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A DYNAMIC REFERENCE SYSTEM FOR SPIN QUANTIZATIONJ.T. Donohue ^{*)} and H. Høgaasen

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A B S T R A C T

Among the many reference systems one can use in describing the spin structure of vector particles we pick out a unique one - the one where the "observable" part of the density matrix is diagonal and therefore has elements that are invariants.

We call this reference system the dynamical one and point out some simple aspects of it in the absorption model and how it can be determined directly from experiment.

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The spin population of a resonance with spin S is characterized by the $(2S+1) \times (2S+1)$ density matrix, usually given in a spherical $|jm\rangle$ basis where m is the spin projection along some axis in the rest frame of the particle. For a spin 1 resonance decaying into two spinless particles (assuming parity conservation in production and decay), the density matrix is almost completely determined by the decay distribution. The choice of quantization axis is usually made on the basis of simplicity of the density matrix and hence of the decay distributions. The "most simple" choice of axis depends largely on the dynamics assumed: for reactions dominated by single particle exchange, the direction of one of the incident particles ¹⁾ is often convenient, as in $\pi p \rightarrow \rho N^*$; for photoproduction of vector mesons, the helicity direction seems to be useful, and in the reaction $\pi p \rightarrow \pi^+ N^*$ the Stodolsky-Sakurai model ²⁾ suggests the normal to the production plane as a natural quantization axis. All of these choices will give equivalent results, since the density matrices obtained are related by rotations; it is just that a simple density matrix is more readily understood than a complicated one. With such a criterion, another natural choice for the quantization axis is that for which the density matrix is diagonal, the diagonal elements being the eigenvalues. The usefulness of such a system can only be tested by experience, but we find that for spin 1 particles there is evidence that the diagonal density matrix has some simple properties.

For vector particles, the observable (in spinless two-body decay) part of the density matrix can be diagonalized by a rotation about the normal to the production plane and a change of basis to cartesian axes. The three independent parameters ρ_{00} , ρ_{1-1} and $\text{Re } \rho_{10}$ are replaced by the three non-negative eigenvalues ^{*)} α , β , γ satisfying $\alpha + \beta + \gamma = 1$, and the rotation angle Θ .

^{*)} This is the simple form of the positivity condition ³⁾ of ρ_{ij} in the dynamical frame.

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We refer to this choice of axes as dynamical since the density matrix assumes a simple form and the angle Θ is determined by the dynamics of the reaction, being a function of energy and production angle for a given reaction. Explicitly, starting from the real part of the density matrix as expressed in terms of the t channel axes (also called the Gottfried-Jackson axes):

$$\text{Re } \rho = \begin{pmatrix} \rho_{11} & \text{Re } \rho_{10} & \rho_{1-1} \\ \text{Re } \rho_{10} & \rho_{00} & -\text{Re } \rho_{10} \\ \rho_{1-1} & -\text{Re } \rho_{10} & \rho_{11} \end{pmatrix} \quad (1)$$

the eigenvalues are ^{3) *)} :

$$\beta = \rho_{11} + \rho_{1-1} \quad (2a)$$

$$\alpha = \frac{1}{2} \left(1 - \beta - ((\rho_{00} - \rho_{11} + \rho_{1-1})^2 + 8(\text{Re } \rho_{10})^2)^{1/2} \right) \quad (2b)$$

$$\gamma = \frac{1}{2} \left(1 - \beta + ((\rho_{00} - \rho_{11} + \rho_{1-1})^2 + 8(\text{Re } \rho_{10})^2)^{1/2} \right) \quad (2c)$$

$$\tan 2\Theta = \frac{-2\sqrt{2} \text{Re } \rho_{10}}{\rho_{00} - \rho_{11} + \rho_{1-1}} \quad (2d)$$

Here β is the eigenvalue along the normal to the production plane, Θ is the angle of rotation about the normal from the incident particle direction to the z axis in the dynamical system, and γ is the eigenvalue along that axis. The configuration is shown in Fig. 1. In terms of the diagonal system, the two-particle

*) To diagonalize the full density matrix, including $\text{Im } \rho_{10}$, one replaces $\text{Re } \rho_{10}$ in Eq. (2) by ρ_{10} . Since Θ then becomes complex, the transformation no longer corresponds to a physical rotation.

decay distribution has the form

$$W(\hat{n}) = \frac{3}{4\pi} \left(\alpha(\hat{n} \cdot \hat{x})^2 + \beta(\hat{n} \cdot \hat{y})^2 + \gamma(\hat{n} \cdot \hat{z})^2 \right) \quad (3)$$

where $\hat{n}$ is a unit vector in the direction of decay, and $\hat{x}$, $\hat{y}$ and $\hat{z}$ are unit vectors along the axes. Since the quantity Θ is a function of s and t , the diagonal decay distribution will be practical only if Θ turns out to be a rather slowly varying function.

We have made a study of the behaviour of the angle Θ and the eigenvalues α , β , γ for the reactions $\pi p \rightarrow \rho p$ and $K p \rightarrow K^* p$ using the absorption model. Since the model, despite its theoretical faults, gives reasonable approximations to experimental decay distributions, one may hope that the behaviour of Θ and α , β , γ in the model will be similar to the experimental behaviour. If sufficiently precise experimental data were available, one could dispense with the absorption model and study the diagonal properties directly, but with the present experimental errors this is not feasible. The absorption model calculations give the surprising result that the angle Θ and the eigenvalue α are slowly varying quantities in momentum transfer throughout the region of the forward diffraction peak, aside from a sharp change at very small angles. These results are shown in Figs. 2a and 2b, for the reaction

$\pi p \rightarrow \rho p$ at 8 GeV/c, assuming π exchange, and the reaction $K^- p \rightarrow K^{*-} p$ at 2.64 GeV/c with a mixture of π and vector meson exchange that gives a good fit to the experimental data of Friedman and Ross ^{4), 5)}. The density matrices in terms of the t channel axes are shown, with the corresponding eigenvalues and $\tan \Theta$, as a function of the c.m. angle. For the ρ production, the angle

$\tan \Theta$ is also a slowly varying function of lab energy, being ≈ 0.28 at 3 GeV/c and ≈ 0.35 at 8 GeV/c. For K^* production the energy dependence of Θ depends on the strength of the vector meson exchange; if one adjusts the vector meson exchange so as to get reasonable agreement with experimental density matrix results at high energy, $\tan \Theta$ remains approximately constant, ≈ 0.2 .

These absorption model results may be expressed simply by saying that in vector meson production there is a fixed direction in space (in the vector meson rest frame) along which the decay probability (or equivalently the spin projection) is small, throughout the forward peak. This follows from the fact that Θ is measured from a fixed direction (the incident beam) and that the eigenvalue α is small and slowly varying. It is interesting to compare these results with those of the Born approximation using π and vector meson exchange. In the latter the t channel axes coincide with the diagonal axes, so the angle Θ is zero at all production angles, i.e., the dynamic and t channel frames coincide. For simple π exchange the Born approximation fixes $\alpha = \beta = 0$, but an admixture of vector meson exchange allows these to become non-zero. To make the absorption model calculation one decomposes the Born helicity amplitudes into partial waves, modifies each partial wave by a factor dependent on the total angular momentum and sums the resulting expansion. The modified amplitudes no longer correspond to the exchange of a simple system but have very complicated structure upon crossing. Still, it appears that the Born results survive in vestigial form, since Θ is approximately constant at a non-zero value and the eigenvalue α is also small and slowly varying.

We regard the absorption model result that spin 1 decay distributions may be understood in terms of three parameters, two of which are slowly varying with energy and momentum transfer as intuitively pleasant. This the more so as the parameters are the

eigenvalues (and thus the invariants) of the density matrix and a direction in space.

To compute these parameters from the Eq. (2) is however a poor method for determining the invariants and the angle Θ . As the errors on ρ_{11} , ρ_{1-1} and $\text{Re } \rho_{10}$ are correlated, the use of Eq. (2) will give unnecessarily big errors on α , β and Θ . Evidently the best method of determining α , β and Θ is to fit them directly from the experimental distribution. Suppose an event N is characterized in the t channel frame by its two angles θ and φ . In the dynamic frame these two angles are transformed by the rotation φ into $\theta'(\Theta)$ and $\varphi'(\Theta)$ giving an experimental distribution $W(\theta', \varphi') = W(\theta'(\Theta), \varphi'(\Theta))$. This distribution can then be fitted by the expression (3) by maximizing a likelihood function with respect to α , β and Θ . It would be most interesting to see in this manner to what extent the dynamics of the absorption model corresponds to the dynamics of strong interaction with respect to spin description.

In conclusion we would express a wish that experimenters, at the same time as they present results of the elements of the density matrix elements in their favourite (kinematic) reference system, also make an independent fit to the decay distribution in terms of α , β and Θ . We think that somebody might get a good idea by fixing his attention to the invariants of the density matrix (the eigenvalues) and the concept of a direction in space to characterize a dynamical feature should appeal to most physicists.

It goes without saying that finding the origin of this direction solves part of the strong interaction dynamics.

R E F E R E N C E S

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FIGURE CAPTIONS

Figure 1 The t channel axes (X, Y, Z) and the diagonal axes (x, y, z) for the vector meson in its rest frame. The Z direction is that of the incident beam, and Y is the normal to the production plane, defined by $p_i \times p_f$. The eigenvalues (α, β, γ) correspond to the axes (x, y, z) , and Θ is the rotation angle about the normal.

Figure 2 Absorption model predictions for the density matrix in the t channel axes, $\tan \Theta$, and the eigenvalues (α, β, γ) for the reactions $K^- p \rightarrow K^{*-} p$ at 2.64 GeV/c (Fig. 2a) and $\pi^- p \rightarrow \rho^- p$ at 8 GeV/c (Fig. 2b).

fig.1