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DOUBLE SCATTERING MODEL FOR HIGH ENERGY
BACKWARD PROTON-DEUTERON SCATTERING

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A B S T R A C T

The impulse series for nucleon-deuteron elastic scattering is generalized near the backward direction. It is shown that double scattering, which is known to be small near the forward direction, becomes dominant (as compared to single scattering) near the backward direction.

As an application, pd elastic scattering is considered at 1, 1.3 and 1.5 GeV where a large angle experiment has been recently done. The double scattering contribution accounts for both the angular distribution and the energy dependence of the large angle pd cross-section.

The predicted values of the backward cross-section are also in reasonably good agreement with the experimental results.

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1. INTRODUCTION

Recently an accurate experiment on elastic proton-deuteron scattering at high energies (from 1 to 1.5 GeV) and at high momentum transfer near the backward direction has been performed ¹⁾. This experiment shows a number of interesting features :

- a) the differential cross-section at fixed energy, which was measured for an interval of momentum transfer ranging from $0.2 \leq \tau \leq 1.2$ (GeV/c)² [$\tau = t - t_{\max}$, where $t_{\max} = t$, ($\theta_{\text{c.m.}} = 180^\circ$)] shows a peak near the backward direction; this backward peak has approximately an exponential form

$$\frac{d\sigma}{d\tau} = e^{a - b\tau + c\tau^2} \quad (1.1)$$

where c is small and $b \simeq 5(\text{GeV}/c)^{-2}$. It is remarkable that this peak is extremely broad as compared to the forward elastic pd peak [$(d\sigma/dt) = e^{a+(b+b_1)t}$, with $b+b_1 \simeq 22(\text{GeV}/c)^{-2}$], and its width is of the same order of magnitude as that of the pp elastic forward peak at comparable energies [$b \simeq 6(\text{GeV}/c)^{-2}$].

- b) the absolute value of the backward cross-section decreases very fast with energy; the values of the (extrapolated) cross-section for $\theta_{\text{c.m.}} = 180^\circ$ are

$T_{\text{lab}}(\text{GeV})$	$(d\sigma/d\tau)_{\tau=0} (\mu\text{b}/(\text{GeV}/c)^2)$
1	144
1.3	36
1.5	14.3

(1.2)

For comparison, we notice that the value of $(d\sigma/dt)_{t=0}$ in the same range of energy is almost constant and of the order of $400 \text{ mb}/(\text{GeV}/c)^2$, namely four orders of magnitude higher than the corresponding backward value at 1.3 GeV.

We have looked for possible mechanisms, giving rise to such a peak near the backward direction.

A peak near the backward direction in the elastic proton-deuteron scattering can be understood as an exchange effect, in which the incident proton replaces into the deuteron the target proton, keeping its original direction; we have in this way a final deuteron which travels in the same direction as the incident proton. This exchange effect gives always a peaking near the backward direction.

At low energies, in the MeV region, the favourite candidate for a backward peak is the neutron exchange diagram (pole diagram, Fig. 1), which gives rise to what is called in nuclear physics the Butler stripping mechanism ²⁾. This effect, if the exchanged neutron is treated as an elementary particle (in contrast with a Regge pole description), cannot give - at the energy in which we are interested - at the same time the absolute value, the energy behaviour, and the angular dependence of the differential cross-section. We can refer to the results of Coleman ³⁾, who has discussed such a mechanism. By using different forms for the deuteron wave function, he showed that in any case one meets some difficulty: for instance, using a zero range wave function, the angular dependence of the predicted backward peak agrees reasonably well with the experimental one, but the predicted magnitude exceeds that of the observed peak by a factor five at 1 GeV, a factor nine at 1.3 and a factor 15 at 1.5 GeV; the energy dependence is therefore completely wrong. By changing the wave function, one can ameliorate the energy dependence, but in this case the angular distribution becomes wrong: if one chooses, e.g., a Hulthén wave function, with or without hard core, the predicted magnitude is smaller than the experimental values by almost two orders of magnitude, and the slope of the backward peak too steep. The situation becomes worse if one uses a wave function which is smaller than the Hulthén one at small distances, as the Moravcsik ⁴⁾ analytic fit to the Gartenhaus wave function, in which case the predicted result is almost three orders of magnitude too small ^{*)}.

^{*)} This strong dependence of the cross-section on the kind of wave function is due to the large momentum transfer experienced in the backward direction [of the order of $4(\text{GeV}/c)^2$], which requires the very high frequency components of the deuteron wave function, and is not changed by an appropriate covariant description of the deuteron-proton-neutron vertex.

In general, one can observe that the difficulty of reproducing at the same time the energy and angle dependence of the differential cross-section with the neutron pole model is due to the fact that it predicts the differential cross-section to be essentially only a function of the variable $u = (p_1 - d_2)^2$ (apart from an energy dependent kinematical factor), while experimentally it is not so.

Moreover, the result that the pole model yields a magnitude of the backward peak too small for the most common wave functions is very remarkable, since it is known that at low energies, on the contrary, the pole term gives a too large contribution ⁵⁾, and has to be reduced by some kind of unitarity corrections.

Another possible candidate for a backward peak is the first re-scattering correction to the pole exchange, namely the impulse approximation or single scattering term (Fig. 2). It is known from a long time ago, starting from the pioneer work of Chew ⁶⁾, that the identity of the incoming proton with the target proton gives rise to a "broad, small peak" near the backward direction. This peak can be simply understood in the following way. The impulse approximation expresses the pd scattering amplitude as the product of the deuteron form factor $G(-t/4)$ times the pp scattering amplitude at the same value of t and at a suitable value of the energy

$$T_{pd}(s, t) = G(-t/4) T_{pp}(s, t) B(s, t)$$

where $B(s, t)$ is a kinematical factor (see Section 2).

The peak near the backward direction is due to the fact that at high energies the pp scattering amplitude exhibits the diffractive behaviour, peaked for $\theta = 0$ and $\theta = \pi$ by the identity of the two protons. The small value of the pd cross-section in the backward direction is due to the small value of the deuteron form factor for $t = t_{\max}$; its width is supposed to be smaller than the pp width, since the form factor increases when $|t|$ decreases from $|t_{\max}|$, whereas

the pp scattering amplitude decreases. We shall discuss in Section 2 how with a reasonable behaviour of the form factor at large momentum transfers, it is impossible to get the right order of magnitude of the cross-section at high energies and to reproduce the energy and angle behaviour of the cross-section.

A possible way to investigate this backward peak is to analyze the following terms in the impulse series, starting from the double scattering ^{*)}.

Double scattering is known to be small in the forward direction at high energies [the Glauber correction ⁸⁾ to the total cross-section amounts only to a few per cent of the total pd cross-section]; nevertheless, double scattering has a much more gentle slope with t than single scattering (the impulse approximation), and therefore its relative importance increases at large angles.

An experimental evidence for the importance of double scattering at relatively high energies can be found in the accurate analysis of Franco ⁹⁾ of the $\bar{p}d$ elastic angular distribution for small t in the range of incident momenta between 2.78 and 10.9 GeV/c, and in the experiment by Crewe et al. ¹⁰⁾. In this last experiment (pd elastic scattering at 450 MeV for c.m. angles between 20° and 120°) the experimental values of the differential cross-section are in good agreement with the predictions of the impulse approximation [with the Gartenhaus ⁴⁾ form factor] for values of $|t| \leq 0.6(\text{GeV}/c)^2$, but for larger angles the cross-section, which at smaller angles was very steeply decreasing with $|t|$, flattens and stays much larger than the prediction of any reasonable form factor. This behaviour can be interpreted as an effect of multiple scattering. Very recently, the dip found in the elastic pd scattering for $T = 2$ GeV and at a value of the momentum transfer $|t| \simeq 0.5(\text{GeV}/c)^2$ has also been interpreted by Coleman and Franco ¹¹⁾ as due to the interference between the single and double scattering contributions.

^{*)} The fact that the experiment quoted in Ref. 1) is at rather high energies requires the use of a relativistic model; multiple scattering effects in elastic pd scattering have been studied with non-relativistic models by many authors ⁷⁾.

Double scattering gives also a peak near the backward direction, always via an exchange mechanism. We shall see in Sections 2 and 3 that, using a model for the double scattering, appropriate to high energies and large momentum transfers, it is possible to get reasonable results for the order of magnitude, the energy behaviour and the angular distribution of the observed backward peak.

The reason why double scattering in the backward direction remains much larger than single scattering is in our model essentially due to the fact that it involves the deuteron form factor for the value $G(-t/16)$ [remember that in the impulse approximation it appears the value $G(-t/4)$]. This fact has two remarkable consequences: the first, as we said, is that the contribution of double scattering is larger, since $G(-t)$ is a quickly decreasing function; the second is that, since the relevant values of the momentum transfer are smaller, the results, still being rather different with different choices of the form factor, are less sensitive to the form factor itself. In other words, the result is less sensitive to the behaviour of the deuteron wave function in the unknown inner region.

A general objection against our general procedure is that, on the one hand, the value of the momentum transfer is so high that the basis of the validity of the impulse series can be questioned^{*)}; on the other hand, that the scattering amplitudes which are involved concern particles which are strongly off-mass-shell.

We have no answer to the first objection, apart from the obvious one that we are dealing with a model which is worth testing with experiment; for the second one, we notice that - as we shall see in Section 2 - in our model only one of the particles is strongly off-mass-shell, and the relevant amplitude is a low energy one (at and near threshold). This

*) This hypothesis amounts to take the deuteron as coupled only to the proton-neutron channel and not to other channels, as for instance the proton-neutron-pions channels. This simplification appears to be justified at low momentum transfers (where the mesonic corrections are of the order of a few per cent) and we assume it to be valid also at high momentum transfers.

is a lucky situation, since it was shown by many people ¹²⁾ and, in a general way, by Lovelace ¹³⁾, how to handle off-mass-shell effects when a scattering amplitude is dominated by resonances or bound states, as it is the case in the low energy nucleon-nucleon scattering.

A few words more on the spin effects. We have treated our problem with the relativistic kinematics, but neglecting the spin effects, which are only taken into account in the low energy nucleon-nucleon scattering amplitude, which appears as an ingredient of our model. The reason for this simplification is nothing but the fact that to treat the deuteron and the nucleons as relativistic spin 1 and $\frac{1}{2}$ particles would have made the calculation too complicated; from the apparent independence of the gross features of high energy physics on spin we hope that this simplification will not change the results in any sensible way.

We have stopped our analysis of the terms in the impulse series to the double scattering terms, and this for at least two reasons. The first and most obvious one is that the treatment of the multiple scattering terms by means of the relativistic Feynman diagram technique becomes exceedingly complicated when the number of successive interactions is larger than two. The second one is that it is likely that a multiple scattering yields an angular distribution flatter than that produced by a double scattering; it is therefore improbable that multiple scattering could contribute to the observed backward peak.

2. THE IMPULSE SERIES

The scattering on a deuteron target at high energy (when the wavelength of the incident particle is smaller than the range of the interaction with the nucleons), and near the forward direction seems to be well described by the so-called impulse approximation. This is a semi-classical model, which has been discussed by Glauber⁸⁾, and more recently by Glauber and Franco¹⁴⁾, in a form suitable to the high energy scattering. The model uses the eikonal representation for the scattering amplitude, and yields the following results for the elastic scattering amplitude of an incident particle x on the deuteron^{*)}

$$T_{xd}(s, t) = 2 \left[T_{xp}(s_{xp}, t) + T_{xn}(s_{xn}, t) \right] G(-\frac{t}{4}) + \\ + \frac{i}{8\pi^2 m_n} \int G(p^2) T_{xp}(s_{xp}, -(\frac{\vec{d}}{2} + \vec{p})^2) T_{xn}(s_{xn}, -(\frac{\vec{d}}{2} - \vec{p})^2) d_2 p \quad (2.1)$$

In this formula s_{xp} and s_{xn} are the invariants relevant to xp and xn scattering: their values depend in general on the momentum transfer t (see below); in the high energy limit and near the forward direction, one has $s_{xp} = s_{xn} = s/2$. The vectors $\vec{n}$ and $\vec{d}$ are defined below: $\vec{d}$ is such that $t = -d^2$. The integration over $d_2 p$ is over the two-dimensional plane perpendicular to the beam axis. The function $G(p^2)$ is the deuteron form factor defined as

$$G(p^2) = \int d_3 x e^{-i \vec{p} \cdot \vec{x}} |\psi(\vec{x})|^2 = \int d_3 q \phi(\vec{q} + \frac{\vec{p}}{2}) \phi(\vec{q} - \frac{\vec{p}}{2}) \quad (2.2)$$

where $\psi(\vec{x})$ is the deuteron wave function and $\phi(\vec{q})$ its Fourier transform.

*) We shall normalize our invariant T matrix in such a way that, for scalar particles, the optical theorem reads $\text{Im } T(s, 0) = 2q \sqrt{s} \sigma_T$ (q = c.m.s. momentum). From this normalization of the T matrix, one gets the differential cross-section by the relation $(d\sigma/dt) = 1/(64\pi q^2 s) |T|^2$.

The first two terms of formula (2.1) correspond to the scattering of the incident particle x by one of the nucleons in the deuteron (single scattering). The third term corresponds to double scattering (shadow or Glauber correction). Of course, also triple scattering and so on would contribute to (2.1), but it is hoped that their contribution will be small, at least near the forward direction.

Since in the Glauber method (2.1) is derived using the eikonal approximation, its validity is limited to small values of the momentum transfer. It is therefore very interesting that formula (2.1) can also be obtained from a Feynman diagram approach, as it was shown by the authors of Ref. 15) and by Abers et al.¹⁶⁾ As it will be discussed in more detail later, one starts from the Feynman diagrams where the upper blobs represent two-body scattering amplitudes, and the barred particles are on the mass shell (see Fig. 3). In this way one obtains a result which agrees with the Glauber formula (2.1) at small momentum transfers (see below).

We shall now assume that the impulse series in the number of two-body scatterings, which is useful near the forward direction, can also be applied near the backward direction in pd scattering, and that the first two terms [generalization of the Glauber formula (2.1)] are just obtained by computing near the backward direction the Feynman diagrams of Fig. 3. Since, as we shall see, the approximations one has to perform are quite different near the forward and backward direction, we shall now review in detail the kind of approximations one does in order to get the Glauber formula and we shall generalize them to non small angle scattering. We shall, moreover, generalize the result of Abers et al.¹⁶⁾ (who did obtain with the Feynman diagram technique only the imaginary part of the double scattering term), to obtain the complete double scattering amplitude. We shall also discuss the relevant kinematical difficulties. For simplicity, we shall restrict ourselves to pd scattering. Let us first consider the impulse approximation (single scattering). We shall use from now on the brickwall co-ordinate system, where the four vectors representing the momenta of the colliding particles have the following components :

$$\begin{aligned}
d_1 &= \left(\sqrt{M^2 + \frac{d^2}{4}}, -\frac{\vec{d}}{2} \right) ; & p_1 &= \left(\sqrt{m^2 + n^2 + \frac{d^2}{4}}, \vec{n} + \frac{\vec{d}}{2} \right) \\
d_2 &= \left(\sqrt{M^2 + \frac{d^2}{4}}, \frac{\vec{d}}{2} \right) ; & p_2 &= \left(\sqrt{m^2 + n^2 + \frac{d^2}{4}}, \vec{n} - \frac{\vec{d}}{2} \right)
\end{aligned} \tag{2.3}$$

where M and m are the masses of the deuteron and the proton, respectively, and $\vec{n} \cdot \vec{d} = 0$. In this system, the Mandelstam invariants s, t, u have the expression

$$\begin{aligned}
s &= (d_1 + p_1)^2 = \left(\sqrt{M^2 + \frac{d^2}{4}} + \sqrt{m^2 + n^2 + \frac{d^2}{4}} \right)^2 - n^2 \\
t &= (d_1 - d_2)^2 = -d^2 \\
u &= (d_1 - p_2)^2 = \left(\sqrt{M^2 + \frac{d^2}{4}} - \sqrt{m^2 + n^2 + \frac{d^2}{4}} \right)^2 - n^2
\end{aligned} \tag{2.4}$$

One can notice that in the forward direction ($t=0$), $\vec{d}=0$ and the brick-wall coincides with the lab. system (deuteron at rest): in the backward direction, $\vec{n}=0$ and the brickwall coincides with the c.m.s.

It is useful to introduce now some definitions. We shall introduce the (scalar) proton-neutron-deuteron coupling constant by using the deuteron pole in the s channel of the proton-neutron scattering as

$$T_{np}^{(pole)} = -\frac{G^2}{s - M^2} \tag{2.5}$$

In order to introduce the deuteron form factor (2.2) (and its relativistic generalization), one has to find the relation between G^2 and the coefficient of the asymptotic part of the deuteron wave function

$$\psi(r) = \frac{N}{\sqrt{4\pi}} \frac{e^{-\chi r}}{r} \tag{2.6}$$

where $\chi^2 = mB$, and $B = 2m - M$ is the deuteron binding energy. The

Fourier transform of $\Psi_{as}(r)$ is

$$\phi_{as}(q) = \frac{1}{\sqrt{2} \pi} \frac{N}{q^2 + \chi^2} \quad (2.7)$$

To this wave function corresponds a pole term in the s wave proton-neutron scattering, such that the scattering amplitude $f(q, \cos\theta)$ ^{*)} can be written as ^{**)}

$$f(q) = - \frac{N^2}{\chi^2 + q^2} \quad (2.8)$$

(q = c.m.s. momentum). From the relation $f(q) = 1/(8 \pi \sqrt{s}) T(s)$, one finally gets

$$G^2 = 32 \pi M N^2 \quad (2.9)$$

From (2.7) and (2.9), it follows that the Fourier transform of the asymptotic deuteron wave function is

$$\phi_{as}(q) = \frac{G}{(64 M \pi^3)^{1/2} (q^2 + \chi^2)} \quad (2.10)$$

2A - Single Scattering

Let us now compute the scattering amplitude corresponding to the single scattering diagram (Fig. 3a). Using the standard rules, one has

*) $f(q, \cos\theta)$ is such that $(d\sigma/d\Omega) = |f(q, \cos\theta)|^2$.

**) This relation is explicitly derived in the Appendix by an elegant method originally due to S. Fubini.

$$T_{pd}(s, t) = \frac{i G^2}{(2\pi)^4} \int d_4 q_3 \frac{T_{pp}(s, t) \mathcal{V}(q_1^2, q_3^2) \mathcal{V}(q_2^2, q_3^2)}{(q_1^2 - m^2)(q_2^2 - m^2)(q_3^2 - m^2)} + (T_{pp} \leftrightarrow T_{pn}) \quad (2.11)$$

$$S_i = (p_i + q_i)^2 = (p_i + d_i - q_3)^2$$

where $\mathcal{V}(q_1^2, q_3^2)$ is the covariant deuteron vertex, depending on two off-mass-shell variables, normalized so that $\mathcal{V}(m^2, m^2) = 1$.

We shall now keep in the integral (2.11) only the contribution in which the particle q_3 is on the mass shell (spectator approximation); this means that we have to make in (2.11) the following replacement

$$\frac{1}{q_3^2 - m^2} \rightarrow -i\pi \frac{\delta(q_3^0 - \sqrt{m^2 + \vec{q}_3^2})}{\sqrt{m^2 + \vec{q}_3^2}} \quad (2.12)$$

This approximation amounts to restrict ourselves to take only the contribution of the pole at $q_3^0 = \sqrt{\vec{q}_3^2 + m^2}$ in performing the integral over q_3^0 . It has been shown by Gross¹⁵⁾ that, for $|t|$ small, this pole is the nearest singularity, and that the result one gets by the replacement (2.12) coincides with that obtained by computing the imaginary part of the diagram with the Cutkosky¹⁷⁾ rule and then the full amplitude from a dispersion relation. This result is true if in the dispersion relation one keeps only the anomalous threshold contribution, or equivalently if one assumes that the vertex function is a constant (which means to keep only the asymptotic part of the wave function). Here one assumes that (2.12) is also allowed with an arbitrary vertex function and for large $|t|$.

With the approximation (2.12), the vertex function $\mathcal{V}(q_1^2, m^2)$ is a function of only one variable; the relation between the vertex function $\mathcal{V}(q_1^2, m^2) \equiv \mathcal{V}(q_1^2)$ and the non-relativistic wave function is such that,

in the non-relativistic limit *)

$$\frac{G \phi(q_i^2)}{q_i^2 - m^2} = (16 \pi \pi^3)^{\frac{1}{2}} \phi(\vec{q}) \quad (2.13)$$

The spirit of the impulse approximation is now to suppose that the nucleon-nucleon scattering amplitude, considered as a function of the variables s_1, q_1^2, q_2^2 , is a gentle one, and therefore that the integral (2.11) can be reasonably approximated by taking the function $T(s_1, t; q_1^2, q_2^2)$ outside the integral at fixed values of the variables $s_1 = \bar{s}_1, q_1^2 = \bar{q}_1^2, q_2^2 = \bar{q}_2^2$, appropriately chosen. Then one gets

$$T_{pd}(s, t) = 2 \left[T_{pp}(\bar{s}_1, t; \bar{q}_1^2, \bar{q}_2^2) + T_{pn}(\bar{s}_1, t; \bar{q}_1^2, \bar{q}_2^2) \right] G(\vec{d}_4^2) \quad (2.14)$$

where

$$G(\vec{d}_4^2) = \frac{G^2}{32 \pi^3} \int \frac{d\vec{q}_3 \phi(q_1^2) \phi(q_2^2)}{\sqrt{\vec{q}_3^2 + m^2} (q_1^2 - m^2) (q_2^2 - m^2)} \quad (2.15)$$

$$q_1^2 = (d_1 - q_3)^2$$

$$q_2^2 = (d_2 - q_3)^2$$

Taking into account (2.10), (2.13), one can easily see that, after a non-relativistic approximation **, (2.15) coincides with (2.2). Equation (2.15) is thus a relativistic deuteron form factor. Since near the forward direction $\vec{d}^2 \ll m^2$, this non-relativistic approximation is allowed, and the right-hand side of Eq. (2.14) coincides there with the two first terms of (2.1).

*) $\vec{q}$ is here the momentum of particle q_1 in the frame in which the deuteron is at rest.

**) By non-relativistic approximation, we mean here that the motion of the nucleons inside the deuteron can be treated non-relativistically. This corresponds to having $d^2 \ll m^2$. Of course, this condition does not imply low energy of the incident proton, but only small momentum transfer. Unless explicitly stated, non-relativistic approximation will always be understood in this sense.

Let us now discuss the problem of the definition of the invariant $\bar{s}_1$ and of the virtual variables $\bar{q}_1^2$ and $\bar{q}_2^2$: to our knowledge, this problem has never been fully discussed in the relativistic case.

A possible justification of putting the scattering amplitude T_{pp} outside the $\vec{q}_3$ integral is to assume that it is a slowly varying function of $\vec{q}_3$ ^{*}), as compared to the rest of the integrand (which varies very quickly with $\vec{q}_3$). Accordingly, we are going to define $\bar{s}_1$ as the value of s_1 taken at the value of $\vec{q}_3$ which maximizes the integrand in (2.15). This is of course not the only possible definition, but is certainly in agreement with the general philosophy of taking outside the integral a slowly varying function.

We can first explore what happens when the vertex functions $\varphi(q_1^2)$ and $\varphi(q_2^2)$ are taken to be constant (approximation of the asymptotic wave function ¹⁸⁾). One can then easily see that the maximum of $D = [(q_1^2 - m^2)(q_2^2 - m^2)]^{-1}$ is obtained for a value of $\vec{q}_3$ defined (at first order in χ^2/m^2) by

$$\frac{\vec{q}_3 \cdot \vec{d}}{|\vec{q}_3| |\vec{d}|} = \pm 1 \quad \vec{q}_3^2 = \frac{d^2}{16} - \chi^2 \quad (2.16)$$

for $d^2 \geq \bar{d}^2 = 16\chi^2$. When $d^2 < 16\chi^2$, then (2.16) gives $\vec{q}_3^2 < 0$, and in that case the maximum is reached for $\vec{q}_3 = 0$. Therefore the maximal configuration is, in general, different near the forward direction and at large angles ^{**)}. More precisely, at very small angles ($d^2 < 16\chi^2$), the maximal configuration is such that both the nucleons q_1 and q_2 are out of the mass shell in the same way, and one has

^{*}) Remember that $s_1 = (d_1 + p_1 - q_3)^2$ depends on the variable $\vec{q}_3$, whereas $t_1 = (p_2 - p_1)^2 = t$ is of course independent of $\vec{q}_3$.

^{**)} It is interesting to notice that the value of $t = -d^2 = -16\chi^2$ is the symmetric with respect to zero of the value of the lowest anomalous threshold, $t = 16\chi^2$ ^{17), 19)}. Since this anomalous threshold is just the singularity one encounters in potential theory ²⁰⁾, it is certainly justified to apply the non-relativistic limit within a circle in the t plane of radius $|t| = 16\chi^2$.

$$-(\bar{q}_1^2 - m^2) = -(\bar{q}_2^2 - m^2) = -M^2 + 2m(M^2 + \frac{d^2}{4})^{\frac{1}{2}} \simeq +2\chi^2 + \frac{d^2}{8} \leq 4\chi^2 \quad (2.17)$$

where, since $d^2 < 16\chi^2$, we have been allowed to take the non-relativistic limit on the deuteron.

At large angle, on the contrary, the maximal configuration corresponding to (2.16) is an asymmetric one, such that always one of the two nucleons q_1 and q_2 is almost on the mass shell. If we take, for instance, the lower sign in (2.16), we get, at first order in χ^2 [with the upper sign, one just exchanges $\bar{q}_1^2$ and $\bar{q}_2^2$]

$$\bar{q}_1^2 - m^2 = -2\chi^2 \quad ; \quad \bar{q}_2^2 - m^2 = -2\left(\chi^2 + \frac{d^2}{4}\right) \quad (2.18)$$

For small angles (where the maximal configuration is for $\vec{q}_3 = 0$), one gets *)

$$\bar{S}_1 = S + m^2 - 2m \left(\sqrt{m^2 + n^2 + \frac{d^2}{4}} + \sqrt{M^2 + \frac{d^2}{4}} \right) \quad (2.19)$$

At large angles, one gets from (2.16) the very simple relation

$$\bar{S}_1 = \frac{S - m^2}{2} \quad (2.20)$$

(where terms of the order χ^2 have been neglected).

It is easy to verify that in the high energy limit both definitions give (for fixed t)

*) It is easy to verify that when both $d^2, n^2 \ll m^2$, applying to (2.19) the non-relativistic limit, one gets the relations ¹⁰⁾

$T_1 = T(25 - 7\cos\theta)/18$; $\cos\theta_1 = (25\cos\theta - 7)/(25 - 7\cos\theta)$, where T and T_1 are the kinetic energies of the incident nucleon in the frames of reference in which the deuteron d_1 and the nucleon q_1 are respectively at rest; θ and θ_1 are the c.m.s. angles of the proton-deuteron and proton-nucleon systems.

$$\bar{s}_1 \xrightarrow{s \rightarrow \infty} s/2 \quad (2.21)$$

For a general wave function [namely, for a non-constant $\psi(q^2)$] the previous results will in general be different. However, for small d^2 , one will still have a symmetric maximization, corresponding to $\vec{q}_3 = 0$, and therefore the results (2.17) and (2.19) will there be unchanged ^{*}). For large d^2 , the relation (2.16) giving the maximal configuration will be modified, and so will the value of $\bar{d}^2$ for which the symmetric and asymmetric maximizations coincide; anyhow, due to the small binding energy of the deuteron, the results (2.18) and (2.20) will not be drastically changed. As an example, we can quote the case of the Hulthén wave function $u(r) = N(e^{-\chi r} - e^{-\gamma r})$, where, at first order in the ratio χ^2/γ^2 , (2.16) becomes

$$\vec{q}_3^2 = \frac{d^2}{16} - \chi^2 \left(2 - \frac{\gamma^2}{\gamma^2 + d^2} \right) \quad (2.16')$$

In any case, there will always be a region $d^2 \gg \bar{d}^2$, where the maximum corresponds to $\vec{q}_3^2 = d^2/16$, $(\vec{q}_3 \cdot \vec{d})/(|\vec{q}_3||\vec{d}|) = \pm 1$, resulting again in (2.18) and (2.20).

Another point to be stressed is that in (2.14) the nucleon-nucleon amplitude depends on the off-mass-shell variables $\bar{q}_1^2$ and $\bar{q}_2^2$. As in general one does not know how to handle (at high energies) the off-mass-shell effects, these effects are simply neglected in the impulse approximation, putting into the amplitude $\bar{q}_1^2 = \bar{q}_2^2 = m^2$. Those off-mass-shell effects are of course much more important at large values of t (for $t=0$ one has simply $\bar{q}_1^2 = \bar{q}_2^2 = -2\chi^2$). Note that from (2.18) at large t either particle q_1 or q_2 is almost on the mass shell, $\bar{q}^2 - m^2 = -2\chi^2$, irrespectively of t .

^{*}) The fact that at small d^2 the results will be the same with any choice of the vertex function can be understood by noticing that small values of d^2 are dominated by the external part of the deuteron wave function in co-ordinate space, where all the wave functions coincide with the asymptotic one.

2B - Double Scattering

Let us now discuss the next term in the impulse series, namely the double scattering contribution, always neglecting the complications due to the spin and the identity of the particles.

The scattering amplitude corresponding to the Feynman diagram of Fig. 3b is given by

$$T_d(s, t) = \frac{G^2}{(2\pi)^8} \int d_4 q_3 d_4 q_4 d_4 q_5 \delta^{(4)}(q_3 + q_4 + q_5 - p_1 - d_1) \frac{\mathcal{U}(q_1^2, q_3^2) \mathcal{U}(q_2^2, q_4^2)}{(q_1^2 - m^2)(q_2^2 - m^2)} \cdot \frac{T_1((p_1 + q_1)^2, (p_1 - q_5)^2, q_1^2, q_4^2, q_5^2) \cdot T_2((p_2 + q_2)^2, (p_2 - q_5)^2, q_2^2, q_3^2, q_5^2)}{(q_3^2 - m^2)(q_4^2 - m^2)(q_5^2 - m^2)} \quad (2.22)$$

$$q_1 = d_1 - q_3 \quad ; \quad q_2 = d_2 - q_4$$

Following the same philosophy used in order to compute the single scattering contribution, we shall retain only the contribution in which the intermediate particles q_3, q_4, q_5 are physical; as we shall see, with this approximation, one gets at small angles just the Glauber correction.

This approximation amounts to making the following replacement *)

*) The factor $\frac{1}{2}$ in formula (2.23) has the following origin. We want to keep the singular part of the product

$$\prod_{i=3,4,5} \left[P \frac{1}{q_i^2 - m^2} - i\pi \delta^P(q_i^2 - m^2) \right]$$

This singular part is not simply given by the product of the three

δ function contributions, since, owing to the fact that the variables are not independent, the product of the principal parts contains also a singular term. The singular part of the product of the three propagators is of the form $i\pi^3 \lambda \delta^P(q_3^2 - m^2) \delta^P(q_4^2 - m^2) \times \delta^P(q_5^2 - m^2)$, where λ is a constant [see Ref. 21), note on p.1329]. An easy way to compute the constant λ is just to compute with the Cutkosky rule the imaginary part of (2.22) across the three-particle cut; one gets in this way the imaginary part of (2.24). We thank Dr. A. Pagnamenta for a discussion on this point.

$$\frac{1}{q_3^2 - m^2 + i\varepsilon} \cdot \frac{1}{q_4^2 - m^2 + i\varepsilon} \cdot \frac{1}{q_5^2 - m^2 + i\varepsilon} \rightarrow \frac{i}{2} \pi^3 \frac{\delta(p_3^0 - \sqrt{\vec{q}_3^2 + m^2})}{\sqrt{\vec{q}_3^2 + m^2}} \cdot \frac{\delta(q_4^0 - \sqrt{\vec{q}_4^2 + m^2}) \delta(q_5^0 - \sqrt{\vec{q}_5^2 + m^2})}{\sqrt{\vec{q}_4^2 + m^2} \sqrt{\vec{q}_5^2 + m^2}} \quad (2.23)$$

Inserting (2.23) into (2.22), one gets

$$T_{pd}(s, t) = \frac{i G^2}{16 (2\pi)^5} \int \frac{d_3 \vec{P} d_3 \vec{Q}}{E_3 E_4 E_5} \delta(p_1^0 + d_1^0 - E_3 - E_4 - E_5) \frac{\mathcal{A}(q_1^2, m^2)}{q_1^2 - m^2} \frac{\mathcal{A}(q_2^2, m^2)}{q_2^2 - m^2} \cdot T_1(s_1, t_1; q_1^2) T_2(s_2, t_2; q_2^2) \quad (2.24)$$

$$\vec{P} = \vec{q}_3 + \vec{q}_4; \quad \vec{Q} = \frac{\vec{q}_3 - \vec{q}_4}{2}; \quad q_5 = p_1 + d_1 - q_3 - q_4$$

$$s_1 = (p_1 + q_1)^2; \quad s_2 = (p_2 + q_2)^2; \quad E_3 = \sqrt{\vec{q}_3^2 + m^2}; \quad E_4 = \sqrt{\vec{q}_4^2 + m^2};$$

$$t_1 = (p_1 - q_5)^2; \quad t_2 = (p_2 - q_5)^2; \quad E_5 = \sqrt{\vec{q}_5^2 + m^2}$$

The expression, as it stands, involves five integrations; moreover, it does not allow the introduction of the deuteron form factor [as defined in (2.15)], which appears in the Glauber correction, since :

- the nucleon-nucleon scattering amplitudes depend on the vector $\vec{Q}$;
- the integrations over the vectors $\vec{P}$ and $\vec{Q}$ are not independent, due to the constraint of the δ function.

The problem is simplified, if one introduces the following approximations (analogous to those introduced in the single scattering): since $D(\vec{Q}, \vec{P}) = [\psi_1 \psi_2 / ((q_1^2 - m^2)(q_2^2 - m^2))]^{-1}$ is a very rapidly varying function of the vector $\vec{Q}$, we shall substitute into the scattering amplitudes $T_1(s_1, t_1, q_1^2)$ and $T_2(s_2, t_2, q_2^2)$ and into the δ function the variable $\vec{Q}$ with its constant value $\vec{Q}_m$, such that the remaining part of the integrand, namely the function $D(\vec{Q}, \vec{P})$, is maximum; in this way we take the product $T_1 T_2 \delta(p_1^0 + d_1^0 - E_3 - E_4 - E_5)$ out of the $d\vec{Q}$ integral. This simplification allows moreover the introduction of the deuteron form factor, at least near the forward direction.

Similarly to what happens for the single scattering, the maximal value $\vec{Q}_m$ has two different expressions, depending on the value of d^2 . For small values of d^2 one has

$$\vec{Q}_m = \pm \frac{\vec{P}}{2} \quad (2.25)$$

while at large d^2 one has

$$\vec{Q}_m = -\frac{\vec{d}}{4} \pm \frac{\vec{P}}{2} \quad (2.25')$$

In both these expressions, terms of the order χ^2 have been neglected. One has to notice that in the first case one has a symmetric maximization, whereas in the second one, one has always one of the two unphysical particles $[q_1$ or q_2 , depending on the sign chosen in (2.25')] as near to the mass shell as possible.

In the previous approximations we have assumed that the scattering amplitudes are slowly varying functions of the vector $\vec{Q}$, as compared with $D(\vec{Q}, \vec{P})$; at high energies, however, the scattering amplitudes show the characteristic diffraction peak, namely are strongly varying functions of their momentum transfer. Nevertheless, it can be shown that, at small d^2 , (2.25) gives also the maximization of the product of the two scattering amplitudes. This result arises from the fact that the product of two diffractive amplitudes, such that the total momentum transfer is d^2 , is maximum when the two partial momentum transfers, on which the two amplitudes depend, are equal; and this is the case with the symmetric maximization.

If we look now at large angles, and more precisely to the backward direction ($\vec{n}=0$), the maximization (2.25') yields one of the two scattering amplitudes at high energies and, owing to the forward-backward symmetry of the proton-proton amplitude ^{*)}, in the forward direction, while the other one is at zero kinetic energy ($s_1 = 4m^2$), [see below formulae (2.33)].

*) For these reasons, these results near the backward direction can be only applied to nucleon-deuteron scattering, where one has identity of one of the target particles with the incident projectile. These results cannot be applied, for instance, to antiproton-deuteron or meson-deuteron scattering, where at high energies there is a very small backward peak.

It is easy to verify that the product $T_1 T_2$ takes also its maximum value with the maximization (2.25') and not with (2.25). These considerations support in this way the approximations yielding from (2.24) to (2.26).

With the use of the approximation (2.25)-(2.25'), from (2.24), one gets

$$T_{pd}(s, t) = \frac{i\pi}{8(2\pi)^3} \int d_3 \vec{p} \frac{\delta(p_1^0 + d_1^0 - E_3 - E_4 - E_5)}{E_3 E_4 E_5} T_1(s_1, t_1; q_1^2) \cdot T_2(s_2, t_2; q_2^2) \cdot \tilde{G}(p^2, \vec{p} \cdot \vec{d}) \quad (2.26)$$

where

$$\tilde{G}(p^2, \vec{p} \cdot \vec{d}) = \frac{G^2}{16M\pi^3} \int d_3 \vec{Q} \frac{\varphi(q_1^2) \varphi(q_2^2)}{(q_1^2 - m^2)(q_2^2 - m^2)} \quad (2.27)$$

Of course, in expression (2.26) all the variables which in (2.24) were functions of $\vec{Q}$ have now to be understood as functions of $\vec{Q}_m$.

We can now discuss separately the scattering near the forward direction (small d^2) and near the backward direction (large d^2), where (2.25) and (2.25'), respectively, apply.

Near the forward direction ($d^2 \ll m^2$), the constraint of the δ function reads

$$\sqrt{m^2 + n^2 + \frac{d^2}{4}} + \sqrt{m^2 + \frac{d^2}{4}} - m - \sqrt{m^2 + p^2} - \sqrt{m^2 + n^2 + p^2 - 2\vec{n} \cdot \vec{p}} = 0 \quad (2.28)$$

which gives in general a relation between p^2 and $\vec{n} \cdot \vec{p}$. If, moreover, we are at high energy (which means n^2 of the order of m^2 or larger), this relation is satisfied if we take $p^2 \ll m^2$; then it gives $\vec{n} \cdot \vec{p} = 0$. We can therefore perform the approximate replacement

$$\delta(p_1^0 + d_1^0 - E_3 - E_4 - E_5) \rightarrow \sqrt{m^2 + n^2} \delta(\vec{n} \cdot \vec{p}) \quad (2.28')$$

Taking into account (2.25), (choosing, to be definite, the positive sign) the kinematical variables become

$$\begin{aligned}
 S_1 &= (p_1 + q_1)^2 = \left(m + \sqrt{m^2 + (\vec{n} - \vec{p})^2} \right)^2 - (\vec{n} - \vec{p})^2 \\
 t_1 &= (p_1 - q_5)^2 = \left(\sqrt{m^2 + \left(\frac{\vec{d}}{2} + \vec{n} \right)^2} - \sqrt{(\vec{n} - \vec{p})^2 + m^2} \right)^2 - \left(\frac{\vec{d}}{2} + \vec{p} \right)^2 \simeq - \left(\frac{\vec{d}}{2} + \vec{p} \right)^2 \\
 q_1^2 &= \left(\sqrt{M^2 + \frac{d^2}{4}} - \sqrt{m^2 + p^2} \right)^2 - \left(\frac{\vec{d}}{2} + \vec{p} \right)^2 \simeq m^2 - 2\chi^2 - 2 \left(\vec{p} + \frac{\vec{d}}{4} \right)^2 \\
 S_2 &= (p_2 + q_2)^2 = \left(\sqrt{m^2 + p^2} + \sqrt{m^2 + (\vec{n} - \vec{p})^2} \right)^2 - n^2 \\
 t_2 &= (p_2 - q_5)^2 = \left(\sqrt{m^2 + \left(\frac{\vec{d}}{2} - \vec{n} \right)^2} - \sqrt{(\vec{n} - \vec{p})^2 + m^2} \right)^2 - \left(\frac{\vec{d}}{2} - \vec{p} \right)^2 \simeq - \left(\frac{\vec{d}}{2} - \vec{p} \right)^2 \\
 q_2^2 &= \left(\sqrt{M^2 + \frac{d^2}{4}} - m \right)^2 - \frac{d^2}{4} \simeq m^2 - 2\chi^2 - \frac{d^2}{8}
 \end{aligned} \tag{2.29}$$

The last approximate equalities follow from neglecting terms of higher order in d^2, p^2 . Since both $p^2, d^2 \ll m^2$, we can moreover take the non-relativistic limit under the integral (2.27); indeed, as we have said, the integrand is a strongly varying function of its argument $\vec{Q}$, and, on the maximum, both $\vec{q}_1$ and $\vec{q}_2$ are small.

In this way, the function $\tilde{G}(p^2, \vec{p} \cdot \vec{d})$ becomes independent on $\vec{p} \cdot \vec{d}$ and coincides with the deuteron form factor, as defined in (2.15)

$$\tilde{G}(p^2, \vec{p} \cdot \vec{d}) \rightarrow G(p^2) \quad \text{for } d^2 \ll m^2 \tag{2.30}$$

If one now considers the energy factors E_3, E_4, E_5 as almost constant and given by their values for $\vec{p} = 0$, for which the rest of the integrand is maximum (now with respect to the variable $\vec{p}$)

$$E_3 = E_4 = m, \quad E_5 = \sqrt{m^2 + n^2}$$

one gets

$$T_{pd}^{(double)}(s, t) = \frac{i}{16\pi^2 m n} \int d_2 p \, T_1(s_1, t_1) T_2(s_2, t_2) G(p^2) \quad (2.31)$$

where the integration is over the two-dimensional plane perpendicular to $\vec{n}$.

We have now three possibilities for the product $T_1 T_2$, corresponding to the three diagrams of Fig. 4^{*)}, and one has to add them. In practice, the charge exchange diagram of Fig. 4c is small at high energies, and the other two give the same result. Adding them, and using the approximate kinematics in (2.29) for t_1 and t_2 , one finally gets the Glauber formula (2.1).

Let us now pass to the scattering near the backward direction. To be definite, we shall always choose the upper sign in (2.25') (choosing the other sign amounts just to exchange the roles of the amplitudes T_1 and T_2).

Let us first consider the backward direction ($\vec{n} = 0$). The δ function gives the relation

$$\sqrt{m^2 + \frac{d^2}{16}} + \sqrt{m^2 + \frac{d^2}{4}} = \sqrt{m^2 + p^2} + \sqrt{m^2 + \left(\vec{p} - \frac{\vec{d}}{4}\right)^2} \quad (2.32)$$

We see therefore that the minimum value of p^2 is reached for $z = (\vec{P} \cdot \vec{d}) / (P \cdot d) = -1$, and is $p^2 = d^2/16$.

*) It has been recently noticed by Wilkin²²⁾ that the presence of the third diagram of Fig. 4 is necessary in order to respect charge independence; see also the recent paper by Franco and Glauber²³⁾.

Considering the kinematic variables, we get ^{*)}

$$\begin{aligned}
 s_1 &= (p_1 + q_1)^2 = 2m^2 + \frac{\vec{d} \cdot \vec{p}}{2} + 2\sqrt{m^2 + \frac{d^2}{16}} \sqrt{m^2 + p^2} \\
 t_1 &= (p_1 - q_4)^2 = \left(\sqrt{m^2 + \frac{d^2}{4}} - \sqrt{m^2 + \frac{d^2}{16}} \right)^2 - \frac{d^2}{16} \\
 q_1^2 &= \left(2\sqrt{m^2 + \frac{d^2}{16}} - \sqrt{m^2 + (\vec{p} - \frac{\vec{d}}{4})^2} \right)^2 - \left(\vec{p} + \frac{\vec{d}}{4} \right)^2 \\
 s_2 &= (p_2 + q_2)^2 = \frac{s - m^2}{2} \\
 t_2 &= (p_2 - q_3)^2 = \left(\sqrt{m^2 + \frac{d^2}{4}} - \sqrt{m^2 + (\vec{p} - \frac{\vec{d}}{4})^2} \right)^2 - \left(\vec{p} + \frac{\vec{d}}{4} \right)^2 \\
 q_2^2 &= m^2 - 2\chi^2
 \end{aligned} \tag{2.33}$$

The amplitude $T_2(s_2, t_2)$ is therefore a high energy amplitude; moreover, since s_2 does not depend on $\vec{p}$, and $q_2^2 - m^2 = -2\chi^2 \simeq 0$, T_2 is a on-mass-shell amplitude at fixed energy.

For $z = -1$, we get $t_2 = 0$. Since the high energy nucleon-nucleon scattering amplitude decreases exponentially with the momentum transfer, T_2 reaches its maximum for $z = -1$ and then decreases fast with z .

We can also see from (2.33) that $s_1 = 4m^2$ for $z = -1$. Therefore T_1 is a low energy amplitude for the same values of z for which T_2 is large. As it is known, both the singlet and triplet nucleon-nucleon scattering amplitudes are very large at threshold (owing to the existence of the deuteron bound state and of the singlet virtual level), and then they decrease fast with the energy. Also T_1 is therefore maximum for $z = -1$ and then decreases very fast with z .

^{*)} Note the different definitions of the momentum transfers t_1 and t_2 as compared to (2.29).

Looking now at the function $\widetilde{G}(P^2, \vec{P} \cdot \vec{d})$, strictly speaking we cannot identify it as a deuteron form factor, being a different convolution of the vertex functions: it is nevertheless related to the deuteron form factor, since as we have seen, it goes into the form factor in the non-relativistic limit. In any case, $\widetilde{G}(P^2, \vec{P} \cdot \vec{d})$ is in general a decreasing function of the variable P^2 , and therefore is also maximal for $z = -1$.

As a conclusion, we can say that the whole integrand of (2.26) is dominated by the values of z near $z = -1$; it is therefore legitimate to approximate in the whole range of integration the amplitude T_2 with a (diffractive) high energy amplitude, and the amplitude T_1 with a low energy amplitude, where the kinematical relations are given by (2.32).

All these qualitative conclusions do not change if we take $\vec{n} \neq 0$, but such that $n^2 \ll d^2$ (namely, for angles near the backward direction). In fact, the value of s_2 and q_2^2 are unchanged, so that T_2 is still a on-mass-shell high energy amplitude; only the minimum value of t_2 (always reached for $z = -1$) is changed, being now a negative value, not any more strictly equal to zero.

On the other hand, the least value of s_1 is now bigger than $4m^2$. For n^2 small enough, we can, however, consider T_1 as a low energy nucleon-nucleon scattering amplitude.

3. APPLICATION TO $p\bar{d}$ ELASTIC SCATTERING AT HIGH ENERGIES

We shall now apply the theory developed in Section 2 to the high energy experiment of Coleman et al.¹⁾, quoted in the Introduction.

Let us first consider the single scattering, corresponding to formula (2.14). In order to compute the contribution of (2.14) near the backward direction, we shall replace the off-mass-shell amplitude, which appears there, by the on-mass-shell physical amplitude, even if one of the particles is strongly off-mass-shell. We are forced to do so, since there is no way - to our knowledge - to handle in an unambiguous way the off-mass-shell effects in a high energy scattering amplitude, which is not dominated by only one partial wave in the direct channel.

We shall therefore parametrize the high energy nucleon-nucleon scattering amplitude in the following way^{*)}:

$$\begin{aligned}
 T_{pp}(s, t, m^2) &= 2q\sqrt{s} \sigma_T^{pp}(s) (i + \alpha^{pp}(s)) e^{-\frac{b}{2}(t_1 - t_{1, \max})} \\
 T_{np}(s, t, m^2) &= 2q\sqrt{s} \left[i(\sigma_T^{pp}(s) - \sigma_T^{pn}(s)) + \alpha^{pp}(s) \sigma_T^{pp}(s) - \right. \\
 &\quad \left. - \alpha^{pn}(s) \sigma_T^{pn}(s) \right] e^{-\frac{b}{2}(t_1 - t_{1, \max})} \quad (3.1)
 \end{aligned}$$

^{*)} In this way we assume that the backward np scattering (charge exchange) can be computed from the difference of the coherent pp and pn scattering. We are well aware of the fact that this hypothesis is not true experimentally (the charge exchange scattering in the backward direction greatly exceeds the optical value and the differential cross-section is much more peaked than the elastic pp and pn cross-sections). Nevertheless, the contribution of the charge exchange term to (3.1) is so small that this hypothesis, which allows a simple parametrization of the amplitude, does not change the results.

where $q_1^2 = (s_1 - 4m^2)/4$ and $\alpha(s_1)$ is the ratio between the real and imaginary part of the NN scattering amplitude, experimentally measured in the forward direction ^{*}). From (3.1) and (2.14), it follows that

$$\frac{d\sigma_{pd}}{dt} = \frac{1}{16\pi} e^{-b(t_1 - t_{\max})} \left[\left(2\sigma_T^{pp}(s_1) - \sigma_T^{pn}(s_1) \right)^2 + \right. \\ \left. + \left(2\sigma_T^{pp}(s_1)\alpha^{pp}(s_1) - \alpha^{pn}(s_1)\sigma_T^{pn}(s_1) \right)^2 \right] G^2\left(-\frac{t}{4}\right) \quad (3.2)$$

For the three energies of the experiment of Ref. 1) ($T_{\text{lab}} = 1, 1.3, 1.5$ GeV), one has $s_1^{(1)} = 5.39$, $s_1^{(2)} = 5.955$, $s_1^{(3)} = 6.33$ GeV². For these energies, the values of σ_T^{NN} and α^{NN} , taken from the experiments 24)-26), are given in Table 1. For the value of b we have chosen $b = 6(\text{GeV}/c)^{-2}$, Refs. 27), 28). Since from the pp elastic experiments 27), for $0 \leq |t| \leq 0.6 (\text{GeV}/c)^2$, the experimental distribution for $\ln d\sigma/dt$ is a straight line with a very good approximation, we have not introduced a term of the form $\exp(\frac{b}{2}t + \frac{c}{2}t^2)$ in formula (3.1). Inserting into (3.2) the experimental values for $(d\sigma/dt)_{\text{pd}}$ one gets the following values for $G^2(-\frac{t}{4})$ (Table 3).

As one can see from Fig. 5, where the form factor of Table 2 is plotted, these results are much bigger (by at least two or three orders of magnitude) than any form factor computed from the commonly used non-relativistic wave functions. Moreover, and this is more critical, the values of $G(-\frac{t}{4})$, obtained for the same momentum transfer from different energies, do not overlap, so that it seems that it is impossible to fit the experiment, using formula (3.2), where the form factor $G(-\frac{t}{4})$ is a smooth function of the momentum transfer t .

^{*}) This formula corresponds to the hypothesis that the real and imaginary parts of the scattering amplitude have the same t dependence within the diffraction peak. This hypothesis is unimportant here, since one could substitute to $\frac{1}{16\pi}(\sigma_T^{\text{pp}})^2(1+(\alpha^{\text{pp}})^2) \times \exp(b(t_{\max} - t))$ the experimentally measured differential pp cross-section, at least when one neglects the small charge exchange term. This substitution will not change the results in Fig. 5 in any sensible way. It will, on the contrary, be necessary when we shall compare the double scattering theory with the experimental results.

Of course, the comparison of the values shown in Table 3 with the form factors computed from a non-relativistic wave function is not very meaningful, since the relativistic effects could well change both the order of magnitude and the slope of the form factor for such large values of the momentum transfer; moreover, we do not know in which direction off-mass-shell effects will contribute in formula (2.14). However, the fact that there is not one smooth function $G(-\frac{t}{4})$ which emerges from Table 3, suggests that it is improbable that single scattering will explain the backward pd peak observed experimentally.

As we have discussed in Section 1, from the physical point of view, it is likely that double scattering, which overwhelms single scattering already at $|t| \simeq 1 \text{ (GeV/c)}^2$, will give a bigger result than single scattering near the backward direction.

Let us now therefore apply the double scattering theory of Section 2.

Since, as we have seen in Section 2, one of the two nucleon-nucleon scattering amplitudes appearing in (2.26) is, for $d^2 \gg n^2$, a low energy scattering amplitude, which cannot certainly be considered as spin independent, we have first to discuss the complications which come in taking into account, at least partially, the effects of the spin and identity of the particles (which we have ignored in our discussion of Section 2).

First of all, three different diagrams actually contribute to pd scattering (see Fig. 4), so that the product $T_1 T_2$ in (2.26) has to be replaced by

$$T_1(s_1, t_1; q_1^2) T_2(s_2, t_2) = T_1^{pp}(s_1, t_1; q_1^2) T_2^{pn}(s_2, t_2) + T_1^{pn}(s_1, t_1; q_1^2) T_2^{pp}(s_2, t_2) - \\ - T_1^{np}(s_1, t_1; q_1^2) T_2^{nn}(s_2, t_2) \quad (3.3)$$

where T_{pn} means elastic proton-neutron and T_{np} proton-neutron charge exchange scattering. The minus sign in the third term is a consequence of the Pauli principle; in fact, if one compares the first graph of Fig. 4 with the third one, one can see that in the initial deuteron vertex, the proton and the neutron have been exchanged; since the deuteron wave function is antisymmetric with respect to the exchange of the two nucleons, this exchange amounts to change the sign of the third term of (3.3) with respect to the first one. The second term has the same sign as the first one, since the two nucleons are exchanged in both deuteron vertices.

As far as the spin problem is concerned, we shall assume, as we have already said in the Introduction, that the high energy amplitude is spin independent, at least inside the diffraction peak ^{*}); in other words, we shall assume that the spin dependent amplitudes are either small or have the same t dependence as the spin independent amplitude. For the low energy amplitude, we shall take only the s wave of the low energy nucleon-nucleon scattering; this means that pp scattering will be a pure spin singlet amplitude, and that pn scattering will be the proper combination of singlet and triplet. Equation (3.3) is therefore written as

$$(T_1 T_2)_{\text{singlet}} = T_{1,\text{singlet}}^{pn} (T_2^{pp} + T_2^{pn}) + T_{1,\text{singlet}}^{pp} (T_2^{pp} - 2 T_2^{pn}) \quad (3.4)$$

$$(T_1 T_2)_{\text{triplet}} = T_{1,\text{triplet}}^{pn} (T_2^{pp} + T_2^{pn}) \quad (3.4')$$

Since we have treated non-relativistically the spin properties of the low energy nucleon-nucleon amplitude and we have assumed the spin independence of the high energy amplitude, we shall also describe with a non-relativistic formalism the spin properties of the deuteron; this means that the pd scattering amplitude contains a spin doublet and a spin quartet amplitude. The fact that we have assumed only s wave interaction in the low energy amplitude and spin independence of the high energy amplitude forbids any doublet-quartet transition. We can in this way express the pd differential cross-section in terms of the amplitudes T_{pd}^s and T_{pd}^t , which are obtained from the expression (2.26) in which the product $T_1 T_2$ in the integrand is replaced by (3.4) and (3.4'), respectively. We have in this way :

^{*}) There is experimental evidence of the spin independence of the high energy nucleon-nucleon amplitude, at least for very small t ²⁹⁾; in any case, it is reasonable to think that this approximation will not modify in any essential way at least the angular distribution of the backward pd scattering.

$$\frac{d\sigma_{pd}(s,t)}{d\tau} = \frac{1}{64\pi q^2 s} \left(\frac{2}{3} \left| \overline{T}_{pd}^t(s,t) \right|^2 + \frac{1}{3} \left| \frac{1}{4} \overline{T}_{pd}^t(s,t) + \frac{3}{4} \overline{T}_{pd}^s(s,t) \right|^2 \right) \quad (3.5)$$

Let us now give the exact parametrization of the amplitudes T_1 and T_2 we shall use. For the high energy amplitude T_2 we take, as before for the single scattering, the exponential form ^{*)}

$$T_2^{NN}(s_2, t_2) = 2g_2 \sqrt{s_2} \sigma_T^{NN}(s_2) (i + \alpha^{NN}(s_2)) e^{\frac{b_2}{2} t_2} \quad (3.6)$$

The values of σ_T^{NN} , α and b are given in Table 1.

For the low energy amplitude, as we have said, we shall keep only the s wave part of the amplitude, which we shall describe by a scattering length form

$$\overline{T}_{1 \text{ singlet triplet}}^{NN}(s_1, t_1) = 8\pi \sqrt{s_1} \left(\frac{a_{s_1}^{NN} + i k_1 (a_{s_1}^{NN})^2}{1 + k_1^2 (a_{s_1}^{NN})^2} \right) \quad (3.7)$$

$$k_1^2 = \frac{s_1 - 4m^2}{4}$$

The fact that we keep only the s wave in the low energy amplitude has another advantage; in fact, as we have noticed in Section 2, the value of q_1^2 is highly unphysical, and so is therefore the value of t_1 ; since the s wave is independent of t_1 we are allowed to ignore this problem.

^{*)} We can repeat here the same remarks made before about the t dependence of the real part of the nucleon-nucleon scattering amplitude and about the description of the charge exchange np scattering: in any case, also here the effect of the charge exchange scattering is so small that whatever bad treatment we give of it, the final results will not change.

This does not mean that there are no off-mass-shell effects in the amplitude T_1 (we remember that on the contrary T_2 is, apart from terms of the order of χ^2 , on the mass shell). However, for a low energy amplitude, dominated by a nearby singularity (as it is the case for the triplet s wave, dominated by the deuteron pole, and for the singlet s wave, dominated by the singlet virtual state), one knows how to handle the off-mass-shell effects. According to the general theory of Lovelace¹³⁾, we have to correct (3.7) in the following way^{*)}:

$$T_{1, \text{singlet}}^{NN}(s, q_i^2) = 8\pi\sqrt{s} \left(\frac{a_s^{NN} + i k_1 (a_s^{NN})^2}{1 + k_1^2 (a_s^{NN})^2} \right) \cdot \frac{1}{1 - \frac{1}{2} (b_s^{NN})^2 (q_i^2 - m^2)} \quad (3.8)$$

where b_s^{pp} , b_s^{np} , b_t^{np} are related to the corresponding scattering lengths and effective ranges^{30), 31)}, and are again given in Table 1.

The only unknown function remains therefore the "effective form factor" $\tilde{G}(P^2, \vec{P} \cdot \vec{d})$.

We report in Table 3 - and we show in Fig. 6, together with the experimental results - the differential cross-section $(d\sigma/d\tau)_{pd}^{**})$, computed^{***)} by substituting to $\tilde{G}(P^2, \vec{P} \cdot \vec{d})$ the form factor obtained from the non-relativistic Hulthén wave function (neglecting the d wave)

*) In general, one should multiply the amplitude given in (3.7) by $\psi(q_1^2)$, as defined in (2.13); our choice of $\psi(q_1^2)$ corresponds to a Hulthén-like $\psi(q_1^2)$, $\psi(q_1^2) = (m_0^2 - m_1^2)/(q_1^2 - m_1^2)$, both for the singlet and the triplet amplitudes.

**) These cross-sections are computed only from the double scattering contribution, neglecting the possible interference with the single scattering contribution.

***) All numerical calculations have been made with the CERN 6600 CDC computer. We thank Dr. M. Tomás for help in the programming.

$$G(p^2) = \frac{2\chi\gamma(\chi+\gamma)}{(\chi-\gamma)^2} \frac{1}{p} \left[\arctg\left(\frac{p}{2\chi}\right) + \arctg\left(\frac{p}{2\gamma}\right) - 2\arctg\left(\frac{p}{\chi+\gamma}\right) \right] \quad (3.9)$$

where ⁴⁾ $\chi = 0.232 \text{ f}^{-1}$, $\gamma = 1.202 \text{ f}^{-1}$.

Surprisingly enough, one finds that in this way one reproduces fairly well both the angular distribution and the energy dependence of the backward proton-deuteron cross-section ^{*)}, the absolute normalization being 1.26 times too large at 1.5 GeV, 1.35 times too small at 1.3 GeV, and 1/0.97 times too large at 1.5 GeV, when the form (3.7) (not including the off-mass-shell effects) is used.

When the off-mass-shell corrections of formula (3.8) are included, the angular distribution, once normalized at a given point for each energy, is even better, as shown in Fig. 7. The absolute normalizations are now too small by a factor 20 at 1 GeV, 24.7 at 1.3 GeV and 18.4 at 1.5 GeV.

We have also tried for $\widetilde{G}(p^2, \vec{p} \cdot \vec{d})$ the relativistic expression (2.27), where the vertex function $\varphi(q^2)$ has been taken to be the relativistic generalization of the Hulthén vertex function (always neglecting the D wave)

$$\varphi(q^2) = \frac{-2(\gamma^2 - \chi^2)}{q^2 - m^2 - 2(\gamma^2 - \chi^2)} \quad (3.10)$$

The results [using only the off-mass-shell formula (3.8)] are reported in Table 3 and Fig. 8.

^{*)} We have limited our analysis to values of $\tau \leq 0.7(\text{GeV}/c)^2$; in fact, for larger τ , both the parametrizations (3.6) and (3.7) are questionable, since the values of $t_{2\min}$ and $s_{1\min}$ become outside the range of their applicability. For instance, for $T = 1.3 \text{ GeV}$ and $\tau = 0.7(\text{GeV}/c)^2$, one has : $t_{2\min} = -0.46(\text{GeV}/c)^2$, $(k_1^2)_{\min} = 0.115(\text{GeV}/c)^2$.

The agreement of the angular distribution is still good. The absolute normalizations are now better than using the non-relativistic form factor (always including off-shell effects) and are too small by a factor 6.26 at 1 GeV, 6.04 at 1.3 GeV, 3.94 at 1.5 GeV.

Of course, our choice of the Hulthén form factor does not mean at all that we are concluding that the true relativistic form factor can be computed starting from the Hulthén wave function: if the results of our model are taken seriously, we can conclude that the true "effective form factor" $\tilde{G}(p^2, \vec{p} \cdot \vec{d})$ will have about the same t dependence as the non-relativistic Hulthén form factor in the t interval between about $2.5 \leq |t| \leq 5 \text{ (GeV/c)}^2$ but from $\sqrt{1.2}$ to $\sqrt{20}$ times bigger (depending on whether one includes or not the off-mass-shell effects). As a matter of fact, any "form factor" which does not show a too rapid variation in this interval of t will not appreciably change the angular distribution for the backward pd peak. This is a consequence of the peaked behaviour of the product $T_1 T_2$, which yields the result that the angular distribution of $(d\sigma/d\Omega)_{pd}$ is essentially determined by the value of this product (which is function of d^2) on its maximum and depends very little on the behaviour of the form factor. On the contrary, the energy dependence of the backward cross-section depends mostly on the t behaviour of the "form factor".

A remarkable consequence is that our result seems to exclude form factors computed with wave functions which show a hard core for distances of the order of a few tenths of fermis: in fact these wave functions not only yield form factors which are much smaller (by a few orders of magnitude) than the Hulthén form factor in the region of t we are interested in, but moreover they show a pronounced dip just in the same interval of t : such a dip, resulting in a rapidly varying form factor, will not only alter the angular distribution, but will also give a completely wrong energy behaviour of the backward proton-deuteron cross-section.

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APPENDIX

Although the relation (2.8) is a well-known one, we shall give here a derivation, based on the Jost functions techniques in potential scattering *).

As usual, we shall define the s wave Jost solutions of the Schrödinger equation $f^+(k, r)$ and $f^-(k, r)$ by the conditions

$$\lim_{r \rightarrow \infty} f^{\pm}(k, r) = e^{\mp i k r} \quad (A.1)$$

The physical solution defined by the conditions $\varphi(k, 0) = 0$; $\frac{d}{dr} \varphi(k, 0) = 1$ is related to the Jost solutions by the relation **)

$$\varphi(k, r) = \frac{1}{2ik} [f^+(k) f^-(k, r) - f^-(k) f^+(k, r)] \quad (A.2)$$

where $f^{\pm}(k) = f^{\pm}(k, r=0)$ are the Jost functions. A bound state is characterized by the condition

$$f^-(k) = 0 \quad (A.3)$$

for $k = i\chi$, $\chi > 0$.

*) For simplicity, we shall restrict ourselves to the s wave problem.

**) See, for instance, the review papers in Ref. 32).

Therefore the bound state wave function is

$$\varphi(i\chi, r) = -\frac{1}{2\chi} f^+(i\chi) f^-(i\chi, r) \quad (\text{A.4})$$

The bound state wave function $u(i\chi, r)$, defined by the conditions $u(i\chi, 0) = 0$, $\int_0^\infty u^2(i\chi, r) dr = 1$, will be proportional to $\varphi(i\chi, r)$

$$u(i\chi, r) = C \varphi(i\chi, r)$$

We can identify the asymptotic coefficient of the bound state wave function, defined as

$$\lim_{r \rightarrow \infty} u(i\chi, r) = N e^{-\chi r}$$

with

$$\lim_{r \rightarrow \infty} \varphi(i\chi, r) = -\frac{1}{2\chi} f^+(i\chi) e^{-\chi r}$$

which gives

$$N = -\frac{C}{2\chi} f^+(i\chi) \quad (\text{A.5})$$

Remembering that

$$\begin{aligned} W[f^+(k, r), f^-(k, r)] &= [f^+(k, r) \dot{f}^-(k, r) - \dot{f}^-(k, r) f^+(k, r)] = \\ &= 2ik \quad \left[\dot{f} = \frac{df}{dr} \right] \end{aligned} \quad (\text{A.6})$$

and using (A.3), we can also write

$$N = -\frac{C}{2\chi} f^+(i\chi) = \frac{C}{\dot{f}^-(i\chi)} \quad (\text{A.7})$$

On the other hand, remembering the definitions ^{*)}

$$S(k) = \frac{f^+(k)}{f^-(k)}; \quad t(k) = \frac{S(k) - 1}{2ik} \quad (\text{A.8})$$

and expanding $f^-(k)$ in the vicinity of its zero for $k = i\chi$,

$$f^-(k) = f^-(i\chi) + \left(\frac{df^-(k)}{dk} \right)_{k=i\chi} (k - i\chi) = \left(\frac{df^-(k)}{dk} \right)_{k=i\chi} (k - i\chi) \quad (\text{A.9})$$

we have, for the singular term near the pole at $k^2 = -\chi^2$

$$t(i\chi) = \frac{f^+(i\chi)}{-2\chi \left(\frac{df^-}{dk} \right)_{k=i\chi}} = \frac{f^+(i\chi)}{(k^2 + \chi^2) \frac{df^-(i\chi)}{d\chi}} \quad (\text{A.10})$$

The next step is to connect N , as defined in (A.5), with the residuum of the t matrix on the pole in the k^2 plane

$$\lim_{k^2 \rightarrow -\chi^2} (k^2 + \chi^2) t(k) = \frac{f^+(i\chi)}{\frac{df^-(i\chi)}{d\chi}} \quad (\text{A.11})$$

For this, let us write the Schrödinger equation for two complex momenta, $k = i\chi_1$ and $k = i\chi_2$

$$\begin{aligned} \ddot{f}^-(i\chi_1, r) - V(r) f^-(i\chi_1, r) - \chi_1^2 f^-(i\chi_1, r) &= 0 \\ \ddot{f}^-(i\chi_2, r) - V(r) f^-(i\chi_2, r) - \chi_2^2 f^-(i\chi_2, r) &= 0 \end{aligned} \quad (\text{A.12})$$

Multiplying now the first equation by $f^-(i\chi_2, r)$ and the second one by $f^-(i\chi_1, r)$ and then subtracting, one gets

^{*)} In order to avoid confusions with the Jost functions, we shall denote here by $t(k)$ the S wave scattering amplitude, which is denoted as $f(q)$ in Section 2.

$$\begin{aligned}
& \dot{f}^-(i\chi_2, r) \ddot{f}^-(i\chi_1, r) - \dot{f}^-(i\chi_1, r) \ddot{f}^-(i\chi_2, r) \equiv \\
& \equiv \frac{d}{dr} \left\{ W \left[\dot{f}^-(i\chi_2, r), \dot{f}^-(i\chi_1, r) \right] \right\} = (\chi_1^2 - \chi_2^2) \dot{f}^-(i\chi_1, r) \dot{f}^-(i\chi_2, r) \quad (A.13)
\end{aligned}$$

Integrating (A.13) between $r = 0$ and $r = \infty$, and remembering that

$$\dot{f}^-(i\chi, r) \xrightarrow{r \rightarrow \infty} 0$$

one has

$$W(\infty) - W(0) = - \begin{vmatrix} \dot{f}^-(i\chi_2, 0) & \dot{f}^-(i\chi_1, 0) \\ \ddot{f}^-(i\chi_2, 0) & \ddot{f}^-(i\chi_1, 0) \end{vmatrix} = (\chi_1^2 - \chi_2^2) \int_0^\infty \dot{f}^-(i\chi_1, r) \dot{f}^-(i\chi_2, r) dr \quad (A.14)$$

This relation can be also written

$$(\chi_1^2 - \chi_2^2) \int_0^\infty \dot{f}^-(i\chi_1, r) \dot{f}^-(i\chi_2, r) dr = - \begin{vmatrix} \dot{f}^-(i\chi_2, 0) - \dot{f}^-(i\chi_1, 0) & \dot{f}^-(i\chi_1, 0) \\ \ddot{f}^-(i\chi_2, 0) - \ddot{f}^-(i\chi_1, 0) & \ddot{f}^-(i\chi_1, 0) \end{vmatrix}$$

and, in the limit $\chi_2 \rightarrow \chi_1 = \chi$, imposing that $\dot{f}^-(i\chi_1, 0) = 0$, one obtains

$$\frac{d \dot{f}^-(i\chi)}{d\chi} = 2\chi \left[\ddot{f}^-(i\chi) \right]^{-1} \int_0^\infty \dot{f}^{-2}(i\chi, r) dr \quad (A.15)$$

Remembering now that the physical wave function is normalized to

$$\int_0^\infty \varphi^2(i\chi, r) dr = \frac{1}{c^2}$$

and the relation (A.4), one gets

$$\frac{d \dot{f}^-(i\chi)}{d\chi} = 2\chi \left[\ddot{f}^-(i\chi) \right]^{-1} \frac{4\chi^2}{[f^+(i\chi)]^2 c^2} = - \frac{4\chi^2}{c^2 f^+(i\chi)} \quad (A.16)$$

and finally for the residuum of the t matrix

$$\lim_{k^2 \rightarrow -\chi^2} (k^2 + \chi^2) t(k) = \frac{f^+(i\chi)}{\frac{df^-(i\chi)}{d\chi}} = -N^2 \quad (\text{A.17})$$

A non-trivial example of this relation can be found if one takes just the Hulthén wave function, in which the bound state normalized wave function is

$$u(i\chi, r) = N (e^{-\chi r} - e^{-\gamma r}) \quad (\text{A.18})$$

where

$$N = \left[\frac{(\chi - \gamma)^2}{2\chi\gamma(\chi + \gamma)} \right]^{-\frac{1}{2}}$$

The relevant t matrix has been worked out by Yamaguchi ³⁰⁾, and is given by

$$t(k) = \frac{2\gamma(\chi + \gamma)^2}{(\chi k + \gamma)^2 (\chi k - \chi - 2\gamma)(\chi k + \chi)} \quad (\text{A.19})$$

It is easy to verify here (A.17). If moreover one works out from this t matrix the expressions of the scattering length and effective range, which are given by

$$a = \frac{2(\chi + \gamma)^2}{\chi\gamma(\chi + 2\gamma)} ; \quad r_d = \frac{3\gamma - \chi}{\gamma(\gamma + \chi)} \quad (\text{A.20})$$

one finds the well-known expression ³³⁾ of N^2 in terms of χ and r_d

$$N^2 = \frac{2\chi}{1 - \chi r_d}$$

- TABLE 1 -

$$a_t^{pn} = 27.354 \text{ (GeV)}^{-1} ; a_s^{pn} = -120.01 \text{ (GeV)}^{-1} ; a_s^{pp} = -88.697 \text{ (GeV)}^{-1}$$

$$b_t^{pn} = 3.4955 \text{ (GeV)}^{-1} ; b_s^{pn} = 3.8868 \text{ (GeV)}^{-1} ; b_s^{pp} = 3.8863 \text{ (GeV)}^{-1}$$

$T(\text{GeV})$	1	1,3	1,5
$\sigma_T^{pp}(\text{mb})$	47.53	47.30	46.83
$\sigma_T^{pn}(\text{mb})$	40.02	41.76	42.42
$\alpha^{pp}(\text{mb})$	-0.060	-0.168	-0.205
$\alpha^{pn}(\text{mb})$	-0.396	-0.450	-0.470

- TABLE 2 -

$ t $ (GeV/c) ²	$G^2(-t/4) \times 10^2$ (T = 1 GeV)	$ t $ (GeV/c) ²	$G^2(-t/4) \times 10^2$ (T = 1.3 GeV)	$ t $ (GeV/c) ²	$G^2(-t/4) \times 10^2$ (T = 1.5 GeV)
2.60	82.6 $\pm$ 3.7	3.63	35.5 $\pm$ 3.5	4.34	13.3 $\pm$ 1.3
2.69	52.4 $\pm$ 3.2	3.75	22.1 $\pm$ 2.2	4.47	9.97 $\pm$ 0.85
2.77	39.3 $\pm$ 1.7	3.86	12.3 $\pm$ 1.2	4.60	5.86 $\pm$ 0.55
2.86	24.7 $\pm$ 0.77	3.97	8.80 $\pm$ 0.87	4.73	3.17 $\pm$ 0.35
2.95	16.7 $\pm$ 0.56	4.09	6.54 $\pm$ 0.32	4.87	2.02 $\pm$ 0.23
3.03	13.8 $\pm$ 0.45	4.14	7.10 $\pm$ 0.48	4.93	2.23 $\pm$ 0.22
3.12	11.4 $\pm$ 0.40	4.20	5.16 $\pm$ 0.39	5.00	1.77 $\pm$ 0.18
3.17	10.9 $\pm$ 0.35	4.26	4.32 $\pm$ 0.37		
3.21	9.48 $\pm$ 0.31	4.30	3.67 $\pm$ 0.27		
3.25	8.64 $\pm$ 0.30				
3.25	9.71 $\pm$ 0.31				

- TABLE 3 -

T(GeV)	1	1.3	1.5	1	1.3	1.5	1	1.3	1.5
τ (GeV/c) ²	$\frac{d\sigma}{d\tau}$ (1)	$\frac{d\sigma}{d\tau}$ (1)	$\frac{d\sigma}{d\tau}$ (1)	$\frac{d\sigma}{d\tau}$ (2)	$\frac{d\sigma}{d\tau}$ (2)	$\frac{d\sigma}{d\tau}$ (2)	$\frac{d\sigma}{d\tau}$ (3)	$\frac{d\sigma}{d\tau}$ (3)	$\frac{d\sigma}{d\tau}$ (3)
0	60.6	20.4	10.6	5.64	1.51	0.702	19.1	7.40	4.32
0.1	42.4	14.5	7.62	3.47	0.953	0.450	11.0	4.30	2.52
0.2	30.5	10.6	5.66	2.26	0.631	0.300	6.82	2.69	1.59
0.3	22.5	7.93	4.28	1.54	0.432	0.207	4.54	1.77	1.05
0.4	16.9	6.03	3.29	1.09	0.304	1.145	3.05	1.21	0.711
0.5	12.9	4.63	2.53	0.791	0.219	0.105	2.15	0.832	0.496
0.6	9.97	3.61	1.99	0.586	0.161	0.077	1.54	0.592	0.348
0.7	7.80	2.84	1.56	0.441	0.119	0.057	1.13	0.429	0.253

$\frac{d\sigma}{d\tau}$ (1) are the values of the differential p - d cross-section one gets using the non-relativistic form factor (3.9) and the amplitude (3.7) which does not include the off-mass shell corrections.

$\frac{d\sigma}{d\tau}$ (2) are the corresponding values using (3.9) and the amplitude (3.8) which includes the off-mass shell corrections.

The values $\frac{d\sigma}{d\tau}$ (3) are obtained using the relativistic "form factor" given by (2.27) and (3.10) and the amplitude with off-mass shell corrections (3.8).

All the differential cross-sections are given in $\frac{\mu b}{(\text{GeV}/c)^2}$.

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FIGURE CAPTIONS ---

- Figure 1 The neutron exchange diagram.
- Figure 2 The single scattering diagram (impulse approximation).
- Figure 3 The Feynman diagrams for single (a) and double (b) scattering. The barred particles are on the mass-shell.
- Figure 4 The three Feynman diagrams contributing to p - d double scattering.
- Figure 5 The square of the relativistic form factor (2.15) obtained from the formula (3.2) is plotted against t . For comparison, the squares of the non-relativistic asymptotic and Hultn form factors are also plotted (the last multiplied by ten).
- Figure 6 The values of the differential p - d cross-section $\frac{d\sigma}{d\tau}$ at 1 GeV (Fig. 6a), 1.3 GeV (Fig. 6b) and 1.5 GeV (Fig. 6c) plotted against τ from $\tau = 0$ to $\tau = 0.7$ (GeV/c)² (dotted curve). These values have been obtained using the non-relativistic form factor (3.9) and the amplitude without off-mass-shell corrections (3.7). They are normalized at $\tau = 0.3$. The normalization factors are indicated in the figures. The points with errors are experimental points taken from Ref. ¹⁾. The continuous curves are a fit to these experimental points also given in Ref. ¹⁾.

Figure 7 The same as in Fig. 6, but now using the non-relativistic form factor (3.9) and the amplitude (3.8) which contains the off-mass-shell corrections. The theoretical curves are also normalized at $\tau = 0.3$ and the normalization factors indicated in the figures.

Figure 8 The same as in Fig. 6 and Fig. 7 but now using the relativistic expression (2.27), (3.10) for the "effective form factor" and the amplitude with off-mass-shell corrections (3.8). The theoretical curves are again normalized at $\tau = 0.3$ and the normalization factors indicated in the figures.

fig. 1

fig. 2

fig. 3

(a)

(b)

(c)

fig. 4

Fig. 5

fig. 6 b

fig. 6 c

fig.8a

fig. 8 b

fig.8c