}{(b_s + b_f)^2} k_t^2 \quad (7a)$$

$$\frac{d^4 \sigma}{dt d_3 k} = \frac{\sigma_T^2 N_g^2}{32 \pi^4} \exp \left[(b_s + b_f) t - 2 b_f k_e^2 - 2 b_f k_t^2 \right].$$

$$\cdot \left[\cosh(b_f \vec{k}_t \cdot \vec{\Delta}) - \frac{\sigma_T}{16\pi(b_s + b_f)} \exp \left(-\frac{t}{4} (b_s + b_f) + \frac{b_f^2}{(b_s + b_f)} k_t^2 \right) \right]^2 \quad (7b)$$

From (7a), (7b), one can draw the conclusions :

- a) - the form of the distribution in t and the position of the dip do not depend on k_ℓ ; different values of k_ℓ only change the absolute value of the cross-section, which is a strongly decreasing function of k_ℓ^2 ;
- b) - for a fixed value of k_t , $|t_0|$ increases with $|E_p - E_n| = |\vec{k}_t \cdot \vec{\Delta}|$;
- c) - in the symmetrical configuration, $E_p = E_n$, $|t_0|$ decreases with k_t . However, one cannot take too large values of k_t , both from the requirement of the non-relativistic energy of the proton and the neutron and from the difficulty that they involve very high momentum components of the deuteron wave functions, which are unknown and in any case not of the form (6).
- d) - the value of t_0 for the elastic scattering, computed from the same wave function, is

$$-t_0^d = \frac{4}{b_s + \frac{b_f}{2}} \ln \left[\frac{16\pi}{\sigma_T} \left(b_s + \frac{b_f}{2} \right) \right] \simeq -2t_0(k_e = k_t = 0)$$

since $b_f \gg b_s$. The physical reason of this value twice as big for the elastic scattering is easily understood by noticing that the single scattering in the elastic case goes down as the Fourier transform of the square of the wave function (the form factor), while in the inelastic case the single scattering goes down as the Fourier transform of the wave function itself.

The use of a more proper wave function does not modify qualitatively the preceding results, at least for not too big k_ℓ and k_t ; in Fig. 2 we plot the form of the function

$$F(k_e, \vec{k}_t, t) = \frac{d^4\sigma}{dt d_3k} / \left(\frac{128\pi^4}{\sigma_T^2 (1+d^2)} \psi^2(t=0) \right)$$

as a function of t , both with $\alpha = 0$ and with a common value of $\alpha_p = \alpha_n = -0.2$ and a value of $\sigma_T = 25$ mb, computed from the Gartenhaus wave function,¹²⁾ for the values $k_\ell = k_t = 0$. The dip is now at a value of $-t_0 = 0.26$ (GeV/c)², and the cross-section on the second maximum is five orders of magnitude smaller than at $t = 0$.

From the previous discussion, it is clear why no dip appears in the momentum transfer distribution of the incoherently scattered particles, namely when one sums up over all the final configurations of the proton and the neutron allowed by the kinematics, looking only to the t distribution of x . In fact, all scatterings (elastic or inelastic) in which all the kinematical variables defining a particular final state are specified, is coherent, since one has to add the amplitudes of single and double scattering, and then square the sum in order to get the differential distribution in all the independent variables. If on the contrary one sums together different final configurations, then one has to add incoherently the corresponding partial cross-sections. Since each particular configuration in a final state gives a dip at a different t_0 , the dip will disappear in the incoherent scattering. In general, incoherent scattering is not computed by summing directly each partial distribution, but using the closure approximation⁵⁾; the result is the same, namely the dip is not present; the closure approximation, however, is less affected by the uncertainties connected with the final state interaction, which is already taken into account in the proper wave function of the bound state.

These remarks limit the feasibility of the coherent break-up of nuclei to the case of light nuclei, when one can hope to measure all the variables in the final state^{*)}.

A further difference between the coherent and incoherent break-up is that the coherent break-up of the deuteron predicts that after the dip the slope of the differential cross-section in t will be $b_s/2$, while in the incoherent process the slope at large t is predicted (and measured¹³⁾) to be equal to b_s .

Few words more about final state interactions between the proton and the neutron. Formula (4) was derived by using plane waves for the wave functions of the proton and the neutron in the final state; now it is known that at low relative energies there is a very strong final state interaction due to the large pn scattering lengths, which affects mostly the s wave

*) This would not be the situation, for instance, in the break-up of heavy nuclei, when one of the break-up products has levels which are not kinematically distinguishable from the others.

projection of the relative wave function. Although this effect is a very important one in determining the absolute values of the cross-section, nevertheless it will not affect the form of the t distribution, inasmuch one keeps fixed the value of the relative momentum $\vec{k}$. In this condition, the final state interaction will result only in an over-all multiplicative factor which can be estimated, for instance, using the dispersive approach of Bosco¹⁴⁾.

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FIGURE CAPTIONS

- Figure 1 - Schematic diagram of the deuteron break-up, showing the single and double scattering diagrams.
- Figure 2 - The function $F(t, k_\ell, \vec{k}_t)$, as defined in the text, computed from the Gartenhaus wave function, plotted as a function of $-t$ for the values $k_\ell = 0$, $\vec{k}_t = 0$.
- Figure 3 - The dependence of $-t_0$ upon k_t , in the range $0 \leq k_t \leq 0.2$ GeV/c, for $k_\ell = 0$, $\vec{k}_t \cdot \vec{\Delta} = 0$. The three curves are computed from the Hulthén, Gartenhaus and Gaussian wave functions.

FIG.1

FIG. 2

FIG. 3