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FLUCTUATIONS IN NUCLEAR REACTIONS

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¹ The survey of literature for this review was concluded in April 1966.

1. INTRODUCTION

Diffraction patterns produced by waves of incident particles are well known in elastic scattering on nuclei. Such diffraction effects persist also in simple inelastic excitations of the system ("direct interactions"). The diffraction patterns are determined mainly by the geometrical shape of the scatterer, the wavelength, and the angular momentum transfer; they reflect the spatial coherence of the incident wave. It might be thought that diffractive effects are negligible if the emitted wave corresponds to a very complicated mode of motion, in the extreme case represented by statistical (thermal) emission. This is not so. Provided certain precautions are taken in the observational setup, coherence can be preserved, but the corresponding diffraction patterns are of highly irregular ("random") shape and show extreme sensitivity to the details of the scattering system and to the wavelength. Such random diffraction patterns and random coherence are by no means peculiar to nuclear physics, but for the unusually small wavelength. As early as 1878 a similar phenomenon was found in optics by Exner (1, 2) who observed the irregular granular patterns produced by a tiny frosted-glass disk in coherent light. A closely related effect occurs in astronomy. The uncorrelated sources of an entire radio galaxy emit waves coherently over small time intervals which gives rise to diffraction like intensity correlations in detectors viewing the galaxy under slightly different angles (3, 4).

In nuclear reactions random diffraction phenomena have been observed typically at energies corresponding to 12-20 MeV or higher excitation of the compound system. At lower energies the exciting wave tends to give rise to so many reflections inside the intermediate nucleus that standing waves associated with sharp resonances begin to dominate the diffraction aspects of the problem. So as not to wash out interference patterns, transitions to individual final states must be studied. Further, the great sen-

sitivity to small changes of wavelength necessitates very good energy resolution in the incident beam. In view of these requirements it is not surprising that most experimental studies of this effect in nuclear physics have relied on modern high-resolution accelerators, available only in the last few years at the relevant energies.

The possibility that nuclear reactions exhibit an enormous fine structure of nonresonant type (fluctuations) owing to a randomly coherent effect was first predicted in 1960 by one of the present authors (5, 6). It was first demonstrated experimentally by the Colli & Facchini group in 1962 (7, 8). The description and interpretation of this fine structure is based on simple assumptions about properties of scattering amplitudes which are qualitatively borne out by many experiments.

The aim of the present article is to present the basic concepts of nuclear cross section fluctuations in a way chosen to emphasize the physical rather than the mathematical aspect.

2. STATISTICAL PROPERTIES OF AMPLITUDES AND CROSS SECTIONS

2.1. Properties of amplitudes. - The statistical properties of the scattering amplitude will now be discussed in the absence of all spin effects. Since nuclear reaction theory is equivalent to the theory of wave guides, we want to start our reasoning with the latter so as to better visualize the essential points. We will concentrate on processes in which the incident wave goes through a highly complicated motion in the nucleus. Therefore we will disregard the question of surface reactions and picture the nucleus as a closed cavity with reflecting, but highly irregular walls. The wave entering this cavity has wavelength λ and the cavity has diameter L . The wave in the box will now be reflected a large number of times. After a number of reflections $N \approx (L/\lambda) \gg 1$, the motion begins to repeat so closely that standing waves associated with sharp resonances

appear in the system. The frequencies or energies of these resonances are very densely spaced for $L \gg \lambda$, and the specific positions of the resonance frequencies depend with extreme sensitivity on the detailed shape and form of the system. This resonance phenomenon is a highly nonlinear effect since standing waves build up by repeated reflections on the same irregularities in the cavity. The statistical properties of resonances have been explored in detail by many authors (9).

We will now give the wave the possibility of escaping before standing waves have been formed, for example by drilling a hole of diameter $d \gg \lambda$ in the system. Thus the number of reflections before escape is

$$n = (\text{surface of cavity}) / (\text{surface of hole}) \sim (L/d)^2 \gg 1$$

Since n is proportional to the time spent in the system before escape, we have $n \propto \hbar/\Gamma$, where Γ is the decay width of the system. On the other hand, the number of reflections which occur before the motion repeats itself in a system of level spacing D is, expressed in suitable units, the Weisskopf recurrence time $2\pi\hbar/D$; hence

$$(\pi\Gamma/D) \simeq (d/\lambda)^2 \gg 1.$$

Since the width becomes so much larger than the associated spacing, this indicates that the detailed resonance structure plays a subordinate rôle when a sufficiently large hole has been opened in the wall. The important difference as compared to the resonant case is that the nonlinearity of the system is considerably reduced, since an irregularity does not normally participate more than once in the reflections. This does not imply that resonant effects are completely eliminated, since the wave can be caught accidentally between two nearly parallel "mirrors" somewhere in the cavity.

The amplitude emitted from a system of this type is of course completely well defined and reproducible, provided all conditions remain identical. The number of effective wavelengths ν contained in the box is (nL/λ) ; if this number is changed by the order of unity or more, the phase of the scattered amplitude will change completely, so that it will

have little relation to its previous value. Such changes of phase relations can be produced in many ways, for example by small changes in the incident wavelength or by small changes in the linear dimensions of the box. To produce such an effect the typical change of wavelength is $\delta\lambda \gtrsim \lambda/v$ and the typical change of dimensions is $\delta L \gtrsim \lambda/n$; the changes necessary are thus extremely small for a large system of long lifetime. The importance of this is that the amplitude emitted from the system under conditions which have changed the phase relations completely may be considered to have uncorrelated phases and to constitute an ensemble in the sense of statistical physics.

By observing the complex amplitude $f = \xi + i\eta$ emitted from a very small additional hole in the cavity for various members of the ensemble, we can obtain a distribution of amplitudes within the ensemble. This distribution is described by the probability density $P(\xi, \eta)$ of finding ξ and η per unit interval. Since the phase of the amplitude is expected to be random for various members of the ensemble, there should be no preference with respect to ξ or η , and the mean values of both the amplitude and its square should be zero:

- (a) $\langle f \rangle = \langle \xi \rangle + i \langle \eta \rangle = 0$, i.e. $\langle \xi \rangle = \langle \eta \rangle = 0$;
- (b) $\langle f^2 \rangle = \langle \xi^2 \rangle - \langle \eta^2 \rangle + 2i\langle \xi\eta \rangle = 0$, i.e. $\langle \xi^2 \rangle = \langle \eta^2 \rangle = a$ and $\langle \xi\eta \rangle = 0$;
- (c) since the distribution function should have no directional dependence, it should be invariant with respect to rotations in the variables (ξ, η) , i.e. $P(\xi, \eta) = P(\xi', \eta')$, where $\xi + i\eta = \xi' + i\eta'$.

The condition $\langle \xi\eta \rangle = 0$ suggests that it may be a good approximation (to be verified experimentally and on the basis of models) to replace condition (b) by the stronger condition that ξ and η have independent distribution functions $P(\xi)$ and $P(\eta)$. Hence

$$(b') \quad P(\xi, \eta) = P(\xi)P(\eta).$$

The conditions (b') and (c) are sufficient to specify uniquely the probability distribution, without further assumption:

$$P(\xi, \eta) = \frac{1}{2\pi a^2} \exp\left\{-\frac{\xi^2 + \eta^2}{2a^2}\right\} \quad 1.$$

(Note that this derivation does not apply to the amplitude of the original hole of size d , since flux conservation puts strong limits on the variation of its absolute value).

If the change of wavelength $\delta\lambda$ necessary to produce an uncorrelated phase of the amplitude is equivalently expressed in terms of the corresponding change of energy or frequency, one finds the condition $\delta E \gtrsim \Gamma$. This therefore implies that the intensity of the emitted radiation will exhibit strong variations (fluctuations) over similar energy intervals.

Since amplitudes taken at energies separated by more than Γ are only weakly correlated, this opens the possibility of replacing ensemble averages by energy averages, and, of course, to use an ensemble based on well separated energies to study the distribution of any measurable quantity. This procedure is analogous to the replacement of ensemble averages by time averages in statistical physics invoking the ergodic theorem (10). An individual statistical system will exhibit correlated properties if measurements on it are being made within times considerably shorter than its relaxation time; if taken at times separated by appreciably more than the relaxation time, the properties will be uncorrelated, so that in practice we can use the system itself for sampling the ensemble. In the case of the nucleus this is particularly important since the forces in the nucleus are specific and well defined with perfectly repeatable experiments, so that an ensemble can only be defined in the above sense.

The previous discussion on a somewhat idealized nucleus should be slightly generalized for actual nuclear reactions. The preceding arguments apply unchanged to the inelastic amplitude emitted in a given channel. Further, the dimension L of the cavity is in the nuclear case not only the linear dimension of the system, but refers to a multidimen-

sional space connected with the numerous nuclear degrees of freedom. But for this the preceding discussion is fully applicable.

2.2 Cross-section probability distribution. - The nuclear amplitude for emission at an angle θ is in absence of spins and also in some other cases described by one single amplitude $f(\theta) = \xi + i\eta$. The reaction cross section $\sigma(\theta)$ is then

$$\sigma(\theta) = |f(\theta)|^2 = \xi^2 + \eta^2 \quad 2.$$

Insofar as the amplitude ($f(\theta)$) is statistical in the sense of Equation 1, the cross section $\sigma(\theta)$ is the sum of the squares of two uncorrelated variables with normal distribution and equal dispersion. Therefore the probability distribution of $\sigma(\theta)$ must be a χ^2 distribution of two degrees of freedom which is simply an exponential (11, 12):

$$\left\{ \text{Probability distribution of } x = \frac{\sigma(\theta)}{\langle \sigma(\theta) \rangle} \right\} = e^{-x} \quad 3.$$

This cross section probability distribution predicts extremely strong variation of the cross section around the average; in particular, the number of small cross sections is high. The typical ratio of largest to smallest cross section in a sample of N members is of the order $(2N \ln N)$ which for $N \approx 17$ is 100. Many experiments on statistical cross sections represented by one single amplitude (one-channel reaction) have cross sections probability distributions well described by Equation 3. A typical example with unusually high statistics is shown in Figure 1.

2.3 Effects of random diffraction. - In a nuclear reaction the direction of the incident beam imposes a preferred direction on the outgoing particle. Even in statistical reactions, total angular momentum conservation must be imposed on the system, but the loss of phase information in the amplitude makes it impossible to tell which direction is forward and which is backward. On the average the particles are emitted symmetrically around 90° to the beam in such reactions. At

Fig. 1. Probability distribution of reaction cross sections for $C^{12}(O^{16}, \alpha^5)Mg^{24}$ at about 30 MeV excitation in the compound nucleus [Halbert et al.(13)].

any definite energy, however, there is no reason to expect this symmetry since no average is performed. The only statement then is, that it should be impossible to predict whether the forward or the backward direction is favored. The corresponding angular distributions are usually highly complicated and asymmetric with extreme sensitivity to incident energy; they may be regarded as random diffraction patterns, produced by accidental coherence from different parts of the nucleus. A suitable way to describe these rapidly varying angular distributions is to introduce the angular correlation function $C(\theta, \theta')$ for the emission of a particle in directions θ and θ' . This function is defined by the following ensemble average of the cross sections

$$\langle \sigma(\theta) \rangle \langle \sigma(\theta') \rangle C(\theta, \theta') = \langle \sigma(\theta) \sigma(\theta') \rangle - \langle \sigma(\theta) \rangle \langle \sigma(\theta') \rangle \quad 4.$$

Thus, if the cross sections at θ and θ' are independent of each other, $C(\theta, \theta') = 0$.

It is useful to define a similar correlation function $c(\theta, \theta')$ for the corresponding amplitudes² (assumed to be statistical)

$$\{\langle \zeta(\theta) \rangle \langle \zeta(\theta') \rangle\}^{\frac{1}{2}} c(\theta, \theta') = \langle f(\theta) f^*(\theta') \rangle - \langle f(\theta) \rangle \langle f^*(\theta') \rangle \quad 5.$$

It can be shown that, provided the correlations are described by a normal multivariate distribution with zero mean, the cross section correlation function is related to the amplitude correlation function by (12)

$$C(\theta, \theta') = |c(\theta, \theta')|^2 \quad 6.$$

The qualitative properties and some of the physical origin of the angular-correlation functions (Equation 4 and 5) are pedagogically well illustrated by a surface emission model due to Brink et al. (14). The nucleus is considered to be spherical of radius R with the outgoing wave assumed to be produced by uncorrelated sources of strength $g(\vec{R})$ distributed on the nuclear surface. The correlation function between two sources in the model is therefore

$$\langle g(\vec{R}) g^*(\vec{R}') \rangle = \text{const} \times \delta(\vec{R} - \vec{R}') \quad 7.$$

which insures that the system radiates isotropically in space. The amplitude emitted in the direction $\vec{k}$ is given by

$$f(\vec{k}) = \int_{\text{surface}} g(\vec{R}) e^{(i\vec{k} \cdot \vec{R})} d\vec{R} \quad 8.$$

According to Equations 5 and 7 the amplitude correlation function as a function of $\vec{q} = (\vec{k} - \vec{k}')$ is given by

$$c(\vec{k}, \vec{k}') = \text{const} \times \int_{\text{surface}} e^{[i(\vec{k} - \vec{k}') \cdot \vec{R}]} d\vec{R} = j_0(qR) \quad 9.$$

where $j_0(x)$ is the spherical Bessel function ($\sin x/x$). If α is the angle

² Such correlations are referred to as partial coherence in optics (15).

between $\underline{k}$ and $\underline{k}'$, Equation 9 implies that strong correlations occur only for

$$qR = 2kR \sin \frac{\alpha}{2} < 1 \quad 10.$$

From the surface emission model it is apparent that the correlation function is mainly determined by the size of the emitting system; further, the oscillating dependence on qR is of a shape closely similar to the classical diffraction shape, but the diffraction-like pattern is in the intensity correlation, not the average intensity (= cross section) itself³.

The similarity of the cross section correlation function with that of ordinary diffraction suggests that a sharp cutoff approximation in angular momentum might also be of use in the present context. In the case of average isotropy the statistical assumption for each partial wave independently and the assumption that the average contribution of each partial wave is a constant for $\ell < \ell_c$ and zero for $\ell > \ell_c$ leads to (14)

$$c(\vec{k}, \vec{k}') \approx \frac{2J_1[(\ell_c + \frac{1}{2})\alpha]}{(\ell_c + \frac{1}{2})\alpha} \left(\frac{\alpha}{\sin \alpha} \right)^{\frac{1}{2}} \quad 11.$$

where J_1 is the cylindrical Bessel function of first order. An experimental example of an angular cross correlation is given in Figure 2.

While the preceding two descriptions qualitatively demonstrate the essential features of cross section angular correlations and in some cases even seem to provide semiquantitative descriptions, they leave to be desired on two points:

(a) the directional influence of the incident beam has been neglected, though it seems often to be relatively small;

³ The surface emission model is closely analogous to the coherent emission from a radio galaxy (3, 4). Radio galactic source points emit coherently for short time intervals, though their emission is uncorrelated when averaged in time. Time averages thus replace our energy averages. Observation stations a distance d apart on the Earth view the galaxy at angles differing by $\delta\theta = d/L$ where L is the distance to the galaxy. Correlations in the instantaneously measured intensities are observed when the

Fig. 2. Angular-correlation functions for the cross sections measured in the reaction $\text{Al}^{27}(\alpha, p)\text{Si}^{30}$ at bombarding energies between 7.25 and 8.11 MeV. The formula used for the fit in the figure is essentially a combination of our Equation 6 and 11 (Dearnaley et al.(16)) .

b) the correct weights of given angular momentum amplitudes depend not only on the emitted particle, but also on penetration of the incident wave into the nucleus as well as the competition of all other modes of emission for any given angular momentum.

These additional effects have been treated in detail in the literature, and explicit expressions for $C(\theta, \theta')$ based on partial-wave expansions are given (14). For proper analysis of angular-correlation experiments, distance between the observation posts is sufficiently small. If R is the typical radius of the galaxy the condition for coherence is

$$kR\delta\theta = R\delta/\lambda L \lesssim 1$$

for reasons identical to the similar condition in the surface emission model (Equation 10). This phenomenon is used to deduce source distributions in radio galaxies.

the more exact expressions which include dynamical effects should be used since even the character of the angular correlations changes in extreme cases.

2.4 Energy correlations.- In Section 2.1 we have seen that amplitudes and cross sections vary rapidly over energy intervals larger than the width Γ . As in the case of the angular distributions, such irregular variations are best studied in terms of correlation functions. For simplicity we shall assume that the average cross section $\langle \sigma(E) \rangle$ is independent of energy. The energy-correlation function $C(\epsilon)$ of the cross section is then defined as (5, 17)

$$\langle \sigma \rangle^2 C(\epsilon) = \langle \sigma(E+\epsilon) \sigma(E) \rangle - \langle \sigma \rangle^2 \quad 12.$$

and the amplitude-correlation function ⁴ by

$$\langle \sigma \rangle c(\epsilon) = \langle f(E+\epsilon) f^*(E) \rangle - |\langle f \rangle|^2 \quad 13.$$

Under the same conditions as in Equation 6 $C(\epsilon)$ and $c(\epsilon)$ are related simply by

$$C(\epsilon) = |c(\epsilon)|^2 \quad 14.$$

In obtaining Equation 12 and 13 we have neglected the angular dependence of the amplitude. In principle the angular- and energy-correlation functions are not separable and should be treated jointly as $c(\epsilon; \theta, \theta')$; in practice, however, present experiments have not yet reached the precision at which this separation is called for.

2.5 Model for nuclear reaction amplitude. - The description of fluctuations in the literature has with few exceptions been based on a flexible model (5, 17, 19) for the scattering amplitude $f(E)$, which is suggested by nuclear reaction theory. The conclusions from this model

⁴ The amplitude correlation function $c(\epsilon)$ also occurs in problems involving coherent bremsstrahlung in nuclear reactions (18).

agree closely with those of the preceding sections, but the definiteness of the model has made it possible to test various assumptions numerically. This testing has turned out to be an important practical tool. The essence of the model is the following: The scattered amplitude is expressed as a sum of resonance terms of constant decay width $\Gamma \gg D$, where D is the level spacing. The residues of the resonances are proportional to $(\gamma \times \gamma)_i$, the usual matrix of partial-width amplitudes for the i^{th} resonance. Hence

$$f(E) = \frac{1}{2ik} \sum \frac{(\gamma \times \gamma)_i}{(E - E_i + i\Gamma/2)} \quad 15.$$

Various assumptions can now be made about $(\gamma \times \gamma)_i$. In the simpler versions of the model the partial-width amplitudes are taken to be uncorrelated and stochastic. The same assumption is well known from the description of width distributions for neutron resonances (20, 21). The choice of a constant decay width Γ is justified by the many partial widths contributing to the width Γ and the lack of dominance of any single decay channel at high excitation energy. The amplitude-correlation function corresponding to Equation 15 is given by

$$C(E) = \frac{i\Gamma}{E + i\Gamma} \quad 16.$$

Numerical and analytical studies of the properties of amplitudes of the type (Equation 5) have been made for various assumptions about partial widths and Γ/D . These investigations reveal the following (22-26):

a) The statistical properties of the amplitude $f(E)$ are highly insensitive to detailed assumptions about partial-width amplitude and level distributions as long as (i) $\pi\Gamma/D \gg 1$ and (ii) the signs of the residues $(\gamma \times \gamma)_i$ are uncorrelated.

b) The independence assumption (b') of Section 2.1 for the real and imaginary part of $f(E)$ is fulfilled to a good approximation.

Equation 15 can be used to calculate an "artificial" excitation func-

tion using stochastic numbers under various assumptions for the matrix elements $(\gamma \times \gamma)_i$. A typical example is given in Figure 3. The "resonant"-type fluctuation behaviour is extremely striking, but from the way the cross section has been generated it is perfectly obvious that the peaks are not produced by any individual resonance, since their width is very much larger than the resonant spacing; further the peaks do not have normal Breit-Wigner-type shapes. Actual excitation functions for one channel reactions show strong similarity to Figure 3 in the region of overlapping levels, as exemplified in Figure 4 by the differential cross section for the ground state transition of $\text{Cl}^{35}(p, \alpha_0)\text{S}^{32}$ close to the backward direction at an excitation energy of about 18.5 MeV. The associated curve is the energy-correlation function deduced from the data using Equation 12, which for the model amplitude (Equation 15) is the Lorentzian

$$C(\varepsilon) = \frac{\Gamma^2}{\Gamma^2 + \varepsilon^2} \quad 17.$$

2.6 Time-energy description. - It is interesting to look at the model amplitude (Equation 15) from a different but equivalent viewpoint (28). Since time and energy are conjugate variables, the Fourier transform $\phi(t)$ of the amplitude $f(E)$ can be looked at as the instantaneous amplitude density received at a detector at time t for a reaction induced by a short pulse. From the definition of the Fourier amplitude

$$\phi(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} f(E) e^{-iEt} dE \quad 18,$$

it is easy to show, using Equation 15, that

$$\langle \phi(t) \phi^*(t') \rangle = \frac{1}{(2\pi)^2} \iint_{-\infty}^{+\infty} \langle f(E) f^*(E') \rangle e^{-iEt + iE't'} dE dE' = \langle \sigma \rangle P(t) \delta(t - t') \quad 19,$$

where the power function $P(t)$ is the Fourier transform of the amplitude correlation function

$$P(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} c(\varepsilon) e^{-i\varepsilon t} d\varepsilon \quad 20.$$

Fig. 3 Artificial excitation function. The curve was calculated from Equation 15 under the following assumptions. For the matrix elements $(\gamma \times \gamma)_i$, stochastic numbers were chosen which were real and had a normal distribution of mean zero and standard deviation unity. Equal level spacing was assumed. Curves are quite similar for Γ/D values equal to 100, 50, or 2 [Dallimore & Hall (25)].

For the correlation function (Equation 16) this gives

$$P(t) = \Gamma e^{-\Gamma t}.$$

The structure of Equation 19 reflects the fact that time amplitudes are uncorrelated for $t \neq t'$, which can be interpreted as a very short relaxation time of the system. The amplitudes $\phi(t)$ have probability distributions analogous to those discussed previously (Equation 1). The power function $P(t)$ describes the average intensity received at the detector at a time t . The time development of this power function is related to the energy-correlation function by the Wiener-Khintchine theorem (Equation 20) in analogy to similar problems in noise theory (28a). The idealized description of fluctuations in nuclear cross sections can thus be equivalently summarized in the statement: nuclei have short relaxation times.

Fig. 4 Upper part: excitation function for the reaction $\text{Cl}^{35}(\text{p}, \alpha_0)\text{S}^{32}$ leading to the ground state in S^{32} at an excitation energy of 18.5 MeV in the compound nucleus A^{36} observed at 170° . Lower part: the corresponding energy-correlation function calculated from Equation 12 [Brentano et al. (27)]. The dashed line is a fit using Equation 17.

The time description of the amplitude indicates clearly that $t = 0$ plays a very special rôle since no relaxation processes have had time to occur. The instantaneously scattered amplitude (direct interactions) is therefore of nonstatistical nature. In the energy representation this manifests itself through a constant non-zero component in the amplitude, so that $\langle \hat{f} \rangle \neq 0$. For the model amplitude (Equation 15) this is described by small systematic deviations of $(\chi \times \chi)$ from random sign leading to $f = \langle f \rangle + \hat{f}$. The statistical properties of the remaining amplitude $\hat{f}$ are little affected. This will be discussed further in Section 4.

2.7 Other statistical properties of cross sections. - The probability distribution of maxima and minima in the cross section can be derived from the following two assumptions: (a) a knowledge of the energy-correlation function; (b) the assumption that the joint probability distribution of amplitudes at three neighbouring energies E_1 , E_2 , E_3 are described by a multivariate normal distribution (12). The results for the Lorentzian correlation function (Equation 16) are that the minimum distribution is close to a χ^2 distribution of one degree of freedom, while the maxima distribution resembles a χ^2 distribution of three degrees of freedom (10).

The probability distribution of the total cross section is expected to be normal for the quantity $(\sigma_{\text{tot}} - \langle \sigma_{\text{tot}} \rangle)$ (29). In this case the dominant part of the amplitude is a constant, since in Equation 15 we get $(\chi \times \chi)_i > 0$ because of the autocorrelation effect in incident and outgoing channels.

An anomalous behaviour of the correlation function (Equation 16) for transitions inhibited by nearly exact conservation laws (like isospin conservation) has been analysed by Bizetti (29a). The result can be interpreted in the time representation as the increasing impurity developing in an initially pure state.

3. ANALYSIS OF DATA

The proper statistical treatment of experimental data is of greatest importance in a fluctuation analysis. Though considerable effort has been devoted to this problem, we will only mention some essential points.

As illustrated in Figure 4, a fit of the measured energy-correlation function $C(\epsilon)$ with the Lorentzian curve (Equation 17) is a convenient way to determine Γ from an experimental excitation function. We see in the figure that at larger ϵ/Γ the experimental correlation function shows strong oscillations around the abscissa. These are due to the finite

energy range of the sample from which our correlation function was calculated. In a fluctuation analysis, the errors produced by "finite range deviations" of this kind (FRD errors) are usually much larger than the ordinary statistical errors due to counting statistics. In the excitation functions, cross sections spaced less than $\pi\Gamma$ are strongly correlated. The number of statistically independent data points is therefore given by the measured energy range I divided by $\pi\Gamma$. Therefore the quantity $n = I/\pi\Gamma$ defines the "sample size". The FRD errors will then be essentially proportional to $(1/n)^{\frac{1}{2}}$. The theory of FRD errors has been treated by several authors and explicit formulas have been derived to calculate these errors for the quantities considered most frequently in fluctuation theory, like average cross sections or the coherence width Γ determined from the correlation function (26, 30-36).

A number of additional methods has been developed for the determination of Γ . The simplest one is counting the number of maxima per unit energy interval, ν , in the excitation function (12, 26). Equation 16 for the correlation function leads to a coherence width $\Gamma = 0.55/\nu$. More advanced methods include an analysis of the oscillations in the correlation function (35, 36) and a Fourier frequency analysis of the excitation function (37, 38).

In addition to finite range deviations other effects such as a varying average cross section (30, 39, 40), nonstatistical structures in the excitation function (41-44), and the finite experimental energy resolution have to be taken into account in the analysis of experimental data. It has been shown that it is possible to deduce the coherence widths even in experiments with an energy resolution much smaller than Γ (46). Some experimentally determined values of Γ will be given later in Figure 7.

A careful investigation of the statistical properties of the fluctuating excitation functions is also necessary to insure the statistical nature of

the observed structures. The study of cross correlation functions between excitation functions leading to different final states, for example, is a valuable method to prove the statistical independence of the transitions. The observation of "modulations" in the excitation functions (41, 42) may help to detect structures of nonstatistical origin like isobaric analogue states or "intermediate structures" (43, 47, 48).

4. DAMPING PHENOMENA AND GENERALIZATIONS

Till now we have discussed only the simplest case of statistical reactions depending only on a single complex scattering amplitude (one-channel case). In most nuclear reactions several amplitudes contribute simultaneously owing to the additional freedom associated with the spins of the particles and the nuclei. In this case the magnitude of fluctuation effects is often reduced, since the observed cross section is the sum of several partial cross sections. Additional reduction of fluctuation effects occurs when direct-interaction phenomena contribute importantly, but this will be discussed later.

The spins of the projectile, emitted particle, initial nucleus, and final nucleus are denoted by i, i', I , and I' respectively with projections $\beta = (\mu, \mu', M, M')$ on the incident direction. The differential cross section to a specific final state can then be expressed as a sum of "basic" partial cross sections σ_{β} (49, 50)

$$\sigma = \sum_{\beta} \sigma_{\beta} \quad 21.$$

The partial cross sections are in general not independent. In particular, it follows from parity conservation that $\sigma_{\beta} = \sigma_{-\beta}$. It is therefore easily seen that the maximum number of independent partial cross sections consistent with this restriction is related to the total spin weight $g = (2i + 1)(2i' + 1) \cdot (2I + 1)(2I' + 1)$ of the system by

$$N^{\max} = \frac{1}{2} g \quad \text{for } g \text{ even}$$

$$N^{\max} = \frac{1}{2} (g + 1) \quad \text{for } g \text{ odd} .$$

For the following two cases N becomes unity at 0° and at 180° :

(a) Consider a reaction with $i=l=0$ or $i'=l'=0$ and with one of the remaining spins to be 0 or $\frac{1}{2}$.

The z axis is chosen in the direction of the incoming particle. Then we have $\ell_z = \ell'_z = 0$. If $i' = l' = 0$, conservation of angular momentum requires for the z direction $\mu + M = \mu' + M' = 0$, hence $M = -\mu$. Thus only

$$\beta = (\mu, -\mu, 0, 0)$$

and

$$-\beta = (-\mu, \mu, 0, 0)$$

contribute.

Since $|\mu|$ can take only one value and $\sigma_\beta = \sigma_{-\beta}$, we have $N = 1$.

(b) Assume $i = i' = 0$ and l or $l' = \frac{1}{2}$. Since $\mu = \mu' = 0$, we have $M = M'$. For $|M| = \frac{1}{2}$ there are only the two possibilities

$$\beta = (0, \frac{1}{2}, 0, \frac{1}{2})$$

and

$$-\beta = (0, -\frac{1}{2}, 0, -\frac{1}{2})$$

which leads to the same conclusion as above.

If the cross section σ of Equation 21 were the sum of N independent partial cross sections with the same variance, then the corresponding probability distribution would be a χ^2 -distribution of $2N$ degrees of freedom. The shape of these distributions as a function of N is shown in the upper half of Figure 5 from which it is evident that the probability of small cross sections is strongly reduced with increasing N . This is a natural consequence of the fact that the excitation curve for a reaction with N channels is the average of N statistically independent one-channel excitation curves. The corresponding cross section corre-

Fig. 5 Calculated probability distributions as a function of channel numbers N (upper part) and for varying direct-reaction contribution (lower part) [Temmer (51) Ratcliff (52)] .

lation function for $\xi = 0$ is (cf. Equation 12)

$$C(\varepsilon) = \frac{\langle \sigma^2 \rangle - \langle \sigma \rangle^2}{\langle \sigma \rangle^2} = \frac{1}{N} \quad 22,$$

which shows that the relative magnitude of the fluctuations decreases in-

versely with N ; the quantity N is sometimes referred to as the fluctuation damping coefficient.⁵

It is therefore useful to define the number of effective channels N in a statistical reaction by Equation 22. This number is generally a function of angle. Using standard compound nuclear theory, Brink et al. (12) have derived an expression for $N(\theta)$; dynamical aspects of the system enter in terms of penetrability factors. For a wide range of angles about 90° it is usually a good approximation to consider the various partial cross sections to be statistically independent and equivalent, so that $N \approx N^{\max}$. A comparison of calculated and experimental values of $N(\theta)^{-1} = C(\varepsilon=0; \theta)$ for the reaction $\text{Cl}^{37}(p, \alpha_c)\text{S}^{34}$ is shown in Figure 6. Since $i' = l' = 0$ in the exit channel, only one channel contributes in the forward and in the backward direction according to (a) above and hence $C(\varepsilon=0, 0^\circ \text{ and } 180^\circ) = 1$. (An analogous one-channel case was used in Figure 1 to illustrate the probability distribution.)

Direct interactions also lead to a reduction of fluctuations. Because of the short time-scale of such processes ($\sim 10^{-22}$ sec) the corresponding amplitudes are nearly constant over the energy intervals usually considered in fluctuation experiments. The reaction amplitude is then approximately the coherent sum of a constant amplitude $\langle f_\beta \rangle$ and a statistical amplitude $\hat{f}_\beta$ as discussed in Section 3. The corresponding cross section becomes

$$\sigma = \sum_{\beta} |\langle f_\beta \rangle + \hat{f}_\beta|^2 \quad 23.$$

For one-channel reaction this leads for the normalized variance to

$$C(\varepsilon=0) = 1 - \gamma^2 \quad 24,$$

where γ is the relative contribution of the direct amplitude to the cross section given by

$$\gamma = \frac{|\langle f \rangle|^2}{\langle \sigma \rangle} = \frac{\sigma^{\text{direct}}}{\langle \sigma \rangle} \quad 25.$$

⁵An elegant general discussion of fluctuation damping under weaker conditions has recently been given by Böhning (52a).

Fig. 6 Normalized variance $C(\varepsilon=0; \theta) = 1/N(\theta)$ versus reaction angle θ for $\text{Cl}^{37}(p, \alpha_0)\text{S}^{34}$. The dotted line marked $1/N^{\text{max}}$ corresponds to the maximum number of effective channels as given in Section 4 [v.Witsch et al. (53)] .

The corresponding probability distributions (11, 12) are plotted in the lower part of Figure 5. We note that the shape of the probability distributions is hardly changed for direct contributions even as large as 50 per cent. For one-channel reactions, Equation 24 or a fit to the probability distribution may be used to determine the direct part, but the accuracy is small for $y \lesssim 0.5$.

Equation 22 for the correlation function for N reaction channels can be generalized to include direct effects (12, 52). Provided the relative amount of direct interactions in each channel is equal, the result is

$$C_{(\varepsilon=0)}^y = \frac{1}{N}(1 - y^2) \quad 26.$$

Here N is the number of channels calculated for a pure compound reaction. Comparison with Equation 22 shows that now also the direct amplitude contributes to the fluctuation damping. The validity of the assumptions leading to Equation 26 must be established for each case separately.

Only then can y be determined from the observed normalized variance by the help of Equation 26. The direct to compound ratio has been determined for several reactions in this way (16, 38, 53-55).

Types of experiments. - In this article the numerous experimental studies of fluctuation phenomena in nuclear reactions cannot be treated in detail. Our illustrations were restricted to a few particularly simple examples. For an experimental investigation the following conditions must be fulfilled: high energy resolution (usually of the order of a few keV), an excitation energy in the compound system high enough to be in the region of overlapping levels (usually 18-20 MeV or more), a sufficiently light nucleus to expect a Γ large enough to be measured with present experimental techniques (compare Figure 7), and a reaction mechanism which provides a predominant compound reaction. The most extensive work has therefore been done on (p, α) reactions (27, 38, 41, 42, 46, 51, 56-59) and on (d, α) reactions (60-67). One (α, p) reaction was studied (16). Neutron-induced reactions were used in a large number of experiments (7, 68-73a) and some photonuclear reactions of the type (p, γ) (74-75) and (γ, p) (76) were investigated. The fluctuation behaviour of the angle integrated and the total cross section was studied in a number of experiments (64, 70, 71).

A series of beautiful experiments has been performed by several groups on heavy-ion reactions like $C^{12}(O^{16}, \alpha)Mg^{24}$ (13, 77-79). They are characterized by large angular momenta of the compound system and high excitations. Of particular interest is the reaction $C^{12}(C^{12}, \alpha)Ne^{20}$ which was carefully analysed (33, 49, 80-81) in view of the possible existence of quasi-molecular nuclear states (82, 83) since a large part of the observed structures can be understood as fluctuations. Another group of experiments concerns fluctuations in the elastic scattering cross section for protons, for which potential scattering is important (54, 55, 84-96). From such experiments the ratio of direct to compound reac-

tions can be determined. Structures of nonstatistical origin are evident in some of these experiments (43, 85).

5. COMPARISON TO "CLASSICAL" COMPOUND-NUCLEUS THEORY

The assumption of a random scattering amplitude does not by itself imply predictions nor of magnitudes of cross sections nor of relative cross sections for transitions to different final states. Predictions of average cross sections and of average angular distributions are however made by the conventional compound nuclear theory, which is based on a picture of the nuclear reaction with an intermediate step of highly complicated motion. It is therefore natural to assume that the compound nucleus theory in addition to predictions of average cross sections also includes implicitly both a quantitative prediction of the associated fluctuations and their distribution, as well as a prediction of the decay width of the intermediate system. This means that the fluctuation effects should be regarded as a necessary (though not sufficient) condition for a quantitative application of compound nuclear theory in the region of high excitation. The usual theory results after averaging.

This is illustrated by the behaviour of angular distributions for transitions to individual final states. Observed with good energy resolution, these often show rapidly varying shapes as a function of energy and they have no fore-aft symmetry. After averaging over large energy intervals or over many final states the angular distributions result with fore-aft symmetry and with quantitative agreement with compound nuclear theory (16, 42, 43, 57, 61, 62, 64). The symmetry results in this theory from the neglect of interference between the partial waves. Further, under favorable conditions it is even possible to determine the relative amount of compound amplitude in the presence of a direct amplitude by their mutual interference.

The information obtainable from compound nuclear reactions at high excitation will now briefly be discussed. An expression for the average cross section leading from an entrance channel c to an exit channel c' , with all possible channels denoted by c'' , is obtained from detailed balance (i.e. time reversal) and the Bohr assumption: factorizability of the process into two steps each depending only on c and c' , i.e. $\sigma_{cc'} = A_c \times B_{c'}$. The average cross section is then immediately expressible in terms of the average transmission coefficients $T_{c''}$ (approximately obtainable from optical potentials, for example):

$$\langle \sigma_{cc'} \rangle \propto T_c \times \frac{T_{c'}}{\sum_{c''} T_{c''}} \quad 27,$$

which is the Hauser-Feshbach equation (88). Two independent tests of the compound nuclear assumption are 1) that ratios of partial cross sections to c'_1 and c'_2 depend only on the respective transmission coefficients $T_{c'_1}$ and $T_{c'_2}$ and 2) that the fluctuations are properly described. If these tests give acceptable results, the coherence width observed in the rapid energy fluctuations can be used to determine the mean level spacing D in the intermediate system by the relation

$$\Gamma = \frac{D}{2\pi} \sum_{c''} T_{c''} \quad 28.$$

Equation 28 expresses that the width Γ is the effective number of decay channels multiplied by the natural energy unit of the decaying system. It is thus possible to use a cross section described by Equation 27 to eliminate the sum over transmission coefficients from Equation 28 which determines the level spacing D . The quantity can be compared to the empirical level density of the same nucleus at lower excitation in order to test theoretical models of the nuclear level density. In particular, the level density parameter a which is proportional to the one-particle level density at the Fermi surface of independent particle models of the nucleus can be determined in this way. This parameter is related to the nuclear temperature T in the effective nuclear excitation energy U by (6)

$$U = a T^2.$$

In practice an analysis according to these lines must take into account that both Γ and D depend on the angular momentum J of the intermediate state (6, 89, 90), though the dependence of Γ on J is in many cases weak (6, 38, 89). It is now necessary to take into account also the spin distribution of the highly excited nuclear states, which can be described by a spin-cut-off factor G^2 . The analysis can still be performed as above provided information from angular distributions is used simultaneously (38, 42, 43, 53, 87). A particularly complete analysis of this kind has been performed by the Saclay group on the reactions $Al^{27}(d, \alpha)Mg^{25}$ and $Mg^{24}(d, \alpha)Na^{23}$ (61-64, 91, 92).

It is interesting to ask how the coherence width Γ varies with nuclear mass number. A guide to this can be obtained by the following simple argument.

The width Γ is proportional to the probability of emission from the nucleus. If the nucleus is considered to be a classical body of temperature T and with a work function W , this probability is proportional to $\exp \{ -W/kT \}$. The nuclear work function is essentially the binding energy of the neutron B_n which to a first approximation may be considered to be independent of mass number A . The main dependence of the nuclear temperature T on mass number and excitation energy U is given by the Fermi gas model for which

$$T = \text{const } (U/A)^{\frac{1}{2}} \text{ MeV}.$$

The coherence width is thus expected to be dominated by a Boltzmann factor $\exp \{ -B_n / T \} = \exp \{ -\text{const } (A/U)^{\frac{1}{2}} \}$. The experimental behaviour of Γ versus $(A/U)^{\frac{1}{2}}$ for excitation energies around 19 MeV is shown in Figure 7, in which the dependence on U gives only a small correction. The points in the figure follow roughly a straight line, indicating that our simple argument is essentially correct and that effects of nuclear shell structure are not of major importance.

Fig. 7 Plot of Γ versus $(A/U)^{1/2}$ for various compound nuclei at excitation energies between 17 and 21 MeV. The numbers in brackets refer to the literature cited at the end of the article. The points marked with squares are measured by an indirect method. The point size does not express errors, which are usually quite large.

A more detailed, but in principle equivalent approach is to derive Γ from compound-nucleus theory and calculate it explicitly in terms of nuclear level densities and penetrabilities (6). This problem as well as the connected one of level densities, has been studied in relation to fluctuation phenomena in several papers (89, 96-101).

6. CONCLUSION

It is experimentally well established that nonresonant rapid variations occur in the excitation functions of many nuclear reactions on light and medium-weight nuclei. These variations, as well as the associated angular distributions, are typical of cross section fluctuations. The generality of this phenomenon is emphasized by the variety of projectiles (γ , p, d, α , heavy ions) for which it occurs.

In the special case of purely statistical reactions, detailed quantitative comparisons can be made between theory and experiments. Such comparisons have in some cases met with remarkable success. This provides nontrivial and severe tests of the statistical model of nuclear reactions, and further demonstrates the importance of coherent wave phenomena in statistical reactions also. The field of observation of nuclear fluctuation phenomena is limited by experimental difficulties. There is no particular reason to doubt that similar effects occur also in heavy elements and in other nuclear reactions, such as electron-induced reactions or fission. With increasing energy the wave spends increasingly shorter times in the nucleus. Therefore the approximation of negligibly short relaxation time becomes more and more questionable as the energy increases. Consequently the emitted wave becomes more and more directly connected with the properties of the exciting wave. Fluctuation effects of the type discussed here will still be present, but their relative importance will then decrease.

Finally, at very high energies, in the many-GeV range, fluctuations in nuclear cross sections are of little interest. In view of the generality of the phenomenon, however, it must be seriously asked whether random coherence does not also play an important rôle in elementary particle physics (28, 102). Its origin and its description would in essentials parallel the nuclear case. The obvious place to search for this effect is in effective two-body reactions with high momentum transfers.

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