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Abstract

The Coulomb energy is evaluated by a harmonic oscillator wave function. It explains the experimental parity-dependence of the Coulomb mass shifts of $A = 17$ isoquartets, suggesting a level-dependent nuclear radius. It also suggests that the two protons of the ground state of ^{17}Ne occupy the $1s_{\frac{1}{2}}^{+}$ -orbit.

The radius of the nucleus attracts our attention recently in relation to the neutron halo [1] etc. In this article, we discuss a simple relation between the radius and the mass shift, and it is shown that a simple model of nuclear radius gives a parity-dependence of the Coulomb mass shifts. And the model is found to explain a recent experiment. The long-range Coulomb force seems sometimes to provide a simple structure of the nuclear masses, which represents the global structure of the nuclear wave functions, like the radius. It is in contrast to the short-range forces, which relate to the fine structures of the wave function.

An approximation with the harmonic oscillator wave function gives the mean-square radius dependent on the major shell, namely, $\frac{M\omega^2}{2}\langle r^2 \rangle = \frac{1}{2}(n + \frac{3}{2})\hbar\omega$, where M is the nucleon mass and $\hbar\omega$ is the oscillator energy constant, and $n = 0$ to the 0s-orbit, $n = 1$ to the 0p-orbit, and $n = 2$ to the 1s0d-orbit. Then, the mean-square radius of the nucleus is given by the sum of the oscillator energy quanta, n , of all nucleons.

The Coulomb energy of the proton in a nucleus may be approximately determined by the mean-square radius of the proton. Assuming a charge distribution of a uniform sphere of radius R , we have the “inside” Coulomb potential $\frac{Ze^2}{2R}[3 - r^2/R^2]$ ($r < R$). It gives a term of mean-square radius, $-\frac{Ze^2}{2R^3}\langle r^2 \rangle$, neglecting the tail of the proton wave function of the region of $r > R$. Thus, the oscillator energy quantum determines both the nuclear radius and the Coulomb energy.

Figure 1 shows the positive and negative parity states of ^{17}Ne and their mirror states of ^{17}N . In the positive parity state, two 0p-nucleons and one 1s0d-nucleon change the charge between the mirror states, while, in the negative parity state, one 0p-nucleon and two 1s0d-nucleons change it. The sum of the oscillator energy quanta of these three nucleons is smaller in the positive parity state $[\sum n]_{\nu+} = 4$ than in the negative parity state $[\sum n]_{\nu-} = 5$. Thus, the three nucleons in the positive parity state has a smaller mean-square radius, $\langle r^2 \rangle_{\nu+} < \langle r^2 \rangle_{\nu-}$, and a larger Coulomb mass difference than the negative parity state, $\langle V_{\text{Coul}} \rangle_{\nu+} > \langle V_{\text{Coul}} \rangle_{\nu-}$.

It is found that the experimental results support the discussion above. The group which includes the two of the present authors have determined the spin and parity of the low-lying states of ^{17}Ne by the reaction $^{20}\text{Ne}(^3\text{He}, ^6\text{He})^{17}\text{Ne}$ [2]. Taking account of the previous data of the isobaric analog states in ^{17}F , ^{17}O and ^{17}N nuclei, we obtain six isospin multiplets all members of which are known. Their mass shifts, i.e., the coefficients, $b(\nu, T)$, $c(\nu, T)$ and $d(\nu, T)$, are given from the equation,

$$E(\nu, T, T_z) = a(\nu, T) + b(\nu, T)T_z + c(\nu, T)T_z^2 + d(\nu, T)T_z^3, \quad (1)$$

where $E(\nu, T, T_z)$ is the mass of the state, and ν represents the quantum numbers except the isospin T and its third component T_z . The notation is that the proton isospin is $T_z = +\frac{1}{2}$. It is found that the experimental mass shifts have a parity-dependent behaviour [2]. The isovector mass shift, $b(\nu, T)$, and the isotensor mass shift, $c(\nu, T)$, of the positive parity, $\frac{1}{2}^+$ and $\frac{5}{2}^+$ isoquartets, are larger than those of the negative parity, $\frac{1}{2}^-$, $\frac{3}{2}^-$, $\frac{5}{2}^-$ and $\frac{7}{2}^-$ isoquartets.

In order to compare the theoretical magnitude with the experiment, a shell model calculation is made, with a harmonic oscillator wave function whose oscillator energy constant is determined by the charge root-mean-square radius of ^{17}O [4]. ^{12}C is treated as a core and yields the Coulomb potential to the valence protons in $0p\frac{1}{2}^-$, $0d\frac{5}{2}^+$ and $1s\frac{1}{2}^+$ -orbits. The valence-core and valence-valence Coulomb interactions are considered [5]. The exchange term, other electromagnetic interactions and the nuclear charge-symmetry-breaking and charge-independence-breaking interactions are neglected, while the Pauli-violated wave functions are excluded. The calculations are made on the states whose configurations are expected to dominate the low-lying states. Since we do not determine the exact configuration of the real low-lying states, several calculated states, which have same spin and parity, may correspond to one experimental state. Generally, the real low-lying states are expressed by linear combinations of these calculated wave functions.

A positive parity state can be obtained by the configuration shown in fig. 2, as well as the fig. 1(a). In this configuration, three $1s0d$ -nucleons change the charge between the mirror states, $[\sum n]_{\nu^+, \text{fig.2}} = 6$, and it predicts an opposite parity-dependence. But this configuration does not gain the binding energy, and it may cause a favour of fig. 1(a).

However, fig. 1(a) is not an isospin eigen function. The isospin eigen function must have a mixed fraction of fig. 2, in some cases, e.g., the state of the two $0d$ -protons and the one $1s$ -neutron. In fig. 3, we show the results of minimal mixing of the configuration of fig. 2 (open diamonds) and maximal one (open squares) for the positive parity states, as well as the result of negative parity states (filled diamonds). The isovector mass shifts, $b(\nu, T)$, are given in fig. 3(a) and the isotensor mass shifts, $c(\nu, T)$, are in fig. 3(b).

It is found that $b(\nu^+, \text{fig.1(a)}) > b(\nu^-, \text{fig.1(c)}) > b(\nu^+, \text{fig.2})$, corresponding to the order of the radii, $\langle r^2 \rangle_{\text{fig.1(a)}} < \langle r^2 \rangle_{\text{fig.1(c)}} < \langle r^2 \rangle_{\text{fig.2}}$. It is found that the magnitude of the calculated parity-dependence of $b(\nu, T)$ is about 50 keV, as can be seen from the difference between the open and filled diamonds in the figure. It agrees with experiment. The parity-dependence of $c(\nu, T)$ is, however, not reproduced by the calculation that predicts a too small deviation.

The nuclear charge-independence-breaking force has large contributions to the $c(\nu, T)$, and its dependences on the spin and other quantum numbers are large [8]. The conclusive discussion on $c(\nu, T)$ may be made after taking account of the nuclear force and reducing the experimental error. At least, the experimental parity-dependence of the isovector mass shift, $b(\nu, T)$, can be recognized as the effect of the difference of the oscillator energy quanta, implying that the nuclear radius of the low-lying states depends on the parity.

We should comment on the structure of the ground state. The previous studies, i.e., the analyses of the measurement of the interaction cross section of ^{17}Ne [9] and the calculation by a three-cluster generator coordinate model [10], suggest that the ground state is a $1s_{\frac{1}{2}}^{1+}$ -state, namely, the two protons occupy the $1s_{\frac{1}{2}}^{1+}$ -orbit, instead of the $0d_{\frac{5}{2}}^{5+}$ -orbit. The present calculation supports the above discussions. It is found in fig. 3(a) that the ground state, $\frac{1}{2}^{-}$, has a relatively large mass shift experimentally, and found a calculated state which looks corresponding to this state, namely, one calculated value is close to the experimental value of the ground state in the figure. The configuration of this calculated state has the structure of $[1s_{\frac{1}{2}}^{1+} \times 1s_{\frac{1}{2}}^{1+}]_0$.

According to our analysis, such a configuration always yields a large mass shift. As discussed above, with the harmonic oscillator wave function, the three orbits of the $1s0d$ -shell have same mean-square radius, $\langle r^2 \rangle$, and therefore, have same matrix elements of the Coulomb potential inside the nucleus, $r < R$. The wave function of the $1s_{\frac{1}{2}}^{1+}$ -orbit spreads to the “out of nucleus”, $r > R$, compared with the $0d$ -orbits, and the correction of the region of $r > R$ causes an increase of the Coulomb energy. Namely, something like the “nucleon skin” of the $1s_{\frac{1}{2}}^{1+}$ -orbit may be the source of the large Coulomb energy of $1s_{\frac{1}{2}}^{1+}$ -protons, and the large mass shift of this configuration.

In summary, we discussed here that the Coulomb mass shift has a simple relation to the radius of the nuclear wave function. The harmonic oscillator model predicts a parity-dependence of radius of $A = 17$ isoquartets and that of mass shift. The negative parity state has the odd number of nucleons in $0p$ -shell, and the mean-square radius $\langle r^2 \rangle$ of the single-particle orbit depends on its major shell, i.e., the $0p$ - or $1s0d$ -shells. From the similarity between the Coulomb potential inside the nucleus and the operator of the square radius, r^2 , we expect that the Coulomb energy depends on the parity.

On the above lines, a shell model calculation is made, and found to explain experimental behaviour of isovector and isotensor mass shifts. Although the magnitudes of the parity-

dependence in isotensor mass shift disagrees with experiment, it may be attributed to the nuclear charge-independence-breaking force, which has large state-dependent contributions [8]. Relying on the agreement in the isovector mass shifts, we conclude that the parity-dependence is due to the difference of radii between the 0p- and 1s0d-shell orbits. Thus, the present study suggests that further investigations of the Coulomb energies of isospin multiplets will provide information on the radii of the excited states, which are difficult to be determined by the usual experimental procedures.

The ground state is found to have a large isovector mass shift experimentally, and it is consistent with the large value predicted for the configuration which contains the structure $[1s_{\frac{1}{2}}^{+} \times 1s_{\frac{1}{2}}^{+}]_0$. In our analysis, the large mass shift is due to that the wave function of the $1s_{\frac{1}{2}}^{+}$ -orbit spreads out of the surface of the nucleus more than the 0d-orbits. Such a configuration is also suggested in the previous studies [9, 10].

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Figure captions

Fig. 1. The schematic configurations of the positive parity states of the ^{17}Ne shown in (a), and ^{17}N in (b), and the negative parity states of ^{17}Ne in (c), and ^{17}N in (d).

Fig. 2. Another configuration of the positive parity states of ^{17}Ne .

Fig. 3. The isovector, shown in (a), and isotensor in (b), mass shifts. The open and filled circles denote the experimental mass shifts [2] of the positive and negative parity states, respectively. The open and filled diamonds are the calculated ones of the positive and negative parity states, respectively. The open square denotes the configuration of the positive parity state shown in fig. 2. The calculated isovector mass shifts in (a) are given after subtracting 50 keV [6] and n- ^1H mass difference of 782 keV from the Coulomb matrix elements.

Fig. 1

Fig. 2

Fig. 3(a)

Fig. 3(b)