CM-P00057354

REGGE RECURRENCES AND π p ELASTIC SCATTERING AT HIGH-
MOMENTUM TRANSFERS

A. Białas and E. Białas
Institute of Physics, Jagellonian University, Cracow
and CERN - Geneva

O. Czyżewski
Institute of Nuclear Research, Cracow
and CERN - Geneva

A. Kotański
Institute of Physics, Jagellonian University, Cracow

A B S T R A C T

The influence of the resonance formation on the high-momentum transfer elastic π p scattering is investigated. A new model, in which the known Regge recurrences are extrapolated to higher energies, is proposed. It is shown that the model can account for the present data on the π p elastic scattering at high momentum transfers. Some implications of such a mechanism are discussed.

This paper discusses the possibility of a new mechanism for explaining the π p elastic and charge-exchange scattering at high momentum transfers. By high momentum transfers is meant the angular region near 90° in the c.m. system where one expects the forward and backward peaks to play only a secondary role (provided the energy is high enough, say greater than 5 GeV/c of the laboratory momentum ¹⁾).

We investigate here the idea that the high momentum transfer elastic and charge-exchange scattering is caused mainly by the formation of the resonance in the direct π p channel. To be more specific, we discuss the possibility that there exist π p resonances up to highest available energies. The formation of these resonances, and their subsequent decay into the π p channel give a contribution to large angle scattering, which we are trying to estimate.

The importance of the direct formation of resonances for the behaviour of the π p elastic and charge-exchange scattering in the intermediate energy region (laboratory momentum ≤ 5 GeV/c) has been realized already by many authors ¹⁾⁻⁷⁾. The resonance effects are, in particular, very clearly seen in the recent data of Coffin et al. ⁴⁾. The importance of the study of 90° region was pointed out by Kemp and Muirhead ⁷⁾.

We would like to discuss in our paper the high-energy region, (laboratory momentum ≥ 5 GeV/c) pointing out the importance of the large angle region (outside of the forward and backward peaks) for the possible investigation of the resonance formation at very high energies.

The idea that the π p resonances may show up even at very high energies is strongly suggested by the recent investigations of Barger and Cline ¹⁾ and Barger and Olsson ²⁾ which tend to show that there exist (at least) two families of resonances (each of them containing five particles) with the properties characteristic for Regge recurrences. In particular, the relation between the mass M and the angular momentum J of a resonance has been suggested in the form

2.

$$\Pi^2 = x + y J \quad (1.1)$$

where x and y are constants within a family. As there are no indications that this law is violated for higher resonances, it may be not unreasonable to believe that the straight line (1.1) on the Chew-Frautschi plot ⁸⁾ will continue up to higher energies.

Once such an assumption is made, one can investigate its consequences for the resonance formation in the πp interaction. The natural place one can look for the resonance effects will be the 90° region. The reason is that, as we will show in the following Sections, the formula (1.1) makes it very likely that the probability of the formation of these πp resonances must decrease at least exponentially with increasing c.m. energy. This result may provide a possible explanation of the approximately exponential decrease of the 90° elastic πp scattering observed in high-energy experiments. Furthermore, since both the forward and the backward peaks are known to behave like some powers of energy, the effects of the resonance amplitude will quickly become negligible in this angular region.

There exist two, rather fundamental, differences between our approach and other proposed explanations ⁹⁾⁻¹²⁾ of the large angle scattering at high energies. The first difference is that most of the existing explanations refer the elastic scattering at high-momentum transfers to the so-called central collisions, i.e., the collisions with the small impact parameters. In our approach the scattering at large angles is caused by an extremely peripheral mechanism, as can be seen from the formula (1.1). In fact, the impact parameter ρ at which the resonance of mass M is produced

$$\rho = \frac{J}{k} \approx \frac{2J}{M} \approx \frac{2M}{y} \quad (1.2)$$

where k is the c.m. momentum, increases with increasing resonance mass. This is the main reason why the probability of the resonance formation must decrease very quickly with increasing energy.

The second point where we differ from other approaches is that our argument is valid only for $\pi^\pm p$ (and possibly K^-p) interaction, but not for pp and K^+p scattering. All other models of large angle scattering at high energy emphasize rather the similarity between pp and πp scattering and attempt to build some universal theory. This point may represent an important test for the validity of our model. The present πp large angle data at high energies are not accurate enough to allow for the precise comparison with the pp scattering. There seems, however, to exist a definite difference between the angular distributions in these two cases. The mechanism we propose suggests that such a difference should persist even at highest energies.

The proposed model does not exclude that other possible mechanisms (in particular the multiple scattering) can play an important role in the large angle region at high energies. We would like to point out only that the resonance effects give the contributions which are of the correct order of magnitude and therefore must be taken into account in a complete theory of high momentum transfer scattering.

In the second Section we discuss the notation and kinematics. Section 3 gives some estimations of the effects of the resonance formation on the large angle scattering at intermediate energies. The details of the model with infinite Regge recurrences are given in Section 4. In Section 5 we present the results of the numerical calculations and compare them with the experimental data. Our conclusions are listed in the last Section.

4.

2. NOTATION AND KINEMATICS

We use the helicity formalism of Jacob and Wick¹³⁾. The scattering amplitude

$$T_{\lambda\mu}(s, \cos \Theta) = \frac{1}{k} \sum_J \left(J + \frac{1}{2} \right) T_{\lambda\mu}^J(s) d_{\lambda\mu}^J(s) \quad (2.1)$$

where λ, μ refer to the proton helicities, is related to the differential cross-section by the formula

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} \sum_{\lambda\mu} |T_{\lambda\mu}|^2 \quad (2.2)$$

Here $s = (\text{total c.m. energy})^2$ and Θ is the c.m. scattering angle. The sum in the formula (2.1) runs over all resonances of the πp system.

The resonant amplitude $T_{\lambda\mu}^J(s)$ was taken in the form

$$T_{\lambda\mu}^J(s) = C_{\lambda\mu}^I X F \frac{1}{\epsilon - i} \quad (2.3)$$

where $C_{\lambda\mu}^I$ is the factor taking into account the dependence on isotopic spin and helicities,

X is the elasticity parameter of a given resonance,

F is a form factor describing the deviations of the shape of a resonance from the Breit-Wigner curve, and

$$\epsilon = \frac{s - M^2}{M \Gamma} \quad (2.4)$$

Here Γ is the total width of a resonance.

3. THE EFFECT OF THE RESONANCE FORMATION IN THE INTERMEDIATE ENERGY REGION

In the low and intermediate energy region (up to ~ 5 GeV/c laboratory momentum) the effect of the resonance formation on the large angle scattering can be estimated by introducing into the formulae (2.1) - (2.4) the parameters of all known resonances. The region of the validity of the estimation is limited by the fact that the highest resonances which have been identified up to now lie approximately at the laboratory momenta around 5 GeV/c. At the energies much higher than these, the behaviour of the cross-section is determined mainly by the form factor F which is not known.

In the actual calculations we have used two sets of data

- a) the tables of Rosenfeld et al. ¹⁴⁾,
- b) the estimates Barger and Cline ¹⁾.

In the cases where not all parameters are given in the Rosenfeld table we have used the numbers from the paper of Barger and Cline.

The form factor F has been taken in the form

$$F = e^{-\beta \frac{(E - M)^2}{E + M}} \quad (3.1)$$

where β is a parameter and E is the total c.m. energy of the system. The parameter β was varied in the region $0 - 2$ (GeV)⁻¹. Up to the laboratory energies ~ 4.5 GeV the results have been found insensitive to its value.

Figures 1a and 1b show the typical curves for 90° scattering and their comparison with the experimental data ^{4),15)}. It is clear that the resonance amplitude is quite important in this region of energy. Up to 4 - 4.5 GeV/c primary momentum the resonant amplitude gives a

6.

cross-section which is comparable to the actually observed one. Let us point out once more that the resonances which have been used for calculations are more or less well established and, in that sense, our estimations are simple consequences of the known data.

One can conclude that the resonance formation in that energy region must certainly be taken into account in any attempt of explaining the large angle data.

4. REGGE RECURRENCES

As was shown in the preceding Section, the formation of resonances in the direct channel plays a rather important role in the large angle π p scattering in the intermediate energy region. One can then ask the question, whether the resonant amplitudes may be important at higher laboratory momenta, say, of the order 10 - 20 GeV/c, or even higher.

The answer to that question is not known at the moment and therefore one is led to speculations. Essentially two possibilities appear :

- a) there exists an upper limit for the mass of the π p resonances, and,
- b) the π p resonances continue up to very high "infinite" masses.

The possibility a) implies that in the high-energy region one can see only the tails of the low-energy resonances. As we really know very little about the behaviour of the resonance tail far away from the peak, the contribution cannot be evaluated in this case.

The possibility b), which will be discussed in more detail, must be based on some extrapolation of the existing data. As the basis of that extrapolation we have used the results of recent investigations of Barger and co-workers which suggest that there exist (at least) two families of resonances which show the characteristic features of the so-called Regge recurrences. Each family contains five particles. It seems very likely that such series of particles continue up to higher energies. It is also quite likely that the formula (1.1) which relates the angular momentum and the mass of the particle will be (at least approximately) valid also for these higher resonances. Very little can be said of course on the problem at which energy the empirical formula (1.1) suggested by Barger and Cline becomes a poor approximation.

In this paper we investigate the attractive possibility that such a behaviour continues up to the highest possible energies. We assume the existence of two infinite series of resonances, all members of a series having the same quantum numbers except spin. The spin of resonances within a recurrence is given by the formula

$$J = 2n - \frac{1}{2}, \quad n = 1, 2, 3, \dots \quad (4.1)$$

The mass of a resonance is related to its angular momentum by the formula (1.1).

In calculations we have used the following parameters :

$$\begin{array}{ll} \text{for } \Delta_8 & \text{trajectory : } x = -0.05 \text{ (GeV)}^2, \quad y = 1.084 \text{ (GeV)}^2 \\ \text{for } N_Y & \text{trajectory : } x = 0.60 \text{ (GeV)}^2, \quad y = 1.160 \text{ (GeV)}^2. \end{array}$$

In order to evaluate the resonant scattering amplitude one must guess also the dependence of other parameters of resonances on the energy of the system and on the resonance mass.

Let us first discuss the elasticity parameter X , that is, by definition, the ratio between the elastic and total rates of decay. By time-reversal invariance this quantity is also proportional to the probability of the formation of given resonance in the πp scattering.

As already mentioned in the Introduction, relation (1.1) implies that the resonances we consider must be produced by very peripheral collisions. This fact is clearly seen in formula (1.2) which shows that the corresponding impact parameter is proportional to the resonance mass.

If (as we believe) the range of the πp forces is finite, the strong centrifugal barrier prevents incoming particle to penetrate inside the "potential well". In consequence, the formation of these higher resonances is strongly suppressed. This suppression can be roughly estimated from the assumption that the probability of forming

a resonance is approximately proportional to the strength of the interaction for the given partial wave. Since for large impact parameter the partial wave amplitude behaves approximately as $a_l \sim e^{-\mu\rho}$, we get, using formula (1.2) :

$$X \approx \phi(\rho) e^{-2\mu \frac{M}{Y}} \quad (4.2)$$

where $\phi(\rho)$ is a slowly varying function of impact parameter.

Equation (4.2) shows that the probability of the formation of a given resonance from Regge recurrence decreases very fast with increasing resonance mass.

This result seems to be very encouraging in our attempt of explaining the large angle πp scattering at high energies. As it is well known, the high-energy experiments suggest the exponential decrease of the large angle elastic cross-section with increasing energy.

Since the estimation of the proportionality coefficient in the exponent in formula (4.2) is very crude, in the actual calculations we have used the elasticity parameter in the form

$$X = e^{-\alpha(M - 1.25)} \quad (4.3)$$

where α is a free parameter of the dimension $(\text{GeV})^{-1}$, M is measured in GeV. The form (4.3) ensures that $X(1.25) = 1$, as it should be for the $(3,3)$ resonance. The value of the parameter α depends on the model used for the interacting potential.

We must introduce also the dependence of the resonance width on the resonance mass. As we do not know anything on its behaviour at high energies, we simply tried two different assumptions

$$\begin{aligned} \text{a)} \quad \Gamma &= \Gamma_0 \text{ independent of the resonance mass,} \\ \text{b)} \quad \Gamma &= \gamma M, \text{ where } \gamma \text{ is a constant.} \end{aligned} \quad (4.4)$$

The detailed behaviour of the large angle cross-section seems to depend rather significantly on the behaviour of Γ at high resonance masses. The more detailed description of these effects will be given in the next Section.

The last problem we have to face is the high-energy behaviour of the form factor. We have used the form (3.1) with parameter β varying in some, rather large, range. It seems that the energy dependence of the cross-section does not depend very strongly on its actual value. In the next Section we discuss this problem in more detail.

5. THE DISCUSSION OF THE NUMERICAL RESULTS

We have investigated numerically the energy and angular dependence of the large angle cross-section up to 8 GeV total c.m. energy. We would like to present in this Section :

- a) general features of the obtained result and their sensitivity on the parameters of our model,
- b) the relation of the results to the existing experimental data.

Let us start with the presentation of the energy dependence of the elastic scattering at the fixed c.m. scattering angle $\Theta = 90^\circ$. In the limits of variation of the parameters of our model for Γ given by formula (4.4b) the general properties of the energy dependence of the cross-section are as follows (Fig. 2a). In the region of small and intermediate energy (up to ~ 3.5 GeV in c.m. system) one can clearly see the bumps in the cross-section, corresponding to the assumed family of resonances. As the energy increases the maxima become less and less significant and the curve becomes almost purely exponential. If the energy increases still further the structure reappears, the oscillations of the cross-section become very pronounced (1 - 2 orders of magnitude) and after that they become once more less and less significant. In the last region also the over-all energy dependence of the cross-section (i.e., behaviour of the average over oscillations) is less steep than in the lower energy region. The energy at which the oscillations of the cross-section reappear is related to the ratio of the parameters α/β , defined in Eqs. (3.1) and (4.3). It increases when α/β decreases. For $\alpha/\beta \leq 1$ we have not found the reappearance of oscillations up to 30 GeV/c lab. momentum. The appearance and the detailed character of oscillations depend strongly on the parameters of the model. In particular the behaviour of the width Γ and the form factor F are at high energies very important ; for $\Gamma = \text{constant}$ the bumps survive up to highest investigated energy (Fig. 2b). One can easily find such sets of parameters for which the oscillations will not appear at any

reasonable energy. As nothing is known on the actual behaviour of these parameters in the high-energy region one cannot make a definite prediction concerning the structure in the energy dependence. The comparison of the π^+p and π^-p elastic scattering indicates that for the same set of parameters the π^+p cross-section is larger than π^-p cross-section by a factor of about two. It would be interesting to check this relation experimentally in the high-energy region.

The average value of the cross-section decreases approximately exponentially with increasing energy, and it is quite easy to choose the parameters of the model so that one finds the agreement with the experimental points at 8 GeV/c and 12 GeV/c primary momentum. This point was not investigated in detail because we feel that our argument is at present only qualitative and much better knowledge of the πp resonant states is needed in order to bring our model into more quantitative form.

The angular dependence [Figs. 3, a) and b)] has a rather complicated structure, especially at high energy. This structure consists of oscillations of rather high frequency. The average distance between two peaks is approximately fitted with a formula

$$\Delta\theta \approx \frac{2}{J} \approx \frac{2}{E^2} \quad (5.1)$$

where J is the angular momentum of the resonance at the given c.m. energy E . The value of $\Delta\theta$, given by this formula, is at high energy much smaller than that predicted for fluctuations by the usual "compound intermediate state" model of elastic scattering¹⁶⁾, where the relevant value of angular momentum is supposed to be proportional to the c.m. energy of the system.

In the investigated energy region oscillations in the π^-p elastic scattering are several times higher than in the case of π^+p . The presence of strong oscillations is related therefore to the presence of two trajectories in the case of π^-p .

6. CONCLUSIONS

We have tried to estimate the contribution of the resonance formation to the large angle π p elastic scattering and charge exchange. At the lower energy (up to 4 GeV/c primary momentum) the estimation confirms the conclusions of other authors ²⁾⁻⁷⁾ that the resonant amplitude is very important for the description of the data. In fact using the tables of Rosenfeld et al. ¹⁴⁾ or the estimations of Barger and Cline ¹⁾ one gets the contribution to the large angle scattering which is comparable to the observed value of the cross-section. Of course some interference between the resonant and non-resonant amplitudes must appear and the actual situation in that energy region is rather complicated.

The extension of the estimation of the resonance production to the higher energies was carried out on the basis of the extrapolation of the concept of Regge recurrences investigated recently by Barger and co-workers ^{1),2)}. It was assumed that there exist two series of π p resonances which have the properties of Regge recurrences and for which the Regge trajectories can be represented by straight lines on the Chew-Frautschi plot. The relation between spin and the mass of the resonances is assumed to be of the form given by formula (1.1).

The consequences of this assumption have been investigated. It was shown in particular that it implies very fast decrease of the elasticity parameter of the resonances with the increasing resonance mass. This implies that at high energy the resonance contribution can be important only in the region of large angles. If one assumes, furthermore, that the resonance production is the only important effect in that angular region, one gets an approximately exponential decrease of the π p elastic scattering and charge exchange cross-section at large angles with increasing energy. The experimental data are compatible with that statement, but they are certainly not conclusive.

The numerical calculations performed on the basis of the presented model have shown that the resonance contribution to the large angle scattering is in fact of the correct order of magnitude. For details we refer to Section 5.

In conclusion, our calculations show that the resonance formation may play a very important role, and possibly even dominate, the π p elastic scattering and charge exchange at large angles and high energies.

We would like to comment now on the difference between the mechanisms of the pp and π p scattering at large angles. No resonances have been observed in the pp system. Since the possibility, that pp large angle elastic scattering can be described in the frame of the statistical model, seems to be ruled out by the experiments of Allaby et al.¹⁷⁾ and of Akerlof et al.¹⁸⁾ it follows that the large angle pp scattering is probably caused by multiple scattering. On the other hand, the multiple scattering seems to be comparatively less effective in the π p case, because of the probability of formation of a resonance. This argument suggests that the pp and π p elastic scatterings at large angle are caused by rather different mechanisms. Such a point of view is supported by the fact that the angular dependence in the high-momentum transfer region differs essentially in two cases. The pp cross-section follows a smooth exponential line, whereas the situation in π p case is much less simple. Similar difference should occur between K^+p scattering, which should be similar to pp case, and K^-p scattering, which should correspond to π p scattering.

In conclusion, we expect a marked difference between the reactions like πp and K^-p scattering, where resonances may be formed in the direct channel, as compared to pp and K^+p cases. The first group of reactions is expected to reveal a more complicated structure both in energy and in angular dependence.

A final remark is that our calculations should be considered only as qualitative indications of the proposed mechanism and by no means as quantitative predictions. This limitation is mainly caused by our still very limited knowledge of the properties of higher mass resonances.

ACKNOWLEDGEMENT

The authors would like to thank Professors D. Amati, G.F. Chew, T. Ericson, R. Hagedorn, A. Martin, L. Van Hove and V.F. Weisskopf, as well as Drs. Ch. Chiu, Z. Chylinski, D.R.O. Morrison, R.J.N. Phillips and K. Zalewski for very helpful discussions.

Two of us (A.B. and O.C.) would like to thank also Professors J. Prentki and C. Peyrou for the kind hospitality extended to them at CERN, where part of this work was done.

REFERENCES

- 1) V. Barger and D. Cline, Phys.Rev.Letters 16, 913 (1966), and "Backward π^+p elastic scattering from a direct channel resonance model", University of Wisconsin preprint, August (1966) ; and Ch.B. Chiu, private communication.
- 2) V. Barger and M. Olsson, "Interference of Regge rho exchange with direct channel fermion resonances", University of Wisconsin preprint (1966), and "Regge pole model and total cross-sections in the intermediate laboratory momentum range 2-6 BeV/c", University of Wisconsin preprint (1966).
- 3) R.J.N. Phillips, Lectures in Erice Summer School, June (1966) and private communication ;
R.K. Logan and L. Sertorio, "A Regge pole analysis of π^-p charge-exchange polarization", University of Illinois preprint (1966).
- 4) C.T. Coffin, N. Dikmen, L. Ettlinger, D. Meyer, A. Saulys, K. Terwilliger and D. Williams, Phys.Rev.Letters 15, 838 (1965) ;
C.T. Coffin, N. Dikmen, L. Ettlinger, D. Meyer, A. Saulys, K. Terwilliger and D. Williams, Phys.Rev.Letters 17, 458 (1966).
- 5) S.W. Kormanyos, A.D. Krisch, J.R. O'Fallon, K. Ruddick and L.G. Ratner Phys.Rev.Letters 16, 709 (1966).
- 6) J. Baacke et M. Yvert, "Regge poles and resonances in π^-p charge-exchange scattering between 2.5 and 6 GeV/c", paper presented at the Berkeley Conference, September (1966).
- 7) M.A.R. Kemp and H. Muirhead, "Elastic meson-nucleon scattering at 90° and the properties of the baryon resonances", University of Liverpool preprint (1966).
- 8) G.F. Chew and S. Frautschi, Phys.Rev.Letters 8, 42 (1962).
- 9) For the statistical approach we refer to the paper by R. Hagedorn, Nuovo Cimento 35, 216 (1965).

- 10) The potential theory is summarized in W.N. Cottingham and N. Dombey, Nuovo Cimento 43A, 397 (1966).
- 11) The "minimal interaction" idea is presented in some detail in A. Martin, Nuovo Cimento 37, 671 (1965).
- 12) For the "Incoherent droplet" model see T.T. Wu and C.N. Yang, Phys.Rev. 137, B708 (1965) ; and K. Huang, Phys.Rev. 146, 1075 (1966).
- 13) M. Jacob and G.C. Wick, Ann.Phys. 7, 404 (1959).
- 14) A.H. Rosenfeld, A. Barbaro-Galtieri, J. Kirz, W.J. Podolsky, M. Roos, W.J. Willis and C.G. Wohl, UCRL 8030 preprint (1966).
- 15) The experimental data were taken from the papers :
 M.L. Perl, L.W. Jones and C.C. Ting, Phys.Rev. 132, 1252 (1963) ;
 M.L. Perl, Y.Y. Lee and E. Marguit, Phys.Rev. 138, B707 (1965) ;
 J. Orear, R. Rubinstein, D.B. Scarl, D.H. White, A.D. Krisch, W.B. Frisken, A.L. Read and H. Rudermann, Phys.Rev.Letters 15, 309 (1965), and Ref. ⁴⁾.
- 16) T. Ericson, "Fluctuation phenomena and statistical reactions", CERN preprint TH. 406 (1964) ;
 G. Cocconi, "Elastic scattering at large angles", invited paper presented at the Stony Brook Conference on Two-Body Processes (1966).
- 17) J.V. Allaby, G. Belletini, G. Cocconi, M.L. Good, A.N. Diddens, G. Matthiae, E.J. Sacharidis, A. Silverman and A.M. Wetherell, "Search for fluctuations in the angular distribution of proton-proton scattering at 16.9 GeV/c", CERN preprint (1966).
- 18) G.W. Akerlof, R.H. Hieber, A.D. Krisch, K.W. Edwards, L.G. Ratner and K. Ruddick, "Proton-proton elastic scattering at 90° and structure within the proton", preprint (1966).

FIGURE CAPTIONS

- Figure 1 Energy dependence of π p scattering cross-section at 90° in c.m.s. in the region of known resonances,
a) π^+ p elastic scattering ;
b) π^- p elastic scattering.
- Figure 2 Energy dependence of π p scattering cross-section at 90° in c.m.s., calculated using "infinite" series of resonances,
a) π^+ p elastic scattering ;
b) π^- p elastic scattering.
- Figure 3 Angular distributions of π^- p large angle scattering, calculated using "infinite" series of resonances.

Fig. 1a

Fig.1b

Fig. 2 a

Fig. 2 b

Fig. 3