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THEORY OF OCTET ENHANCEMENT AND THE PSEUDOSCALAR
VECTOR MESON INTERACTION

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ABSTRACT

The possibility of a dynamical octet enhancement in the perturbation patterns of the vector meson octet is studied by means of the Dashen-Frautschi technique. For the case in which the vector octet corresponds to a resonance in pseudoscalar-pseudoscalar scattering, the formalism pertinent to this study is developed in detail. Application of an unperturbed $SU(3)$ symmetric amplitude taken from a self-consistent model investigated by the authors shows that the relevant mass coupling shift matrix has an eigenvalue very close to unity both in the eight- and 27-representation ($\alpha_8 = 0.98$, $\alpha_{27} = 0.92$). An appropriate parametrization of the model indicates no qualitative difference in the magnitude of the important eigenvalues of these two representations, thus giving no unambiguous enhancement of the eight over the 27.

1. INTRODUCTION

All the violations of $SU(3)$ symmetry caused by semi-strong electromagnetic and weak interactions seem to follow an octet pattern. This may be due to definite transformation properties of the corresponding terms in the fundamental Lagrangian. However, an alternative explanation, based on a dynamical octet enhancement, has been proposed ¹⁾. In particular, if the dynamics is of the self-consistent type, it can easily provide a possible mechanism through which such an enhancement might come out ^{1),2)}: the bootstrap restrictions imposed by the very strong, $SU(3)$ symmetric, interactions could be able to "choose" among the various possible transformation properties of weaker ones, enhancing only their octet part.

To support the general idea of a bootstrap origin of octet enhancement, Dashen and Frautschi have applied their S matrix perturbation technique ³⁾ to evaluate the semi-strong and electromagnetic mass shifts among the baryon octet and decuplet ⁴⁾, and the parity violating part of the non-leptonic hyperon decays ⁵⁾. These evaluations are very rough, since for instance the static model is used for pseudoscalar meson-baryon scattering in exact $SU(3)$ symmetry, when the ratio of the mean masses of the two multiplets is not so small ($\simeq 0.36$). Furthermore, in contrast to the two aforementioned calculations, which actually led to octet enhancement, a similar one concerning the parity conserving baryon-meson couplings ⁶⁾ gave more ambiguous results. Nevertheless, owing in particular to the fact that the parity conserving coupling problem is much more complicated than the two other ones, one can regard the situation of the bootstrap octet enhancement theory as reasonably good, insofar as baryons are concerned.

One would of course like to know whether a similar statement can be made for vector mesons, and it is the purpose of the present work to give a hint concerning this question. To do so, the possible perturbation patterns of the vector meson bootstrap will be studied, this bootstrap being considered in its simplest possible form, i.e., as operating in PP scattering (P is the pseudoscalar octet).

2.

Apart from the general interest just mentioned, such a study can have bearings in particular on the $\omega - \varphi$ mixing theory. In fact, the usual $\omega - \varphi$ mixing scheme ⁷⁾ leans on the hypothesis that semi-strong interactions have octet transformation properties, but, unfortunately, it does not provide any a posteriori check of this hypothesis.

To be more precise, let us briefly describe this scheme. Adopting for simplicity an effective Hamiltonian, we will write :

$$H = H_1 + H_8 + H_{27} \quad (1.1)$$

where H_1 conserves $SU(3)$ and H_8 (respectively H_{27}) transforms under this group like the centre of an octet (respectively a 27-) representation. The eigenstates of H_1 having $Y=I=0$ are a unitary singlet ω_0 , with mass m_s , and a member of an octet, φ_0 , with mass m . Treating as usual H_8+H_{27} as a perturbation, and keeping only first order terms, one will obtain the masses of the physical ω and φ particles by diagonalizing the mass matrix :

$$M = \begin{vmatrix} m + \langle \varphi_0 | H_8 | \varphi_0 \rangle + \langle \varphi_0 | H_{27} | \varphi_0 \rangle & \langle \varphi_0 | H_8 | \omega_0 \rangle \\ \langle \omega_0 | H_8 | \varphi_0 \rangle & m_s \end{vmatrix} \quad (1.2)$$

since, due to the fact that ω_0 is a singlet :

$$\langle \varphi_0 | H_{27} | \omega_0 \rangle = \langle \omega_0 | H_8 | \omega_0 \rangle = \langle \omega_0 | H_{27} | \omega_0 \rangle \quad (1.3)$$

But the masses of the other octet members (K^* and ρ) will at the same time be given by :

$$m_{K^*} = m + \langle K^* | H_8 | K^* \rangle + \langle K^* | H_{27} | K^* \rangle$$

(1.4)

$$m_\rho = m + \langle \rho | H_8 | \rho \rangle + \langle \rho | H_{27} | \rho \rangle$$

[Here, $|K^*\rangle$ and $|\rho\rangle$ represent pure SU(3) states, as did $|\varphi_0\rangle$ and $|\omega_0\rangle$ above.] The matrix elements of H_8 and H_{27} that appear in (1.4) are respectively proportional to $\langle \varphi_0 | H_8 | \varphi_0 \rangle$ and $\langle \varphi_0 | H_{27} | \varphi_0 \rangle$, the proportionality factors being just ratios of two Clebsch-Gordan coefficients. If then one postulates that H_{27} is negligible against H_8 , the knowledge of m_{K^*} and m_ρ allows m and $\langle \varphi_0 | H_8 | \varphi_0 \rangle$ to be calculated, and the (1,1) element of the $\mathcal{M}$ matrix is fixed. One is finally left with two parameters, m_s and $\langle \varphi_0 | H_8 | \omega_0 \rangle$, to fit the two physical masses m_ω and m_φ ; no test of this octet hypothesis is then possible⁸⁾. On the other hand, if H_{27} happened to be of the same order of magnitude as H_8 , one would have more parameters than experimental numbers, and nothing definite could be said.

The relations between an eventual octet enhancement in the vector meson bootstrap and $\omega - \varphi$ mixing are quite intimate. In this respect, an important drawback of the dynamical model which will be used here is that it deals only with the vector octet, since the singlet is not linked to two pseudoscalar mesons (the representation 1 formed with two P octets is symmetric under their interchange, and is then forbidden by Bose statistics in the $= 1$ wave). From the sketch of the $\omega - \varphi$ mixing scheme that was just given, one sees that the eventual existence of a 27-term in the Hamiltonian affects only the (1,1) element of the mass matrix [see formulae (1.2) and (1.3)], i.e., the mass formula inside the octet. One can then hope that a separate study of symmetry breaking within this octet is meaningful, and look whether a dynamical

octet enhancement comes out of such a study, in support of the currently accepted hypothesis. However, one should bear in mind that the situation could be exactly reversed : perhaps, even in a bootstrap model, octet enhancement is due to the mixing of the octet and singlet, in which case we would miss it here.

The main difficulty, in applying the Dashen-Frautschi perturbation theory ³⁾, is that the unperturbed, SU(3) invariant, bootstrap solution of the problem, which forms the basis of the calculation, is actually not known. In the case of baryons, provided that one is prepared to accept the static model as a reasonable approximation, it seems that the results do not strongly depend on the detailed properties of this unperturbed solution; so that, in the calculations of Refs. ^{4),5)} and ⁶⁾, a very simple form was taken for it. On the contrary, it is very unlikely that such a simple form can be used in the present case of PP scattering. Therefore, we have first investigated the SU(3) invariant vector meson bootstrap ⁹⁾, in order to get a reasonable unperturbed amplitude.

In these bootstrap models, the possibility of a dynamical enhancement is linked to the interplay of crossed and direct channels. If we symbolically write $\delta\mu$ and δm for the mass shifts produced by semi-strong interactions in the vector and pseudoscalar meson octets, respectively, and δg for the corresponding coupling shifts, we will obtain equations of the form

$$\begin{aligned}\delta m &= \alpha \delta m + \beta \delta g + \gamma \delta\mu + \delta \\ \delta g &= \alpha' \delta m + \beta' \delta g + \gamma' \delta\mu + \delta'\end{aligned}\tag{1.5}$$

and δ and δ' are eventual "driving terms" ³⁾. We will treat here $\gamma\delta\mu$ and $\gamma'\delta\mu$ as effective driving terms, that is consider $\delta\mu$ as given in the evaluation of δm and δg . It should, however, be stressed that such a procedure is difficult to justify. Furthermore, it has an important drawback : the component of $\delta\mu$ along 27 is

not zero (since the Gell-Mann-Okubo mass formula is not so well obeyed within the pseudoscalar octet), but it is certainly much smaller than the octet one; it is then not evident that the inhomogeneous terms $\gamma\delta_\mu + \delta$ and $\gamma'\delta_\mu + \delta'$ are of the same order in 8 and 27, since radically different situations could show up according to the relative magnitudes of $\gamma\delta_\mu$ ($\gamma'\delta_\mu$) and δ (δ'). If it were possible, one should supplement the system (1.5) by a third equation giving δ_μ , i.e., obtained from a dynamical model of the pseudoscalar mesons. However, it is not very clear of which particles the P's are essentially bound states, so that one does not know how to write such an equation, furthermore, it would not necessarily "close" the system, since it could bring in new shifts. This is why we will restrict ourselves here to an investigation of equations (1.5) and study the matrix ¹⁰⁾:

$$M = \begin{vmatrix} \alpha & \beta \\ \alpha' & \beta' \end{vmatrix} \quad (1.6)$$

Octet enhancement, if it comes from system (1.5), will manifest itself by an eigenvalue of this matrix close to 1 in the octet case, and not in the 27 one ²⁾.

Another point should be mentioned before describing the calculations: in mesonic supermultiplets, the mass shifts are so large that the validity of first order perturbation theory can be seriously questioned. However, besides the arguments usually invoked in this respect ¹¹⁾, one can add here that, being essentially interested in eventual instabilities of the vector meson system with respect to SU(3) perturbations, we can treat these in first order even if the final departure from the symmetry is not small.

The material contained in this paper is presented in the following way: Section 2 displays the general Dashen-Frautschi formulae for the case of a resonance and applies them to our specific problem. Section 3 deals with the calculation of the matrix elements of M [Eq. (1.6)];

they are obtained as products of a generalized $SU(3)$ crossing coefficient and a dynamical factor, the value of which depends on the specific model considered. Section 4 gives the final numerical results when the dynamical factors are calculated from the bootstrap solution of Ref. ⁹⁾. Section 5 gathers the main conclusions and discusses them. Finally, the details of the calculations are described in two Appendices.

2. BASIC FORMULAE

When considering the vector meson bootstrap in the limit of exact SU(3) symmetry, one has to face the problem of defining unperturbed masses for the relevant multiplets. In the spirit of the considerations which will be developed in Section 3, it seems that we should take here the mean mass μ of the P octet as given by :

$$\mu^2 = \frac{1}{8} [4 m_K^2 + 3 (m_\eta^2 + m_\eta'^2)] \quad (2.1)$$

which results in :

$$\mu \approx 410 \text{ Mev} \quad (2.2)$$

For the vector meson octet, one finds the additional difficulty that the mass of its $Y=0$, $I=0$ member is unknown. If one takes it as verifying the Gell-Mann-Okubo mass formula, one gets for the mean mass :

$$m^2 = \frac{1}{8} [4 m_K^2 + 3 m_\rho^2 + m_{\phi_0}^2] = \frac{1}{3} (2 m_K^2 + m_\rho^2) \quad (2.3)$$

So that numerically :

$$m \approx 850 \text{ Mev} \quad (2.4)$$

It therefore seems that, even in the SU(3) symmetric problem, the vector mesons are resonances in PP scattering with :

$$\frac{m^2}{\mu^2} \approx 4.3 \quad (2.5)$$

If this is so, one has to adapt the Dashen-Frautschi formulae ³⁾ to the case of a resonance. To see how this can be done in a simple way, we will derive them by a method slightly different from the one originally used.

8.

A) General formulae for a resonance (one-channel case)

Consider the elastic partial wave scattering amplitude $A(s)$ verifying the unitarity condition :

$$\text{Im } A(s) = -\rho(s) \quad \text{for } s \geq s_t \quad (2.6)$$

where s_t is the scattering threshold and $\rho(s)$ a known kinematical function. In the unperturbed situation, the discontinuity of A across its left-hand cut is a given function $\mathcal{G}(s)$:

$$\text{Im } A(s) = \mathcal{G}(s) \quad \text{for } s \leq s_L \quad (2.7)$$

s_L being the beginning of the left-hand cut.

One writes $A(s)$ in the usual form :

$$A(s) = N(s)/D(s) \quad (2.8)$$

the imaginary parts of N and D being :

$$\text{Im } N(s) = \mathcal{G}(s)D(s) \quad \text{for } s \leq s_L \quad (2.9a)$$

$$\text{Im } D(s) = -\rho(s)N(s) \quad \text{for } s \geq s_t \quad (2.9b)$$

Suppose that one introduces a perturbation, so that $\mathcal{G}(s)$ and $\rho(s)$ respectively become $\mathcal{G} + \delta\mathcal{G}$ and $\rho + \delta\rho$. Equations (2.9) are then modified to :

$$\text{Im } (N + \delta N) = \mathcal{G}D + \mathcal{G}\delta D + D\delta\mathcal{G} \quad (2.10a)$$

$$\text{Im } (D + \delta D) = -\rho N - \rho\delta N - N\delta\rho \quad (2.10b)$$

If one defines

$$\bar{J}(s) = D(s) \delta N(s) - N(s) \delta D(s) \quad (2.11)$$

the imaginary parts of this new function on the left- and the right-hand cuts are, respectively :

$$\text{Im}_L \bar{J}(s) = D^2(s) \delta \sigma(s) \quad (2.12a)$$

$$\text{Im}_R \bar{J}(s) = N^2(s) \delta \rho(s) \quad (2.12b)$$

Formulae (2.12) enable one, in principle, to calculate $J(s)$ from the dispersion relation :

$$\bar{J}(s) = \frac{1}{\pi} \int_L ds' \frac{D^2(s') \delta \sigma(s')}{s' - s} + \frac{1}{\pi} \int_R ds' \frac{N^2(s') \delta \rho(s')}{s' - s} \quad (2.13)$$

Let us call $F_R(s)$ and $F_I(s)$ the real and imaginary parts of the function $F(s)$ respectively. We will define the position s_R and reduced width R of a resonance occurring in the unperturbed amplitude in such a way that :

$$D_R(s_R) = 0 \quad (2.14a)$$

$$R = - \frac{N(s_R)}{D'_R(s_R)} \quad (2.14b)$$

where D'_R denotes the derivative of D_R with respect to s . [If one has to deal with a bound state, $s_R < s_t$ and D_R coincides with D itself.]

After the perturbation is switched on, the new position $s_R + \delta s_R$ and new reduced width $R + \delta R$ will be given by :

$$D_R(s_R + \delta s_R) + \delta D_R(s_R + \delta s_R) = 0 \quad (2.15a)$$

$$R + \delta R = - \frac{N(s_R + \delta s_R) + \delta N(s_R + \delta s_R)}{D'_R(s_R + \delta s_R) + \delta D'_R(s_R + \delta s_R)} \quad (2.15b)$$

To first order in the perturbation, these equations write :

$$\delta s_R = - \frac{\delta D_R(s_R)}{D'_R(s_R)} \quad (2.16a)$$

$$\delta R = - \frac{N(s_R)}{D'_R(s_R)} \left[\left(\frac{N'(s_R)}{N(s_R)} - \frac{D''_R(s_R)}{D'_R(s_R)} \right) \delta s_R + \frac{\delta N(s_R)}{N(s_R)} - \frac{\delta D'_R(s_R)}{D'_R(s_R)} \right] \quad (2.16b)$$

It is then straightforward, using the real part of (2.11) and its derivative with respect to s , to transform equations (2.16) into :

$$\delta s_R = \frac{J_R(s_R)}{N(s_R) D'_R(s_R)} = - \frac{1}{R} \frac{J_R(s_R)}{D'^2_R(s_R)} \quad (2.17a)$$

$$\delta R = \frac{D''_R(s_R)}{D'^3_R(s_R)} J_R(s_R) - \frac{J'_R(s_R)}{D'^2_R(s_R)} \quad (2.17b)$$

Formulae (2.17) are exactly the same as in Ref. 3). With the definitions (2.14), the bound state and resonance cases are completely similar, and one only has to take the real parts of the various functions involved, to go from the former to the latter.

B) Application to the vector meson bootstrap

The problem we want to study here is not, strictly speaking, a one-channel one. In fact, let us consider PP scattering in the P wave and in a given state of isospin I and hypercharge Y , $i = (Y, I)$. Although the corresponding vector meson appears only in the representation 8_A , scattering can also occur in other representations β (10 and $\overline{10}$). In the unperturbed $[SU(3) \text{ invariant}]$ situation, however, these various possible channels are completely decoupled, so that the N and D matrices are diagonal. Taking for simplicity the case in which only one representation β can be realized besides 8_A , we will have :

$$D^i(s) = \begin{vmatrix} D_8(s) & 0 \\ 0 & D_\beta(s) \end{vmatrix} \quad (2.18)$$

and a similar expression for $N(s)$. [Note that D_8 and D are independent of i .]

We know that

$$D_{8_R}(s_R) = 0 \quad (2.19)$$

When $SU(3)$ is no longer strictly conserved, 8 and β are coupled, and :

$$D(s) + \delta D(s) = \begin{vmatrix} D_8(s) + \delta D_8^i(s) & \delta D_{8\beta}^i(s) \\ \delta D_{\beta 8}^i(s) & D_\beta(s) + \delta D_\beta^i(s) \end{vmatrix} \quad (2.20)$$

The shift δs_R^i in the position of the resonance in state i is then determined by the condition :

$$\det [D_R + \delta D_R^i]_{s=s_R + \delta s_R^i} = 0 \quad (2.21)$$

which immediately leads to a formula analogous to (2.17a), D being replaced by D_g :

$$\delta s_R^i = - \frac{\delta D_{gR}^i(s_R)}{D_{gR}^i(s_R)} = - \frac{1}{R} \frac{J_{gR}^i(s_R)}{D_{gR}^i(s_R)} \quad (2.22)$$

where

$$J_g^i(s) = \frac{1}{\pi} \int_L ds' \frac{D_g^2(s') \delta \phi_g^i(s')}{s' - s} + \frac{1}{\pi} \int_R ds' \frac{\eta_g^2(s') \delta \phi_g^i(s')}{s' - s} \quad (2.23)$$

In a similar way one can obtain the perturbed residue for the (g, g) matrix element by an expression analogous to (2.17b) :

$$\delta R_g^i = 2g \delta g_g^i = \frac{D_{gR}^{\prime\prime}(s_R)}{D_{gR}^{\prime 3}(s_R)} J_{gR}^i(s_R) - \frac{J_{gR}^{\prime i}(s_R)}{D_{gR}^{\prime 3}(s_R)} \quad (2.24)$$

(g is the unperturbed coupling constant, δg_g^i being its perturbation in the i channel).

Besides these modifications, the vector meson i acquires a first order coupling to the state (β, i) which is given by :

$$\delta R_{\beta}^i = g \delta g_{\beta}^i = - \frac{J_{\beta R}^i(s_R)}{D'_{\beta R}(s_R) D_{\beta R}(s_R)} \quad (2.25)$$

with :

$$J_{\beta}^i(s) = \frac{1}{\pi} \int_L ds' \frac{D_R(s') D_{\beta}(s') \delta \sigma_{\beta R}^i(s')}{s' - s} + \frac{1}{\pi} \int_R ds' \frac{N_R(s') N_{\beta}(s') \delta \sigma_{\beta R}^i(s')}{s' - s} \quad (2.26)$$

Formulae (2.22), (2.24) and (2.25) will be used in computing the elements of the matrix M defined above [Eq. (1.6)].

3. THE MATRIX M FOR THE 8 AND THE 27 CASES

A) Notation

We have to consider two-index and three-index quantities, which we will respectively denote by $\delta_{m_{ij}}$ and $\delta_{g_{ijk}}$ [i, j, k are $SU(3)$ indices, the conventions being defined in Figs. 1 and 2; δ_m stands for the shift of the square of the mass].

Fig. 1

$\delta_{m_{ij}}$

Fig. 2

$\delta_{g_{ijk}}$

As we know that the various possible $SU(3)$ transformation properties of these quantities will be separated in the equations ³⁾, it is convenient to expand $\delta_{m_{ij}}$ and $\delta_{g_{ijk}}$ in irreducible tensors of $SU(3)$:

$$\delta_{m_{ij}} = \sum_{N, \Gamma, n} e_{ij}^{N, \Gamma, n} \delta_m^{N, \Gamma, n} \quad (3.1a)$$

$$\delta_{g_{ijk}} = \sum_{\beta\gamma, N, \Gamma, n} e_{ijk}^{\beta\gamma, N, \Gamma, n} \delta_{g, \beta\gamma}^{N, \Gamma, n} \quad (3.1b)$$

As the first index of each quantity represents an outgoing particle (see Figs. 1 and 2), one has

$$e_{ij}^{Nr,n} = (-1)^i \begin{vmatrix} 8 & 8 & Nr \\ -i & j & n \end{vmatrix} \quad (3.2a)$$

$$e_{ijk}^{\beta\gamma,Nr,n} = (-1)^i \sum_{\gamma} \begin{vmatrix} 8 & 8 & \beta\gamma \\ j & k & \gamma \end{vmatrix} \begin{vmatrix} 8 & 8 & Nr \\ -i & \gamma & n \end{vmatrix} \quad (3.2b)$$

$$\begin{vmatrix} \mu_1 & \mu_2 & \mu_3 \\ \nu_1 & \nu_2 & \nu_3 \end{vmatrix}$$

stands for the $SU(3)$ Clebsch-Gordan coefficient combining the number ν_1 of the representation μ_1 with ν_2 of μ_2 to give ν_3 of μ_3 , and :

$$(-1)^i = (-1)^{i - Y_i + I_{3i}} \quad (3.3)$$

More generally, we will adopt here the notations and conventions of Ref. 12).

Conservation of electric charge implies :

$$(-1)^n = 1 \quad (3.4)$$

Furthermore, we are essentially interested here in semi-strong violations of $SU(3)$ symmetry, in which case :

$$Y_n = I_n = 0 \quad (3.5)$$

and for simplicity we will denote the corresponding quantities by δ_m^{Nr} and $\delta_{\beta\gamma}^{Nr}$.

B) Independent parameters

Not all of the parameters δm^{Nr} and $\delta g_{\beta\gamma}^{Nr}$ are independent because of the restrictions imposed by conservation laws. For δm , charge conjugation implies that :

$$\delta m_{-i-i} = \delta m_{i-i} \quad (3.6)$$

which can immediately be translated into :

$$\delta m^{Nr} = \xi_3(8, 8, Nr) \delta m^{Nr} \quad (3.7)$$

As a consequence ¹³⁾ :

$$\delta m^{8_A} = 0 \quad (3.8)$$

and the only free parameters are δm^{8_S} and δm^{27} .

To derive the equivalent conditions on the δg 's, it is convenient to use an effective Hamiltonian, which we will take as :

$$\delta H = i\sqrt{12\pi} \sum_{i,j,k} \delta g_{ijk} [V_i^\mu]^\dagger P_j \partial_\mu P_k \quad (3.9)$$

First, the two pseudoscalars, being identical and occurring in a P wave, one must impose

$$\delta g_{ijk} = -\delta g_{ikj} \quad (3.10)$$

which means that only the antisymmetric representations $\beta\gamma = 8_A, 10, \overline{10}$ are allowed :

$$\delta g_{\beta\gamma}^{Nr} = -\xi_1(8, 8, \beta\gamma) \delta g_{\beta\gamma}^{Nr} \quad (3.11)$$

In the same way, Hermiticity, T and C invariance of the Hamiltonian (3.9) are ensured if the δg 's are real and :

$$\delta g_{j\bar{k}} = -\delta g_{-j, -\bar{k}} \quad (3.12)$$

Indeed, with the phase conventions of Ref. ¹²⁾, the pseudoscalar and vector meson octet members verify :

$$P_j^\dagger = (-1)^j P_{\bar{j}} \quad [V_i^\mu]^\dagger = (-1)^i V_{-\bar{i}}^\mu \quad (3.13a)$$

$$C P_j C^{-1} = (-1)^j P_{\bar{j}} \quad C V_i^\mu C^{-1} = (-1)^{i+1} V_{-\bar{i}}^\mu \quad (3.13b)$$

Now, from (3.1b), one knows that :

$$\begin{aligned} \delta g_{-i, -\bar{j}, -\bar{k}} &= (-1)^i \sum_{\beta' \gamma' N' \Gamma'} \begin{vmatrix} 8 & 8 & \beta' \gamma' \\ -j & -k & -i \end{vmatrix} \begin{vmatrix} 8 & \beta' & N' \Gamma' \\ i & i & 0 \end{vmatrix} \delta g_{\beta' \gamma' N' \Gamma'}^{N' \Gamma'} = \\ &= (-1)^i \sum_{\beta' \gamma' N' \Gamma'} \zeta_3(8, 8, \beta' \gamma') \zeta_2(8, \beta', N' \Gamma') \begin{vmatrix} 8 & 8 & \beta' \gamma' \\ j & k & i \end{vmatrix} \begin{vmatrix} 8 & \beta' & N' \Gamma' \\ -i & i & 0 \end{vmatrix} \delta g_{\beta' \gamma' N' \Gamma'}^{N' \Gamma'} \end{aligned} \quad (3.14)$$

Comparing this expression with Eq. (3.1b), one sees that condition (3.12) writes :

$$\delta g_{\beta' \gamma' N' \Gamma'}^{N' \Gamma'} = - \zeta_3(8, 8, \beta' \gamma') \zeta_3(8, \beta', N' \Gamma') \delta g_{\beta' \gamma' N' \Gamma'}^{N' \Gamma'} \quad (3.15)$$

Table I of Ref. ¹²⁾ can then be used to find that :

$$\delta g_{8_A}^{8_A} = 0 \quad ; \quad \delta g_{10}^8 = \delta g_{\bar{10}}^8 \quad ; \quad \delta g_{10}^{27} = -\delta g_{\bar{10}}^{27} \quad (3.16)$$

One is then left with two parameters for $N=8$, $\delta g_{8_A}^{8_S}$ and $1/\sqrt{2}(\delta g_{10}^8 + \delta g_{\bar{10}}^8)$ and two for $N=27$, $\delta g_{8_A}^{27}$ and $1/\sqrt{2}(\delta g_{10}^{27} - \delta g_{\bar{10}}^{27})$.

c) SU(3) crossing coefficients

$\delta_{m_{ii}}$ is the quantity that was called δs_R^i in Section 2, and is consequently given by formula (2.22). As for δg_8^i and δg_β^i appearing in (2.24) and (2.25), they describe the perturbed couplings of vector meson i to the $(8_{A,i})$ and (β,i) PP states respectively, so that they correspond to :

$$\delta g_\beta^i = \sum_{jk} \begin{vmatrix} 8 & 8 & \beta \\ j & k & i \end{vmatrix} \delta g_{jk}^i \quad (3.17)$$

(a similar expression holds for δg_8^i).

Since we are interested in the influence of δm and δg coming from the crossed channels on those appearing in the direct one, in expressions (2.23) and (2.26) only the integrals over the left-hand cut need be considered. The calculation of the relevant $\delta\sigma$'s is given in detail in Appendix I and applied in Appendix II to the evaluation of the matrix M . The results are :

$$\frac{\delta m^N}{m^2} = I^{mm} A^N \frac{\delta m^N}{m^2} + I^{gg} \sum_\beta B_\beta^N \frac{\delta g_\beta^N}{g} + D^N \quad (3.18a)$$

$$\frac{\delta g_\beta^N}{g} = I_\beta^{mg} B_\beta'^N \frac{\delta m^N}{m^2} + I_\beta^{gg} \sum_{\beta'} C_{\beta\beta'}^N \frac{\delta g_{\beta'}^N}{g} + D_\beta'^N \quad (3.18b)$$

with

$$I^{mm} = \frac{6}{D_{8R}^{13}(v_R)} \int_L \frac{dv'}{v' - v_R} D_8^2(v') \frac{1}{v'^2} \left(5 + \frac{m^2 + 2\mu^2}{v'} \right) \quad (3.19a)$$

$$I^{gm} = \frac{12}{m^2} \frac{1}{D_{8R}^{13}(v_R)} \int_L \frac{dv'}{v' - v_R} \frac{D_8^2(v')}{v'} \left(1 + \frac{m^2}{2v'} \right) \left(4 + \frac{m^2 + 4\mu^2}{2v'} \right) \quad (3.19b)$$

$$I_8^{mq} = \frac{3}{4} \frac{m^2}{D_{8R}^{13}(v_R)} \int_L \frac{dv'}{(v' - v_R)^2} D_8^2(v') \frac{1}{v'^2} \left(5 + \frac{m^2 + 2\mu^2}{v'} \right) - \\ - \frac{3}{4} m^2 \frac{D_{8R}''(v_R)}{D_{8R}^{13}(v_R)} \int_L \frac{dv'}{v' - v_R} D_8^2(v') \frac{1}{v'^2} \left(5 + \frac{m^2 + 2\mu^2}{v'} \right) \quad (3.19c)$$

$$I_\beta^{mq} = \frac{3}{2} \frac{m^2}{D_{8R}^{13}(v_R) D_{\beta R}(v_R)} \int_L \frac{dv'}{v' - v_R} D_8(v') D_\beta(v') \frac{1}{v'^2} \left(5 + \frac{m^2 + 2\mu^2}{v'} \right) \quad \beta \neq 8 \quad (3.19d)$$

$$I_8^{qq} = \frac{3}{2} \frac{1}{D_{8R}^{13}(v_R)} \int_L \frac{dv'}{(v' - v_R)^2} D_8^2(v') \frac{1}{v'} \left(1 + \frac{m^2}{2v'} \right) \left(4 + \frac{m^2 + 4\mu^2}{2v'} \right) - \\ - \frac{3}{2} \frac{D_{8R}''(v_R)}{D_{8R}^{13}(v_R)} \int_L \frac{dv'}{v' - v_R} D_8^2(v') \frac{1}{v'} \left(1 + \frac{m^2}{2v'} \right) \left(4 + \frac{m^2 + 4\mu^2}{2v'} \right) \quad (3.19e)$$

$$I_\beta^{qq} = \frac{3}{D_{8R}^{13}(v_R) D_{\beta R}(v_R)} \int_L \frac{dv'}{v' - v_R} D_8(v') D_\beta(v') \frac{1}{v'} \left(1 + \frac{m^2}{2v'} \right) \left(4 + \frac{m^2 + 4\mu^2}{2v'} \right) \quad \beta \neq 8 \quad (3.19f)$$

ν_R is the position of the resonance in the $\nu \equiv q^2$ plane :

$$\nu_R = \frac{m^2}{4} - \mu^2 \quad (3.20)$$

The coefficients A, B, B' and C have the following expressions (see Appendix II for a detailed derivation) :

$$A^N = \sum_{i \nu a b c d} \frac{(-1)^{i+\nu}}{i \nu a b c d} \begin{vmatrix} 8 & 8 & N \\ -i & i & 0 \end{vmatrix} \begin{vmatrix} 8 & 8 & 8_A \\ a & b & i \end{vmatrix} \begin{vmatrix} 8 & 8 & 8_A \\ c & d & i \end{vmatrix} (-1)^{b+c} \begin{vmatrix} 8 & 8 & 8_A \\ a & -c & \nu \end{vmatrix} \begin{vmatrix} 8 & 8 & 8_A \\ b & -d & -\nu \end{vmatrix} \begin{vmatrix} 8 & 8 & N \\ -\nu & \nu & 0 \end{vmatrix} \quad (3.21a)$$

$$B_b^N = 2 B_\beta^N = 2 \sum_{i \nu a b c d} \frac{(-1)^{i+\nu}}{i \nu a b c d} \begin{vmatrix} 8 & 8 & N \\ -i & i & n \end{vmatrix} \begin{vmatrix} 8 & 8 & 8_A \\ a & b & i \end{vmatrix} \begin{vmatrix} 8 & 8 & 8_A \\ c & d & i \end{vmatrix} (-1)^{b+c} \begin{vmatrix} 8 & 8 & 8_A \\ a & -c & -\nu \end{vmatrix} \begin{vmatrix} 8 & 8 & \beta \\ b & -d & \nu \end{vmatrix} \begin{vmatrix} 8 & 8 & N \\ -\nu & \nu & n \end{vmatrix} \quad (3.21b)$$

$$\frac{1}{\beta \beta'} A^N = 2 \sum_{i \nu a b c d} \frac{(-1)^{i+\nu}}{i \nu a b c d} \begin{vmatrix} 8 & \beta & N \\ -i & i & 0 \end{vmatrix} \begin{vmatrix} 8 & 8 & \beta \\ a & b & i \end{vmatrix} \begin{vmatrix} 8 & 8 & 8_A \\ c & d & i \end{vmatrix} (-1)^{b+c} \begin{vmatrix} 8 & 8 & 8_A \\ a & -c & -\nu \end{vmatrix} \begin{vmatrix} 8 & 8 & \beta' \\ b & -d & \nu \end{vmatrix} \begin{vmatrix} 8 & \beta' & N \\ -\nu & \nu & 0 \end{vmatrix} \quad (3.21c)$$

They can easily be calculated from formula (AII.6) of Appendix II: the results are :

$$A^8 = 0 \quad A^{27} = -\frac{5}{18}$$

$$B^8 = 2 \quad B'^8 = 0$$

$$B^{27} = 2 \quad B'^{27} = -\frac{5}{9}$$

$$B_{\frac{10+\overline{10}}{\sqrt{2}}}^8 = 2 \quad B_{\frac{10+\overline{10}}{\sqrt{2}}}^{'8} = -\frac{1}{\sqrt{2}}$$

$$B_{\frac{10-\overline{10}}{\sqrt{2}}}^{27} = 2 \quad B_{\frac{10-\overline{10}}{\sqrt{2}}}^{'27} = -\frac{2\sqrt{2}}{9}$$

$$C_{88}^8 = 0$$

$$C_{88}^{27} = -\frac{5}{9}$$

(3.22)

$$C_{8 \frac{10+\overline{10}}{\sqrt{2}}}^8 = C_{\frac{10+\overline{10}}{\sqrt{2}}}^8 = -\frac{1}{\sqrt{2}}$$

$$C_{8 \frac{10-\overline{10}}{\sqrt{2}}}^{27} = C_{\frac{10-\overline{10}}{\sqrt{2}}}^{27} = -\frac{2\sqrt{2}}{9}$$

$$C_{\frac{10+\overline{10}}{\sqrt{2}} \frac{10-\overline{10}}{\sqrt{2}}}^8 = 0$$

$$C_{\frac{10-\overline{10}}{\sqrt{2}} \frac{10-\overline{10}}{\sqrt{2}}}^{27} = -\frac{5}{18}$$

4. NUMERICAL RESULTS

The matrices we want to study are the following : for $N=8$:

$$M^8 = \begin{vmatrix} 0 & 0 & -\frac{1}{\sqrt{2}} I^{gm} \\ 0 & 0 & -\frac{1}{\sqrt{2}} I^{gg} \\ -\frac{1}{2\sqrt{2}} I_{10}^{mg} & -\frac{1}{\sqrt{2}} I_{10}^{gg} & 0 \end{vmatrix} \quad (4.1)$$

where the parameters are taken in the order $(\delta m^8/m^2)$, $(\delta g_{8A}^8/g)$ and

For $N=27$

$$M^{27} = \begin{vmatrix} -\frac{5}{18} I^{mm} & -\frac{5}{9} I^{gm} & -\frac{2\sqrt{2}}{9} I^{gm} \\ -\frac{5}{18} I^{mg} & -\frac{5}{9} I^{gg} & -\frac{2\sqrt{2}}{9} I^{gg} \\ -\frac{\sqrt{2}}{9} I_{10}^{mg} & -\frac{2\sqrt{2}}{9} I_{10}^{gg} & -\frac{5}{18} I_{10}^{gg} \end{vmatrix} \quad (4.2)$$

the order being $\frac{\delta m^{27}}{m^2}$, $\frac{1}{g} \delta g_{8A}^{27}$, $\frac{1}{g} \delta g_{\frac{10-10}{\sqrt{2}}}^{27}$

The three eigenvalues of M^8 are

$$\alpha_1^8 = 0, \quad \alpha_3^8 = -\alpha_2^8 = \left[\frac{1}{2} I_{10}^{gg} I_8^{gg} + \frac{1}{4} I^{gm} I_{10}^{mg} \right]^{1/2} \quad (4.3)$$

Those of M^{27} are the roots of a third equation :

$$\alpha^3 + a\alpha^2 + b\alpha + c = 0 \quad (4.4)$$

with

$$a = \frac{5}{18} [I^{mm} + I_{10}^{qq} + 2 I_8^{qq}]$$

$$b = \frac{1}{324} [18 I_{10}^{qq} I_8^{qq} + 25 I^{mm} (I_{10}^{qq} + 2 I_8^{qq}) - 2 I^{qm} (8 I_{10}^{mq} + 25 I_8^{mq})]$$

$$(4.5)$$

$$c = \frac{5 I_{10}^{qq}}{324} [I^{mm} I_8^{qq} - I_8^{mq} I^{qm}]$$

In Ref. ⁹⁾, a model for the unperturbed vector meson bootstrap was described. The input information was a left-hand discontinuity created by the exchange of a vector meson, with a sharp cut-off at $\nu = -\Lambda$ in order to have a Fredholm integral equation. The first point of interest here is that the projection of the vector meson exchange diagrams on the 10 and $\overline{10}$ direct channels vanishes, as is well known ¹⁴⁾. This means that, in this rough model, N_{10} is identically zero, so that $D_{10}(\nu) \equiv 1$. All the other quantities appearing in expressions (3.19) can be taken from the results of Ref. ⁹⁾ [in particular, $D_{8R}'(\gamma_R) \simeq -0.33$ and $D_{8R}'' \simeq 0.45$], which gives the following numerical values for the various I 's :

$$I^{mm} \simeq -4.68$$

$$I^{qm} \simeq 3.83$$

$$I_8^{mq} \simeq -2.70$$

$$I_{10}^{mq} \simeq 3.66$$

$$I_8^{qq} \simeq 2.53$$

$$I_{10}^{qq} \simeq -2.00$$

(4.6)

Replacing these into formulae (4.3) and (4.4) yields the eigenvalues :
for M^8 :

$$\begin{aligned}\alpha_1^8 &= 0 \\ \alpha_3^8 &= -\alpha_2^8 \simeq 0.98\end{aligned}\tag{4.7}$$

and, for M^{27} :

$$\begin{aligned}\alpha_1^{27} &\simeq -0.56 \\ \alpha_2^{27} &= 0.09 \\ \alpha_3^{27} &\simeq 0.92\end{aligned}\tag{4.8}$$

Taken literally, these results would imply that the shifts transforming according to the 8 representation receive a significant enhancement over the 27 ones ($1-\alpha_3^8 \simeq 0.02$ and $1-\alpha_3^{27} \simeq 0.08$). However, the relevant eigenvalues of M^8 and M^{27} differ by less than 10%; since in our dynamical model important contributions have been left out, this difference cannot be claimed to imply octet enhancement in a reasonably unambiguous way.

The inadequacy of our model is clearly manifest in the fact that the self-consistent coupling exceeds the experimental value by more than a factor of two [see Ref. ⁹⁾]. Since this coupling is inversely proportional to $D'_{8R}(\gamma_R)$, one possibility is that the model underestimates, to some extent, the value of this derivative.

The dynamical factors which enter the present calculation [Eqs. (3.19)] depend on $D'_{8R}(\gamma_R)$, $D''_{8R}(\gamma_R)$ and on the function $D_8(\gamma)$ along the left-hand cut. General arguments suggest that for a class of asymptotically vanishing left-hand discontinuities [like that of Ref. ⁹⁾], the shape of $D_8(\gamma)$ is the same. On the other hand, formulae (3.19)

remain unaltered if the D function is multiplied by a constant. The conclusion is that a reasonable way to generalize our dynamical model is to consider $D'_{8R}(\nu_R)$ and $D''_{8R}(\nu_R)$ as two independent parameters, keeping $D_8(\nu)$ unchanged along the left-hand cut. But varying these parameters and studying the corresponding variation of α_3^8 and α_3^{27} , we hope to see whether the similarity of behaviours already found between $N=8$ and $N=27$ is more than an accident :

$$D'_{8R}(\nu_R) = -0.33 d' \quad (4.9)$$

$$D''_{8R}(\nu_R) = 0.45 d''$$

and have varied d' and d'' independently between 0.5 and 2 [$d'=d''=1$ corresponds to no modification in the solution of Ref. 9)]. The variations of α_3^8 and α_3^{27} were found to be quite similar : their values always remained close to each other. In particular, it was impossible to obtain a situation in which α_3^8 lie close to 1 and α_3^{27} different from it by more than 10%.

5. CONCLUSIONS

The investigation carried out in this paper gives no unambiguous indication that an eventual bootstrap origin of the vector meson octet could favour any particular transformation properties for the corresponding semi-strong $SU(3)$ breaking effects. As the dynamical enhancement arguments are "universal" ³⁾ [the matrices M^N that were studied above are independent of the particular member n of the representation N according to which the shifts transform], the same statement holds for electromagnetic violations of $SU(3)$ symmetry.

This conclusion was reached in the following way : in a simple, first order perturbation model as the one used here, the elements of the relevant matrices M^N can be separated ³⁾ into a numerical coefficient, which depends only on group properties [$SU(3)$ Clebsch-Gordan coefficients], multiplying a dynamical factor, the calculation of which necessitates the knowledge of the unperturbed amplitude (more precisely, the features of this unperturbed amplitude that are needed are the value of the D function along the left-hand cut together with the first and second derivatives of its real part at the resonance position). We have evaluated the dynamical factors by using a solution previously obtained in a study of an $SU(3)$ symmetric bootstrap model for the vector meson octet ⁹⁾. The largest eigenvalues were found to be 0.98 for $N=8$ and 0.92 for $N=27$; as these do not differ significantly, it was concluded that the octet representation does not stand out as a more likely transformation property for the perturbation. Furthermore, varying the values of the derivatives of $\text{Re } D$ at resonance confirms the similarity between the 8 and 27 equations : it is not possible, within this model, to have an eigenvalue near 1 in the octet case and not in the 27 one.

As discussed in the introduction, these results are not very satisfactory, from the point of view of a general octet enhancement theory, and therefore also from that of the usual $\omega - \varphi$ mixing scheme. In fact, they fail to support definitely the hypothesis that the relevant semi-strong $SU(3)$ breaking Hamiltonian transforms like the centre of an octet, i.e., that the Gell-Mann - Okubo formula can be used to find the mass of the $Y=I=0$ member of the vector octet.

However, we could not claim that this hypothesis is ruled out by the above considerations. Indeed, even if one believes in the dynamical nature of octet dominance, its origin could very well lie in effects that were not taken into account in the simplified model considered here.

One important such effect is precisely $\omega - \varphi$ mixing itself. Because only the vector meson octet could be obtained in an $SU(3)$ symmetric bootstrap calculation of PP scattering in the $\ell = 1$ wave, we have taken the point of view that, roughly speaking, $\omega - \varphi$ mixing takes place "after" the perturbation of this bootstrap. One cannot, however, exclude the possibility that octet enhancement shows up only when the vector singlet and octet are treated simultaneously on the same footing.

Furthermore, a strong bias can come in through the influence of the external pseudoscalar masses that, due to the lack of a reliable dynamical model for pseudoscalar mesons, was considered here as part of the "driving terms". In fact, if this influence dominates, the effective driving terms will be much smaller in the 27 representation than in the 8 one, and octet dominance in the vector meson system can be just a reflection of the pseudoscalar meson properties. However, if the largest eigenvalues of M^8 and M^{27} are actually near to 1, as was found above, it will be rather difficult to calculate effectively the vector meson mass shifts and the VPP coupling shifts in terms of the pseudoscalar mass differences; in fact, one will need a high precision in the knowledge of the eigenvalues α in order that the relative uncertainty on the quantity $(1-\alpha)$, which enters the calculation, be reasonable. In particular, a very small difference between the largest eigenvalues of M^8 and M^{27} could compensate for the disproportion of the driving terms.

A conservative over-all conclusion of the present work could then be that, in contrast to the case of baryons ⁴⁾, the simplest possible bootstrap model does not lead to a clear-cut octet enhancement for the vector meson system.

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A P P E N D I X I

PERTURBATION OF THE LEFT-HAND DISCONTINUITY

To be able to evaluate the influence of δm and δg appearing in the exchange diagrams on the same quantities coming from the perturbation of the resonance in the direct channel, one has to compute the functions $\delta \sigma_8^i(s)$ and $\delta \sigma_8^i(s)$ [see formulae (2.23) and (2.26)], which represent the modifications of the left-hand discontinuities of the processes $(P+P)_{8A,i} \rightarrow (P+P)_{8A,i}$ and $(P+P)_{\beta,i} \rightarrow (P+P)_{8A,i}$ respectively.

We will write the effective Hamiltonian as :

$$H + \delta H = i\sqrt{12}\pi \sum_{ijk} f_{ijk} [V_\mu^c]^\dagger P_i \gamma_\mu P_k \quad (\text{AI.1})$$

where

$$f_{ijk} = g_{ijk} + \delta g_{ijk} \quad (\text{AI.2})$$

δg_{ijk} is the same as was defined in formulae (3.1b) and (3.9) and g_{ijk} is the SU(3) symmetric unperturbed coupling constant :

$$g_{ijk} = g \begin{vmatrix} 8 & 8 & 8_A \\ j & k & i \end{vmatrix} \quad (\text{AI.3})$$

Let us first consider the direct (s) channel for the scattering

$$P_a + P_b \rightarrow P_c + P_d \quad (\text{AI.4})$$

where the scattering angle θ is defined between the three-momenta of P_a and P_c , and calculate the residue at the vector meson pole (see Fig. 3).

Fig. 3

From the Feynman rules, one obtains the S matrix element :

$$\begin{aligned}
 S^{ab \rightarrow cd} &= \frac{i}{(2\pi)^2} \frac{\delta(P_a + P_b - P_c - P_d)}{4\sqrt{P_a^0 P_b^0 P_c^0 P_d^0}} i\sqrt{12\pi} \left[f_{cab} \epsilon P_b^\mu + f_{cba} \epsilon P_a^\mu \right] \times \\
 &\times \frac{g_{\mu\nu} + \frac{K_\mu K_\nu}{m^2}}{K^2 + m^2} i\sqrt{12\pi} \left[f_{-i-c-d} (-iP_d^\nu) + f_{-i-d-c} (-iP_c^\nu) \right] \quad (A1.5)
 \end{aligned}$$

where summations over μ, ν and i are to be performed. Due to the identity of the scattered particles, if one defines the scattering matrix $\mathcal{T}$ by :

$$S^{i \rightarrow f} = \delta^{fi} + 4i \frac{\delta(P_a + P_b - P_c - P_d)}{\sqrt{P_a^0 P_b^0 P_c^0 P_d^0}} \mathcal{T}^{i \rightarrow f}(s, t, u) \quad (A1.6)$$

and its partial wave projection T_ℓ by :

$$T_\ell(s) = 2\pi \int_{-1}^{+1} d(\cos\theta) \mathcal{T}(s, t, u) P_\ell(\cos\theta) \quad (\text{AI.7})$$

the elastic unitarity condition writes :

$$\text{Im } T_\ell^{-1} = - \frac{q}{\sqrt{s}} \quad (\text{AI.8})$$

where q is the centre-of-mass momentum. One then finds :

$$\mathcal{T}_{ab \rightarrow cd} \underset{s \approx m^2}{\approx} \frac{3}{16\pi} \frac{t-u}{s-m^2} \sum_i f_{iab} f_{i-c-d} \quad (\text{AI.9})$$

$[m]$ should actually be replaced by $m-i(\Gamma/2)$, where Γ is the width of the vector meson; in the crossed channels, however, one can neglect this width].

Only the $\ell = 1$ projection of this scattering amplitude is different from zero :

$$T_1 \underset{s \approx m^2}{\approx} \frac{q^2}{s-m^2} \sum_i f_{iab} f_{i-c-d} \quad (\text{AI.10})$$

Since we are dealing here with a P wave, we can work with the function :

$$A \underset{ab \rightarrow cd}{=} \frac{1}{q^2} T_1 \underset{ab \rightarrow cd}{=} \quad (\text{A.11})$$

without introducing any pole at threshold. We will do so, because, due to formula (AI.3), if $SU(3)$ is conserved ($\delta H = 0$), the projection of this new amplitude onto the $(8_A, i)$ state :

$$A^{8_A, i} = \sum_{c, b, c, d} \left| \begin{array}{ccc} 8 & 8 & 8_A \\ a & b & c \end{array} \right| \left| \begin{array}{ccc} 8 & 8 & 8_A \\ d & d & c \end{array} \right| A^{a b \rightarrow c d} \quad (\text{AI.12})$$

has the simple form :

$$A^{8_A, i}(s) \simeq - \frac{g^2}{s - m^2} \quad (\text{AI.13})$$

As a consequence, the residue R appearing in formula (2.22) is simply replaced by g^2 . In the same way, one sees that δR_8^i and δR_8^i [Eqs. (2.24) and (2.25)] are equal to $2g\delta g_8^i$ and $g\delta g_\beta^i$ respectively, with :

$$\delta g_\beta^i = \sum_{j, k} \left| \begin{array}{ccc} 8 & 8 & \beta \\ j & k & c \end{array} \right| \delta g_{jk}^i \quad (\text{AI.14})$$

where β can also be 8_A .

To go from the direct to the crossed channels, one has to perform the following changes :

$$\text{t channel : } P_b \leftrightarrow \bar{P}_c \quad s \leftrightarrow t \quad (\text{AI.15a})$$

$$\text{u channel : } P_b \leftrightarrow \bar{P}_d \quad s \leftrightarrow u \quad (\text{AI.15b})$$

When only the mass and couplings of the vector mesons are perturbed, one then immediately deduces from Eq. (AI.9) the modifications of the vector meson exchange Born terms :

$$\begin{aligned}
\delta \mathcal{G}_3^{ab \rightarrow cd} = \frac{3}{16\pi} \left\{ \sum_v \delta m_{vv} \left[(-1)^{b+d} q_{va-d} q_{-vb-d} \frac{s-u}{(t-m^2)^2} + \right. \right. \\
+ (-1)^{b+d} q_{va-d} q_{-v-cb} \frac{t-s}{(u-m^2)^2} \Big] + \frac{s-u}{t-m^2} \sum_v (-1)^{b+c} [q_{va-c} \delta q_{-vb-d} \\
+ q_{-vb-d} \delta q_{va-c}] + \frac{t-s}{u-m^2} \sum_v (-1)^{b+d} [q_{va-d} \delta q_{-v-cb} + \\
+ q_{-v-cb} \delta q_{va-d}] \Big\} \quad (\text{AI.16})
\end{aligned}$$

To obtain $\delta \mathcal{G}_{8\beta}^i$, one must first project the initial state on (β, i) and the final one on $(8_A, i)$:

$$\begin{aligned}
\delta \mathcal{G}^{\beta, i \rightarrow 8_A, i} = \frac{3}{16\pi} \sum_{abcdv} \begin{vmatrix} 8 & 8 & \beta \\ a & b & c \end{vmatrix} \begin{vmatrix} 8 & 8 & 8_A \\ c & d & i \end{vmatrix} \left\{ (-1)^{b+c} \right. \\
\times q_{va-c} q_{-vb-d} \delta m_{vv} \left[\frac{s-u}{(t-m^2)^2} + \frac{t-s}{(u-m^2)^2} \right] + \\
+ (-1)^{b+c} [q_{va-c} \delta q_{-vb-d} + q_{-vb-d} \delta q_{va-c}] \left[\frac{s-u}{t-m^2} + \frac{t-s}{u-m^2} \right] \Big\} \quad (\text{AI.17})
\end{aligned}$$

then

$$\delta \mathcal{G}_{8\beta}^i = I_{m_L} \delta A^{\beta, i \rightarrow 8_A, i} = \frac{1}{q^2} I_{m_L} \delta T_1^{\beta, i \rightarrow 8_A, i} \quad (\text{AI.18})$$

[$\delta \mathcal{G}_8^i$ is given by a similar formula, β being replaced by 8_A].

Let us define :

$$\begin{aligned}\delta\sigma(s) &= I_{m_L} \frac{3}{8q^2} \int_{-1}^{+1} d\cos\theta P_1(\cos\theta) \left[\frac{s-u}{t-m^2} + \frac{t-s}{u-m^2} \right] \\ &= -\frac{3}{4q^2} \pi \left(1 + \frac{m^2}{2q^2} \right) \left(4 + \frac{m^2+4t^2}{2q^2} \right)\end{aligned}\quad (\text{AI.19})$$

and
$$\delta\sigma'(s) = \frac{2}{2m^2} \delta\sigma = -\frac{3}{8q^4} \pi \left(5 + \frac{m^2+2t^2}{q^2} \right)$$

(AI.20)

The expression for $\delta\sigma \frac{i}{8\beta}$ ($\beta = 8_A, 10, \text{ or } \overline{10}$) then writes :

$$\begin{aligned}\delta\sigma_{8\beta}^i &= \sum_{abcdv} \left| \begin{smallmatrix} 8 & 8 & \beta \\ a & b & i \end{smallmatrix} \right| \left| \begin{smallmatrix} 8 & 8 & 8_A \\ c & d & i \end{smallmatrix} \right| (-1)^{b+c} \left\{ q_{va-c} q_{-vb-d} \times \right. \\ &\quad \times \delta_{m_{vv}} \delta\sigma'(s) + [q_{va-c} \delta q_{-vb-d} + q_{-vb-d} \delta q_{va-c}] \delta\sigma(s) \left. \right\}\end{aligned}\quad (\text{AI.21})$$

A P P E N D I X II

CALCULATION OF THE MATRIX ELEMENTS OF M^8 AND M^{27}

When the expression (AI.21) of $\delta\sigma_{8\beta}^i$ and the analogous one for $\delta\sigma_{\beta}^i$ are replaced into formulae (2.23) and (2.26), one has to calculate the following integrals :

$$I_{\beta}^{mm} = -\frac{1}{D'_{8R}(m^2)} \frac{1}{\pi} \int_L \frac{ds'}{s'-m^2} D_8^2(s') \delta\sigma'(s') \quad (\text{AII.1a})$$

$$I_{\beta}^{gg} = -\frac{1}{m^2} \frac{1}{D'_{8R}(m^2)} \frac{1}{\pi} \int_L \frac{ds'}{s'-m^2} D_8^2(s') \delta\sigma(s') \quad (\text{AII.1b})$$

$$I_{\beta}^{mg} = -\frac{m^2}{2} \frac{1}{D'_{8R}(m^2)} \frac{1}{\pi} \int_L \frac{ds'}{(s'-m^2)^2} D_8^2(s') \delta\sigma'(s') + \frac{m^2}{2} \frac{D''_{8R}(m^2)}{D'_{8R}(m^2)} \times \frac{1}{\pi} \int_L \frac{ds'}{s'-m^2} D_8^2(s') \delta\sigma'(s') \quad (\text{AII.1c})$$

$$I_{\beta}^{m\beta} = -\frac{m^2}{D'_{8R}(m^2) D'_{\beta R}(m^2)} \frac{1}{\pi} \int_L \frac{ds'}{s'-m^2} D_8(s') D_{\beta}(s') \delta\sigma'(s') \quad (\text{AII.1d})$$

$$I_{\beta}^{gg} = -\frac{1}{2} \frac{1}{D'_{8R}(m^2)} \frac{1}{\pi} \int_L \frac{ds'}{(s'-m^2)^2} D_8^2(s') \delta\sigma(s') + \frac{1}{2} \frac{D''_{8R}(m^2)}{D'_{8R}(m^2)} \frac{1}{\pi} \int_L \frac{ds'}{s'-m^2} D_8^2(s') \delta\sigma(s') \quad (\text{AII.1e})$$

$$I_{\beta}^{\beta\beta} = -\frac{1}{D'_{8R}(m^2) D'_{\beta R}(m^2)} \frac{1}{\pi} \int_L \frac{ds'}{s'-m^2} D_8(s') D_{\beta}(s') \delta\sigma(s') \quad (\text{AII.1f})$$

Taking into account the definitions (AI.19) and (AI.20), and changing variables from s' to $\gamma' = (s'/a) - \mu^2$ (a similar change is also performed in the derivatives of D_{8R}), one finds the expressions displayed in formula (3.19).

With these notations and the result (AI.21) of the calculations of Appendix I, formulae (2.22), (2.24) and (2.25) become :

$$\frac{\delta m_{ii}}{m^2} = I^{mm} \sum_v A_{ii;vv} + I^{qq} \sum_{vbd} B_{ii;vb-d} \frac{\delta q_{vb-d}}{q} + D_{ii}' \quad (\text{AII.2a})$$

$$\frac{\delta q_{\beta}^i}{q} = I_{\beta}^{mq} \sum_v B'_{i(\beta,i);vv} \frac{\delta m_{vv}}{m^2} + I_{\beta}^{qq} \sum_{vbd} C_{i(\beta,i);vb-d} \frac{\delta q_{vb-d}}{q} + D_{\beta}^{'i} \quad (\text{AII.2b})$$

with :

$$A_{ij;vy} = \sum_{a,b,d} \begin{vmatrix} 8 & 8 & 8_A \\ a & b & j \end{vmatrix} \begin{vmatrix} 8 & 8 & 8_A \\ c & d & i \end{vmatrix} (-1)^{b+c} \begin{vmatrix} 8 & 8 & 8_A \\ a & -c & y \end{vmatrix} \begin{vmatrix} 8 & 8 & 8_A \\ b & -d & -v \end{vmatrix} \quad (\text{AII.3a})$$

$$B_{ij;vb-d} = 2 \sum_{a,c} \begin{vmatrix} 8 & 8 & 8_A \\ a & b & j \end{vmatrix} \begin{vmatrix} 8 & 8 & 8_A \\ c & d & i \end{vmatrix} (-1)^{b+c} \begin{vmatrix} 8 & 8 & 8_A \\ a & -c & -v \end{vmatrix} \quad (\text{AII.3b})$$

$$B'_{i(\beta,i);vy} = \sum_{a,b,c,d} \begin{vmatrix} 8 & 8 & \beta \\ a & b & j \end{vmatrix} \begin{vmatrix} 8 & 8 & 8_A \\ c & d & i \end{vmatrix} (-1)^{b+c} \begin{vmatrix} 8 & 8 & 8_A \\ a & -c & y \end{vmatrix} \begin{vmatrix} 8 & 8 & 8_A \\ b & -d & -v \end{vmatrix} \quad (\text{AII.3c})$$

$$C_i(\beta, j)_{j \nu b-d} = 2 \sum_{ac} \begin{vmatrix} 8 & 8 & \beta \\ a & b & j \end{vmatrix} \begin{vmatrix} 8 & 8 & 8_A \\ c & d & i \end{vmatrix} (-1)^{b+c} \begin{vmatrix} 8 & 8 & 8_A \\ a & -c & -v \end{vmatrix} \quad (\text{AII.3d})$$

When one uses the expansion (3.1) for δ_m and δ_g , one finds that $N=8$ and $N=27$ are decoupled, which is a general property of such equations ³⁾. Due to (AI.14), δg_{β}^i has to be replaced by :

$$\delta g_{\beta}^i = (-1)^i \sum_N \begin{vmatrix} 8 & \beta & N \\ -i & i & 0 \end{vmatrix} \delta g_{\beta}^N \quad (\text{AII.4})$$

Remembering the orthogonality properties of the Clebsch-Gordan coefficients, one is left with expressions of the form :

$$A^N \delta_{NN'} \delta_{nn'} = \sum_{ij \nu \gamma} (-1)^i \begin{vmatrix} 8 & 8 & N \\ -i & j & n \end{vmatrix} A_{ij \nu \gamma} (-1)^v \begin{vmatrix} 8 & 8 & N' \\ -v & \gamma & n' \end{vmatrix} \quad (\text{AII.5})$$

which are the ones defined in formulae (3.21).

To calculate the A's, B's and C's, it is convenient to multiply both sides of (AII.5) by

$$(-1)^{i'} \begin{vmatrix} 8 & 8 & N \\ -i' & j' & n \end{vmatrix}$$

and to sum over N and n :

$$(-1)^{\left| \begin{smallmatrix} i' & 8 & 8 & N' \\ -i' & j' & n' \end{smallmatrix} \right|} A^{N'} = \sum_{v, y} A_{i' j' v y} (-1)^v \left| \begin{smallmatrix} 8 & 8 & N' \\ -v & y & n' \end{smallmatrix} \right| \quad (\text{AII.6})$$

From this and similar formulae, it is straightforward to obtain the results (3.22).

REFERENCES AND FOOTNOTES

- 1) R.E. Cutkosky and P. Tarjanne, Phys.Rev. 132, 1354 (1963);
R.F. Dashen, S.C. Frautschi, M. Gell-Mann and Y. Hara, Report
at the International Conference on High Energy Physics,
Dubna (1964).
- 2) R.F. Dashen and S.C. Frautschi, Phys.Rev. 143, 1171 (1966).
- 3) R.F. Dashen and S.C. Frautschi, Phys.Rev. 137, B1318 (1965).
- 4) R.F. Dashen and S.C. Frautschi, Phys.Rev. 137, B1331 (1965).
- 5) R.F. Dashen, S.C. Frautschi and D.H. Sharp, Phys.Rev.Letters 13,
777 (1964).
- 6) R.F. Dashen, Y. Dothan, S.C. Frautschi and D.H. Sharp, Phys.Rev.
143, 1185 (1966);
B. Diu, H.R. Rubinstein and R.P. Van Royen, Nuovo Cimento 43,
961 (1966).
- 7) J.J. Sakurai, Phys.Rev.Letters 9, 472 (1962).
- 8) In principle, a check could be obtained from the decay properties
of φ and ω . In practice, however, it is not precise
enough to be really convincing.
- 9) A.P. Contogouris, B. Diu and M. Pusterla, "Bootstrap Solutions
in a Vector Meson Model", CERN preprint TH.691 (1966).
- 10) This matrix is not necessarily a 2×2 one, since δ_m and δ_g
can have several components (see Section 3 for a more precise
statement).
- 11) See, e.g., F. Gürsey, T.D. Lee and M. Nauenberg, Phys.Rev. 135,
B467 (1964).
- 12) J.J. de Swart, Revs.Modern Phys. 35, 916 (1963).
- 13) This is well known : the Gell-Mann - Okubo mass formula for mesons
has no term proportional to the hypercharge Y, because of
charge conjugation invariance.
- 14) J.J. de Swart, Nuovo Cimento 31, 420 (1964).