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FURTHER CONSEQUENCES OF THE QUARK MODELIN INELASTIC MESON-BARYON SCATTERING^{*)}J.L. Friar^{**)}

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A B S T R A C T

A model of inelastic meson-baryon scattering in terms of quark scattering is presented. Relations are found between cross-sections for various processes, and predictions are made for density matrix elements where particles of higher spin are produced. A number of predictions are made which do not depend on assumptions about quark dynamics other than those inherent in the quark scattering model. A comparison is made with experimental data available at the present time.

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The success of the quark model in deriving relations between elastic scattering cross-sections at high energies ^{1),2)} has led to increased study of the model as applied to other types of interactions. In particular, it has been applied with varying degrees of success to the problem of $P\bar{P}$ annihilation ³⁾, to neutral meson production ⁴⁾, and to isobar production for non-strange particles ⁵⁾. The results which involve scattering of high energy particles depend, in general, on two basic assumptions. First, one assumes that particles are composed of quarks in the usual way, and, second, one assumes that the particle-particle scattering amplitude is the coherent sum of the various quark-quark amplitudes. The fact that one can successfully calculate some high energy cross-sections on the basis of such simple assumptions naturally leads one to ask how far one can push the model before serious discrepancies between experiment and theory occur. It is the purpose of this note to work out the results of the quark model for a wide class of two-body meson-baryon reactions, and then compare these results with whatever experimental data is available at the present time.

In general, we shall not try to give numerical fits to the experimental data, since in most cases the data exist only up to 8 GeV/c, which is not really asymptotic, and since in general we shall have many more quark amplitudes, which can be regarded as adjustable parameters, than numbers to be fit. It would clearly be pointless in such a case to do any detailed numerical work, since any fit we got would not be unique anyway.

We shall be more concerned with calculating density matrices than with cross-sections for the following reason : some work on cross-sections has already been done ^{4),5)}, and it has been found quite difficult to obtain good tests of the quark hypothesis because of cross terms between different quark amplitudes, and unknown kinematic effects, such as form factors, which must be included. In density matrix elements, on

the other hand, we deal with ratios of amplitudes, rather than with the amplitudes themselves, so that these kinematic effects should largely tend to cancel. In particular, the problems caused by the fact that zero angle scattering corresponds to different momentum transfers for different inelastic production processes (which can only be handled by inserting ad hoc form factors into expressions for the cross-sections ⁴⁾), do not appear when considering density matrices. We compare amplitudes for a single reaction, and hence the amplitudes are all evaluated at the same s and t . In addition, one would hope that other dynamical effects, such as multiple scattering, which are not included in our model would tend to cancel also.

In fact, we shall see that in a certain number of cases, the amplitudes which we calculate will cancel out of the expressions for the density matrices completely, leaving exact predictions which do not depend in any way on the quark amplitudes.

The outline of the presentation shall be as follows : in Part I we present the model and the formalism which we shall use to calculate meson-baryon interactions. In Part II, we calculate various cross-sections and density matrices, and in Parts IV and V we compare these calculations with experiment. Finally, a short discussion of the results is given.

I. - FORMAL DEVELOPMENT

The model which we shall use has already been discussed by Jacob and Itzykson ⁵⁾, and is the obvious extension of the simple additivity model which has been applied to elastic scattering calculations. Let us, as an example, consider the pion. It is a system of two spin $\frac{1}{2}$ particles in an S state, so that in the rest frame of the pion (that is, in the centre-of-mass of the $q\bar{q}$ system), the spin wave function is just

$$\psi_{spin} = \frac{1}{\sqrt{2}} [(+-) - (-+)] \quad (1)$$

If we take the axis of quantization to be the direction of motion of the composite particle, then we can write this wave function in a covariant way simply by changing spin projections to helicities, provided that there is no orbital angular momentum. Thus the π^+ wave function is just

$$\psi_{spin} = \frac{1}{\sqrt{2}} [|P^+\rangle |\bar{n}^-\rangle - |P^-\rangle |\bar{n}^+\rangle] \quad (2)$$

The same process can be carried out for the baryon. In this case, however, the assumption that there is no internal motion of the quarks is much less realistic. We know that the spin-isospin part of the baryon wave function must be symmetric ⁶⁾, so that a wave function in which all the quarks are in relative S states would violate Fermi statistics. If we wish to maintain Fermi statistics, then orbital angular momentum must be introduced between the quarks, and our method will only be valid if the motion due to this orbital angular momentum is very small compared to the motion of the baryon as seen in the total centre-of-mass system. In this case, the helicity, given by $\lambda = \frac{\vec{v} \cdot \vec{p}}{|\vec{p}|}$ will not vary greatly during the time of the interaction, and we can proceed as if the orbital angular momentum were zero.

Then we can write out the wave functions for a given helicity state of any fundamental particle. Such a list of wave functions is given in Appendix I, where, for simplicity, we have written $\xi_i \xi_j$ as $(i \bar{j})$.

The scattering amplitude for meson-baryon scattering can then be written

$$\langle \lambda_4 \lambda_3 | \hat{T} | \lambda_2 \lambda_1 \rangle = \langle \sum_{ij} a'_{ij} \xi_i \bar{\xi}_j \sum_{lmn} b'_{lmn} \xi_l \xi_m \xi_n | \hat{T} \sum_{gr} a_{gr} \xi_g \bar{\xi}_r \sum_{stu} b_{stu} \xi_s \xi_t \xi_u \rangle \quad (4)$$

where the wave function coefficients a and b can be read from Appendix I. Now the additivity assumption states that ξ_q scatters from each of the baryon quarks, and that antiquark $\bar{\xi}_r$ does the same. Using that fact that the baryon wave function is fully symmetric, we find that

$$\langle \lambda_4 \lambda_3 | \hat{T} | \lambda_2 \lambda_1 \rangle = 3 \sum_{se} z_{se} \left\{ w_{g_s} \langle \lambda_4 \lambda_3 | g_s \rangle + \bar{w}_{r_j} \langle \lambda_4 \lambda_3 | \bar{r}_s \rangle \right\}$$

where we have written

$$z_{se} = \sum_{m,n} b'_{elmn} b_{smn}$$

and

$$\begin{aligned} w_{g_s} &= \sum_r a'_{ir} a_{gr} \\ \bar{w}_{r_j} &= \sum_g a'_{g_j} a_{gr} \end{aligned} \quad (5)$$

6.

By changing dummy indices in the second term, we can cast this into the form

$$3 \sum z_{s\ell} \left\{ w_{g\ell} \langle \ell \ell / g s \rangle + \bar{w}_{g\ell} \langle \bar{\ell} \ell / \bar{g} s \rangle \right\}$$

From Eq. (5), and from the fact that for the non-strange mesons we see that

$$a_{ij} = P a_{ji}$$

there $P = \pm 1$, we find that

$$w_{g\ell} = P P' \bar{w}_{g\ell} = \gamma \bar{w}_{g\ell}$$

where the parameter $\gamma = \pm 1$ depends only on the symmetries of the initial and final meson, and is given for initial π states by

final state	π	ρ	ω	η
γ	+1	-1	+1	-1

So that we finally get

$$\langle \lambda_4 \lambda_3 | \hat{T} | \lambda_2 \lambda_1 \rangle = 3 \sum_{s\ell} Z_{s\ell} W_{g\ell} \left\{ \langle \ell \ell | g s \rangle + \gamma \langle \ell \ell | \bar{g} s \rangle \right\} \quad (6)$$

A similar procedure for baryon-baryon scattering gives

$$\langle \lambda_4 \lambda_3 | \hat{T} | \lambda_2 \lambda_1 \rangle = 9 \sum_{a\ell} Z_{a\ell}^{\lambda_2 \lambda_4} Z_{g\ell}^{\lambda_1 \lambda_3} \langle \ell \ell | a g \rangle \quad (7)$$

If we assume isotopic invariance, then all quark amplitudes can be written as A_T^λ where $\lambda = 1 \dots 6$ is the helicity index of Table I, and T refers to the total isotopic spin, so that $T = 1$ or 0 for (non-strange quark)-(non-strange quark) scattering, and $T = \frac{1}{2}$ for (strange quark)-(non-strange quark) scattering.

The problem of calculating amplitudes is thus reduced to the simple problem of calculating the W and Z matrices. These are given in Appendix II for the reactions which we shall consider.

II. - CALCULATION OF RESULTS IN THE MODEL

Using Eq. (6), we can proceed to calculate reactions of the type

$$\begin{aligned}
 \pi p &\rightarrow \pi (N, \Delta) \\
 &\rightarrow \eta (N, \Delta) \\
 &\rightarrow \rho (N, \Delta) \\
 &\rightarrow \omega (N, \Delta)
 \end{aligned}
 \tag{8}$$

as well as strange particle reactions of the type

$$\begin{aligned}
 K p &\rightarrow K \Delta \\
 &\rightarrow K^* p \\
 &\rightarrow \omega \Lambda \\
 &\rightarrow \phi \Lambda
 \end{aligned}
 \tag{9}$$

which are tabulated in Appendix III.

The calculation of the amplitudes is greatly aided by the fact that the additivity assumption forbids reactions of the type

$$\pi^- p \rightarrow \left(\begin{matrix} \pi^+ \\ \rho^+ \end{matrix} \right) \Delta^-
 \tag{10}$$

where double charge exchange must occur. Consequently, by isotopic invariance, there is only one independent amplitude which must describe all reactions of the type ⁵⁾

$$\pi p \rightarrow \pi \Delta
 \tag{11}$$

and another single amplitude for

$$\pi p \rightarrow \rho \Delta$$

Therefore, in Appendix III, we give one example of each of these processes, and a list of the other amplitudes which are related to it by a simple Clebsch-Gordan coefficient.

Unfortunately, the amplitudes in Appendix III cannot easily be put into a form which allows the easy comparison of cross-sections, since the square of each helicity amplitude involves cross terms between different A_{π}^{λ} which are difficult to separate. If one wishes to derive exact expressions, one must resort to inequalities. For example, if we denote by $\nabla(A+B \rightarrow C+D)$ the sum of the squares of the helicity amplitudes suitably corrected for phase space, we see that

$$\begin{aligned} \nabla(\pi^- p \rightarrow \pi^0 n) &> \nabla(\pi^- p_{\frac{1}{2}} \rightarrow \pi^0 n_{-\frac{1}{2}}) \\ &= \frac{25}{8} \nabla(\pi^- p \rightarrow \pi^0 \Delta^0) \end{aligned} \quad (12)$$

an expression given in a slightly different form elsewhere ⁵⁾.

In addition, we can make several statements about the asymptotic limits of various ratios of amplitudes on the basis of assumptions about the dynamics of the quark-quark interaction. We shall show in Part V that, for example, the charge-exchange amplitude

$$\langle p n | n p \rangle + \gamma \langle \bar{n} n | \bar{p} p \rangle \quad (13)$$

which appears in Appendix III can be written asymptotically as

$$\langle p n | n p \rangle [1 + \gamma(\beta)] \quad (14)$$

where β is a phase which depends on the model which we choose for the basic quark interaction. In terms of these parameters, we can easily see that in the asymptotic region

$$\frac{\Gamma(\pi^- B \rightarrow \eta^0 B')}{\Gamma(\pi^- B \rightarrow \pi^0 B')} = \frac{|1-\beta|^2}{|1+\beta|^2} = R_a \quad (15a)$$

$$\frac{\Gamma(\pi^- B \rightarrow \rho^0 B')}{\Gamma(\pi^- B \rightarrow \omega^0 B')} = \frac{|1-\beta|^2}{|1+\beta|^2} = R_b \quad (15b)$$

where B and B' are any baryons which are allowed to appear in the reactions by the usual selection rules. We also find

$$\frac{\Gamma(\pi^- p \rightarrow \eta^0 n)}{\Gamma(\pi^- p \rightarrow \eta^0 n)} = \frac{|1-\beta|^2}{2} = R_c \quad (15c)$$

$$\frac{\Gamma(\pi^- p \rightarrow \pi^0 n)}{\Gamma(\pi^- p \rightarrow \pi^0 n)} = \frac{|1+\beta|^2}{2} = R_d \quad (15d)$$

$$\frac{\Gamma(\pi^+ N \rightarrow \omega \left(\begin{smallmatrix} \Delta \\ N \end{smallmatrix} \right))}{\Gamma(\pi^+ N \rightarrow \omega \left(\begin{smallmatrix} \Delta \\ N \end{smallmatrix} \right))} = \frac{|1-\beta|^2}{2} = R_e \quad (15e)$$

$$\frac{\Gamma(\pi^+ p \rightarrow \pi^0 \Delta^{++})}{\Gamma(\pi^+ p \rightarrow \pi^0 \Delta^{++})} = \frac{|1-\beta|^2}{2} = R_f \quad (15f)$$

Another set of statements can be made about various reactions if we consider the problem of configuration mixing. The ω wave function which is given in Appendix I can be written

$$\omega = (q \bar{q})_{0,1} \quad (16)$$

where q and $\bar{q}$ refer to the non-strange quarks, and the indices refer to total isotopic spin and spin, respectively. This is the usual quark model assignment of the ω . In this model we write

$$\phi = (\lambda \bar{\lambda})_{0,1} \quad (17)$$

If, however, we wish to write the ω and ϕ as

$$\omega = \cos \theta_m (q \bar{q})_{0,1} + \sin \theta_m (\lambda \bar{\lambda})_{0,1}$$

$$\phi = -\sin \theta_m (q \bar{q})_{0,1} + \cos \theta_m (\lambda \bar{\lambda})_{0,1} \quad (18)$$

in the usual SU_3 prescription, then it is clear that the part of the wave functions involving $(\lambda \bar{\lambda})$ cannot contribute to charge-exchange ω or ϕ production, since producing this final state from an incident pion would involve changing both pion quarks simultaneously, and such a process is forbidden by the additivity assumption.

It then follows that

$$\frac{\sigma(M^\pm B \rightarrow \omega B')}{\sigma(M^\pm B \rightarrow \phi B')} = \cot^2 \theta_m \quad (19)$$

where B and B' are as defined above, and $M^\pm$ is any charged meson. We can then regard the result of Eq. (19) as a test of the hypothesis that the ϕ and ω wave functions are not the pure quark model wave functions given in Appendix I, but are mixtures of these wave functions.

In addition to this familiar $\omega - \phi$ mixing, there is evidence from the decay of the 2^+ mesons ⁷⁾ that the η is also not the state

$$\eta = (s\bar{s})_{0,0} \quad (20)$$

as given in Appendix I, but is, in fact, given by the state

$$\eta = \cos \alpha (\lambda\bar{\lambda})_{0,0} + \sin \alpha (s\bar{s})_{0,0} \quad (21)$$

As before, when considering charge-exchange reactions, the $\lambda\bar{\lambda}$ part of the wave function does not contribute, so that Eq. (15a) becomes

$$\frac{\nabla(\pi^- B \rightarrow \eta_0 B')}{\nabla(\pi^- B \rightarrow \pi^0 B')} = \sin^2 \alpha \frac{|1 - \beta|^2}{|1 + \beta|^2} \quad (15a')$$

We shall return to a discussion of the magnitudes of θ_m and α in a later section.

As was mentioned in the introduction, the difficulties encountered in obtaining relations between cross-sections largely disappear when we turn to density matrices. Since the calculation of helicity amplitudes is particularly simple in this model, we can expect to calculate spin correlations easily also.

The method for obtaining the density matrix from the helicity amplitudes has been given elsewhere ⁸⁾. Consider, for example, the reaction $\pi^- p \rightarrow \rho^- p$. We can use the amplitudes in Appendix III to calculate

$$\hat{\rho}_{\mu\mu'} = \frac{1}{N} \sum_{\lambda\lambda'} \langle \mu \lambda | \lambda' \rangle \langle \mu' \lambda | \lambda' \rangle^* \quad (22)$$

where N is determined by the condition $\text{Tr}(\hat{\rho}) = 1$ and where μ and μ' represent the ρ helicity, and λ' and λ the helicities of the initial and final nucleon, respectively. The density matrix is given by

$$\rho_{mm'} = \sum_{\mu\mu'} d_{m\mu}^{\frac{1}{2}10} \hat{\rho}_{\mu\mu'} d_{\mu'm'}^{\frac{1}{2}1-0} \quad (23)$$

We can then label the helicity amplitudes as

$$\langle 1 \frac{1}{2} | \frac{1}{2} \rangle = \langle -1 -\frac{1}{2} | -\frac{1}{2} \rangle = \langle -1 \frac{1}{2} | \frac{1}{2} \rangle = \langle 1 -\frac{1}{2} | -\frac{1}{2} \rangle = B$$

$$\langle 1 -\frac{1}{2} | \frac{1}{2} \rangle = - \langle -1 \frac{1}{2} | -\frac{1}{2} \rangle = C$$

$$\langle -1 -\frac{1}{2} | \frac{1}{2} \rangle = - \langle 1 \frac{1}{2} | -\frac{1}{2} \rangle = D$$

$$\langle 0 \frac{1}{2} | \frac{1}{2} \rangle = - \langle 0 -\frac{1}{2} | -\frac{1}{2} \rangle = E$$

(24)

We note in passing that the same set of helicity amplitudes appears in the reaction $\pi^+ n \rightarrow \omega p$, so that the results for ρ^- production can be carried over to this reaction by letting $B \rightarrow B'$, $C \rightarrow C'$, ..., etc., where the primes refer to the amplitudes associated with $\pi^+ n \rightarrow \omega p$. We then find that

$$\hat{\rho}_{\mu\mu'} = \frac{1}{N} \begin{pmatrix} 2|B|^2 + |C|^2 + |D|^2 & 0 & 2|B|^2 + C^*D + D^*C \\ 0 & 2|F|^2 & 0 \\ 2|B|^2 + C^*D + D^*C & 0 & 2|B|^2 + |C|^2 + |D|^2 \end{pmatrix} \quad (25)$$

where

$$N = 2 [2|B|^2 + |D|^2 + |C|^2] + 2|F|^2 \quad (26)$$

Thus the first prediction of the quark model is that the density matrix is pure real. Since our model is expected to work only at small scattering angles, we calculate the density matrix elements at $\theta = 0$. We find that

$$\begin{aligned} \rho_{00}(0) &= \frac{2|F|^2}{N} \\ \rho_{10}(0) &= 0 \\ \rho_{1-1}(0) &= \frac{2|B|^2 + C^*D + D^*C}{N} \end{aligned} \quad (27)$$

Turning our attention to the reaction

$$\pi N \rightarrow \pi \Delta \quad (28)$$

we can make two important observations immediately. First, the absence of double charge exchange processes in this model implies that the amplitudes for all processes of the above type are proportional. Consequently, the model will give the same density matrix elements for each of them.

Secondly, we see from Appendix III that there is only one amplitude in the problem, namely

$$\frac{1}{\sqrt{3}} \langle \frac{3}{2} \mid \frac{1}{2} \rangle = \langle -\frac{1}{2} \mid \frac{1}{2} \rangle = \tilde{B} \quad (29)$$

This means that the density matrix will be independent of $\tilde{B}$. This reaction, then, should give us a very critical test of the application of the quark model to inelastic interactions. We find

$$\rho_{\mu\mu'} = \frac{\tilde{B} \tilde{B}^*}{8 \tilde{B} \tilde{B}^*} \begin{pmatrix} 3 & 0 & \sqrt{3} & 0 \\ 0 & 1 & 0 & \sqrt{3} \\ \sqrt{3} & 0 & 1 & 0 \\ 0 & \sqrt{3} & 0 & 3 \end{pmatrix} \quad (30)$$

Finally, we turn to the reaction

$$\pi N \rightarrow \rho \Delta \quad (31)$$

where we can again define amplitudes

$$\begin{aligned} \langle 1 \frac{3}{2} \mid \frac{1}{2} \rangle &= B' \\ \langle 1 -\frac{1}{2} \mid \frac{1}{2} \rangle &= C' \\ \langle 0 \frac{1}{2} \mid \frac{1}{2} \rangle &= F' \end{aligned} \quad (32)$$

which lead to the results

$$\begin{aligned}
\rho_{00}^{(0)} &= \frac{2 |F'|^2}{2 |F'|^2 + 4 (|B'|^2 + |C'|^2)} \\
\rho_{10}^{(0)} &= 0 \\
\rho_{1-1}^{(0)} &= \frac{\text{Re} (C' B')}{2 |F'|^2 + 4 (|B'|^2 + |C'|^2)} \\
\rho_{33}^{(0)} &= \frac{1}{4} \\
\rho_{31}^{(0)} &= 0 \\
\rho_{3-1}^{(0)} &= \frac{\text{Re} (C' B')}{2 (|B'|^2 + |C'|^2)} \quad (33)
\end{aligned}$$

The density matrix for

$$\pi N \rightarrow \omega \Delta$$

should be identical to the above except the sign of γ in all of the amplitudes should be reversed.

The density matrices for reactions involving strange particles can be derived simply from the above calculations. For example, from Appendix III, we see that

$$D (K^+ p \rightarrow K^0 \Delta) = D (\pi^+ p \rightarrow \pi^0 \Delta) \quad (34)$$

where $D(A+B \rightarrow C+D)$ stands for any density matrix element of the reaction indicated. This relation follows because the amplitudes for the reaction on the left-hand side of Eq. (34) are precisely those which appear in Eq. (29). The only difference between the two reactions in Eq. (34) is in the composition of these amplitudes in terms of quark amplitudes, which cancel out anyway.

If we consider now reactions of the type

and

we see that there is only one isotopic channel through which they can go, namely the $T = 1$ channel. Consequently, all density matrices corresponding to reactions of the type (35) must be equal, as must the matrices corresponding to (36), although the two need not be equal to each other. Consequently, Eq. (34) holds for all reactions of the type (35).

To get the density matrices of the reaction

we simply note that the W and Z matrices are the same for this reaction as for

except for the fact that in the former, no $\bar{A}$ amplitudes appear. Consequently, we find that

$$\begin{aligned} \rho_{00}(0) &= \frac{2|F'|^2}{2|F'|^2 + 4[|B'|^2 + |C'|^2]} \\ \rho_{10}(0) &= 0 \\ \rho_{1-1}(0) &= \frac{\text{Re}(C'B')}{|F'|^2 + 2[|C'|^2 + |B'|^2]} \end{aligned} \quad (39)$$

where F' , B' , C' , ..., etc., are defined as in Eq. (34), but with $\bar{A} = 0$.

If we turn to the reaction

we find that the process differs fundamentally from the reactions which we have been considering up to now in that it involves A_1 , the (non-strange quark)-(strange quark) scattering amplitude. If we define

$$\begin{aligned} \langle 1 \frac{1}{2} | \frac{1}{2} \rangle &= \langle -1 \frac{1}{2} | \frac{1}{2} \rangle = L \\ \langle 1 -\frac{1}{2} | \frac{1}{2} \rangle &= - \langle -1 -\frac{1}{2} | \frac{1}{2} \rangle = M \\ \langle 0 \frac{1}{2} | \frac{1}{2} \rangle &= R \\ \langle 0 -\frac{1}{2} | \frac{1}{2} \rangle &= Q \end{aligned} \quad (41)$$

we get

$$\begin{aligned} \rho_{00}^{(0)} &= \frac{2|R|^2 + 2|Q|^2}{N} \\ \rho_{10}^{(0)} &= \frac{2 \operatorname{Re}(MQ)}{N} \\ \rho_{1-1}^{(0)} &= \frac{2|L|^2 - 2|M|^2}{N} \end{aligned} \quad (42)$$

where

$$N = 2 \left[|R|^2 + |Q|^2 + 2(|L|^2 + |M|^2) \right] \quad (43)$$

III. - EXACT PREDICTIONS OF THE QUARK MODEL

We shall divide the discussion of the comparison of the quark model with the data into two parts : those which involve no further assumptions than those laid down in Part II, and those which are derived on the basis of some assumption or other about the dynamics of the quark-quark interaction. We shall discuss the former in this section. It is no accident, therefore, that it is the shortest section of the paper.

Starting with the cross-section calculations, we find that data exist for the inequality of Eq. (12) (at 8 GeV/c)⁹⁾, at which point the inequality seems to be slightly violated.

As has been pointed out⁵⁾, the violation is small enough to allow us to hope that at slightly higher energies, it may disappear. Equation (12) contains another interesting prediction. The cross-section for the reaction $\pi^- p \rightarrow \pi^0 \Delta^0$ is equal to the pion-nucleon charge exchange spin flip amplitude, which vanishes at $\theta = 0$ ¹⁰⁾. This means that at high energies the amplitude for Δ production in the forward direction must vanish.

It has been reported⁵⁾ that the production of ϕ mesons is very much suppressed compared to other production processes. From Eq. (19), this means that θ_m , the $\omega - \phi$ quark model mixing angle must be very small. Consequently, it would seem that the pure quark model assignment of the ω in Eq. (16) and Eq. (17) is better than the SU_3 assignment in Eq. (18).

With these few comments, clearly, we have exhausted the model as far as exact results for cross-sections go. As we pointed out in the introduction, however, we expect that the calculation of density matrix elements should be a much cleaner test of the model than cross-sections, since in many cases we can find density matrix elements which are independent of quark parameters. For example, we see that the elements of

$\hat{\rho}_{\mu\mu'}$, in Eq. (30) are completely independent of the single quark amplitude $\tilde{B}$, so that the prediction which is derived from this expression for the elements of $\rho_{mm'}$ will be an exact result. It will involve no assumptions other than those which went into the construction of the model in the first place, and will therefore be a clean test of the quark model as applied to inelastic processes. These results are not however, unique to the quark model, since they can be derived from a combination of the Stodolsky-Sakurai and the quark model, also ¹¹⁾.

In addition, in certain reactions one or more density matrix elements may turn out to be independent of the quark amplitudes [see, for example, Eq. (33)].

In Table II, we present these matrix elements together with the data presently available.

Whether or not this constitutes good agreement or not depends to a certain extent on individual prejudices. However, with the single exception of associated Δ production, the quark model gives a reasonable fit to the data as it exists at the moment. In particular, the signs and relative magnitudes are matched remarkably well, especially when one considers the simplicity of the model. It should be noted, however, that the data involves an average over small angular spreads around $\theta \simeq 0$, while the theoretical prediction is for $\theta = 0$. When better data are available, then, these exact results of the quark model can be evaluated more readily.

Having now exhausted all of the results of the model which do not depend on assumptions about quark dynamics, we turn to the problem of trying to get other predictions from the models with as few extra assumptions as possible.

Reaction	Element	Theory	Experiment at 8 GeV/c	Experiment at 4 GeV/c
$\pi N \rightarrow \pi \Delta$	$\rho_{3 \ 3}$	0.375	0.22 ± 0.06	0.40 ± 0.06
	$\rho_{3 \ -1}$	0.215	0.132 ± 0.07	0.21 ± 0.08
	$\rho_{3 \ 1}$	0	0.066 ± 0.06	-0.03 ± 0.07
$\pi p \rightarrow \omega \Delta$	$\rho_{3 \ 3}$	0.25	0.24 ± 0.08	0.15 ± 0.04
	$\rho_{3 \ -1}$	0	0.017 ± 0.09	0.04 ± 0.04
$\pi p \rightarrow \rho \Delta$	$\rho_{3 \ 3}$	0.25	0.05 ± 0.03	0.08 ± 0.03
	$\rho_{3 \ -1}$	0	0.015 ± 0.028	0.01 ± 0.03
Experiment at 3 GeV/c				
$K^+ p \rightarrow K^0 \Delta$	$\rho_{3 \ 3}$	0.375	0.28 ± 0.06	
	$\rho_{3 \ -1}$	0.215	0.21 ± 0.05	
	$\rho_{3 \ 1}$	0	0.04 ± 0.05	
Experiment at 5 GeV/c				
$K^+ p \rightarrow K^{*} \Delta^{++}$	$\rho_{3 \ 3}$	0.25	0.18 ± 0.09	
	$\rho_{3 \ -1}$	0	0.01 ± 0.02	

- Table II -

Exact predictions of the quark model for density matrices. The data for the first three reactions from Ref. ¹²⁾, the fourth from Ref. ¹³⁾, and the fifth from Table 3 of Ref. ¹⁴⁾.

IV. - OTHER RESULTS OF THE MODEL

As in the previous section, we shall treat cross-sections first, and then density matrices.

It is well known ¹⁵⁾ that it is possible to formulate a Pomeranchuk theorem for inelastic interactions, and that the amplitude for the reaction

$$A + B \rightarrow C + D$$

becomes equal, up to a phase, to the asymptotic amplitude for

$$A + \bar{C} \rightarrow \bar{B} + D$$

If we apply this theorem to the amplitudes in Eq. (13), the unknown phase can be called β , and we get the asymptotic expression in Eq. (14). To go farther, however, it is necessary to make dynamical assumptions about the basic quark interaction to determine β . In Ref. ⁵⁾, for example, it was assumed that the charge-exchange reaction between quarks goes primarily by ρ exchange, so that $\beta = -1$.

On the other hand, it has recently been pointed out ¹⁶⁾ that many properties of high energy elastic scattering can be explained on the basis of the so-called "exchange degeneracy" Regge model, in which one assumes that the residue functions of the ρ and R are equal. In this case,

$$\beta = e^{i\pi\alpha(t)} = i \quad (44)$$

where the second equality holds for high energies and where the momentum transfer associated with forward scattering approaches zero.

We could now turn to Eq. (15) and calculate the asymptotic values of the various ratios R for each model, and then compare with what data are available. Before doing so, however, we wish to point out that the value of

$$R_a = \frac{\Gamma(\pi^- p \rightarrow \eta_0 n)}{\Gamma(\pi^- p \rightarrow \pi^0 n)}$$

is greater than 1 in both models, whereas the experimentally observed value of this ratio ¹⁷⁾ at 18.8 GeV/c (which is, one would hope, in the asymptotic region) is in fact about $\sim \frac{1}{5}$. We can understand this only in terms of the $\eta - X_0$ mixing discussed in connection with Eq. (15a'). In fact, we can turn the problem around and use the experimental value of R_a and R_c to determine α , and see if the conjecture of Ref. ⁷⁾ that the η is primarily $(\lambda \bar{\lambda})_{0,0}$ is verified. In Table III, we present calculations of α from both R_a and R_c for both of the models which we have presented.

Ratio	β	Energy of data	$\sin^2 \alpha$
R_a	-1	18.8	≈ 0
	i	18.8	$\approx \frac{1}{5}$
R_c	-1	9.8	$\approx \frac{1}{10}$
	i	9.8	$\approx \frac{1}{5}$

- Table III -

The data for R_a are taken from Ref. ¹⁷⁾
and for R_c from Ref. ⁹⁾.

We see, then, that the scattering data at present can only be used to estimate α , and that all that can be said is that $\alpha \lesssim 30^\circ$. This is not as small a mixing as has been suggested ⁷⁾, but it should be possible to see whether the α predicted from decay data and the α predicted from scattering data agree when better data become available. The general idea of $\eta - X_0$ mixing, however, seems to be verified. We should also note that we can then say, in analogy to Eq. (19), that at high energies,

$$\frac{\sigma(M^\pm B \rightarrow \eta B')}{\sigma(M^\pm B \rightarrow X^0 B')} = \tan^2 \alpha \ll 1 \quad (45)$$

No data exist on differential cross-sections for reactions of this type, but reaction cross-sections at 3.5 GeV/c ¹⁸⁾ are not in the ratio of $\tan^2 \alpha$. It will be interesting to see if this persists at higher energies.

It must be pointed out, however, that the "mixing" of particles of such widely different masses as the η and X_0 is quite likely to introduce form factors into Eq. (45), since the radial integrals in the two interactions considered are likely to be quite different from each other. This effect is not calculable at the present time ⁶⁾, so that deviations from Eq. (45) would not be surprising.

In Table IV, we present the predictions of the model for the ratios R_b , R_d and R_F , together with some experimental results.

Ratio	β	Theory	Energy (GeV/c)	Experiment
R_b	-1	$\gg 1$	8	10
	i	1		
R_d	-1	0	9.8	1
	i	1		
R_f	-1	2	3.5	~ 0.5
	i	1		

- Table IV -

Data for the first and last reactions from Ref. 14), and for the second from Ref. 9) and Ref. 17).

The agreement here looks pretty bad, but one must exercise caution in interpreting the results of Table IV, since none of the energies are really asymptotic so that Eq. (15) cannot really be applied. Also, the expression for β in Eq. (44) is sensitive to deviations from the asymptotic condition that $t = 0$ at $\theta = 0$.

At any rate, the results of Eq. (15) depend only on the assumptions of the quark model, and there are, at most, two unknowns in the six relations given (the unknowns being α , the $\eta - X_c$ mixing angle, and β , where the condition $|\beta| = 1$ has been used). Consequently, when good data become available, a test of the quark model will be possible in which one looks at the ratios $R_d, \dots, R_f$ and sees whether they can be fit with these two parameters. Since the parameter α can also be determined from decay data, one should be able to have another clean test of the quark model.

In addition, an experimental determination of β would shed some light on the problem of the basic quark interaction.

Finally, we note in passing that the fact that ¹⁴⁾ the reaction

$$K^- p \rightarrow \phi \Lambda \quad (46)$$

has a cross-section comparable to reactions like

$$\begin{aligned} & \rightarrow \pi^- \Sigma^+ \\ K^- p & \rightarrow \pi^- \gamma^{*+} \\ & \rightarrow \rho^- \Sigma^+ \end{aligned}$$

even though in (46) the number of Λ quarks is not conserved on both sides of the equation is not a difficulty in this model, but does contradict the hypothesis of quark rearrangement ³⁾, at least as that hypothesis is applied to peripheral interactions. In fact, if we assume that

$$A_{\frac{1}{2}} \sim \bar{A}_0$$

we find the inequality

$$\sigma(K^- p \rightarrow \omega \Lambda) > \frac{\sigma(K^- p \rightarrow \phi \Lambda)}{2} \quad (47)$$

which is satisfied at 3 and 3.5 GeV/c ¹⁴⁾.

Turning our attention to the density matrices, the fact that quark amplitudes do not cancel out of matrix elements in, for example, the reaction

$$\pi^- p \rightarrow \rho^- p$$

means that a certain amount of fitting will be necessary. Let us consider the ρ density matrix.

The predominant feature is that the ρ meson is preferentially polarized perpendicular to the momentum of the incoming pion, so that $\rho_{00} \simeq 1$ and other matrix elements are quite small. We can reproduce this effect by letting

$$|F|^2 \gg |B|^2, |C|^2, |D|^2 \quad (48)$$

Since this assumption will be used many times in the subsequent analysis, we shall make a few comments on it at this point. We see from Appendix III that there are two general types of quark amplitudes which appear in the helicity amplitudes. First, there are the ordinary helicity flip inelastic amplitudes, such as A_T^4 , and then there are the differences of elastic amplitudes, usually in the form $A_T^3 - A_T^1$. Since there are a large number of individual amplitudes, it would be fruitless to try to discover the nature of each one. Rather, we assume, for want of anything better, that the amplitudes in each of the two classes are roughly the same, and we can regard Eq. (27) as a statement, derived empirically, about the relative magnitudes of the amplitudes belonging to the two classes. This is clearly not a perfect way to proceed, but we shall see that even with this very crude assumption on the quark amplitudes, a great deal of useful information can be derived.

Our procedure shall then be to apply the assumption in Eq. (48) to all other vector meson production processes and see whether the results are in agreement with experiment. We can think of this process in the following way: we "derive" Eq. (48) by fitting the observed decay data for ρ^- production, apply this condition to predict the results of other production processes, and then compare these predictions with experimental data. Such a comparison is given for amplitudes involving non-strange quarks in Table V.

Reaction	Element	Theory	Experiment at 8 GeV/c	Experiment at 4 GeV/c
$\pi p \rightarrow \rho \Delta$	$\rho_0 \ 0$	1	0.77 ± 0.04	0.77 ± 0.04
	$\rho_1 \ -1$	0	-0.035 ± 0.02	-0.04 ± 0.03
	$\rho_1 \ 0$	0	-0.119 ± 0.03	-0.06 ± 0.03
$\pi p \rightarrow \rho p$	$\rho_0 \ 0$	1	0.54 ± 0.07	0.70 ± 0.08
	$\rho_1 \ -1$	0	0.075 ± 0.062	0.17 ± 0.08
	$\rho_1 \ 0$	0	-0.08 ± 0.05	-0.07 ± 0.07
$\pi p \rightarrow \omega \Delta$	$\rho_0 \ 0$	1	0.25 ± 0.10	0.47 ± 0.05
	$\rho_1 \ -1$	0	0.17 ± 0.08	0.13 ± 0.05
	$\rho_1 \ 0$	0	-0.03 ± 0.06	-0.10 ± 0.03
Experiment at 3 GeV/c				
$\pi^+ n \rightarrow \omega p$	$\rho_0 \ 0$	1	> 0.5	
	$\rho_1 \ -1$	0	< 0.2	
	$\rho_1 \ 0$	0	?	
Experiment at 5 GeV/c				
$K^+ p \rightarrow K^* \Delta^{++}$	$\rho_0 \ 0$	1	0.71 ± 0.05	
	$\rho_1 \ -1$	0	0.01 ± 0.02	
	$\rho_1 \ 0$	0	-0.08 ± 0.02	

- Table V -

Data for the first three reactions from
Ref. 12), for the fourth from Ref. 19),
and for the last from Ref. 14)

The agreement is quite good for the ρ meson matrix elements, but not so good for the ω , although even here it does predict which element should be the largest.

Next, we turn to the Δ matrix elements which have not yet been discussed. In this case the physically reasonable assumption that

$$A_T^2 = A_T^4 \quad (49)$$

(that is, that the amplitude to flip both quark helicities is independent of the helicity orientations) gives the results of Table VI.

Reaction	Element	Theory	Experiment at 8 GeV/c	Experiment at 4 GeV/c
$\pi p \rightarrow \omega \Delta$	$\rho_3 \cdot 1$	-0.215	-0.113 ± 0.08	-0.05 ± 0.04
$\pi p \rightarrow \rho \Delta$	$\rho_3 \cdot 1$	-0.215	-0.076 ± 0.03	-0.01 ± 0.03
Experiment at 5 GeV/c				
$K^+ p \rightarrow K^* \Delta^+$	$\rho_3 \cdot 1$	-0.215	0.08 ± 0.02	

- Table VI -

Data taken from same sources as in Table V.

Once again, the Δ matrix elements for associated resonance production are very bad. This leads us to suppose that whatever causes the failure of the model in predicting the Δ density matrix for

$$\pi p \rightarrow \rho \Delta$$

is the same thing that causes the failure in

$$K p \rightarrow K^* \Delta$$

At any rate, this must definitely be viewed as a failure of the model.

Finally, we turn to the interaction

$$K^+ p \rightarrow K^* p$$

This process differs from those previously considered in that amplitudes involving Λ , the strange quark, survive in the helicity amplitudes. If we assume that they behave no differently from the other amplitudes, we can apply Eq. (27) to give the density matrix results in Table VII.

Reaction	Element	Theory	Experiment at 3 GeV/c	Experiment at 5 GeV/c
$K^+ p \rightarrow K^* p$	ρ_{00}	1	0.07 ± 0.06	0.25 ± 0.12
	ρ_{10}	0	0.32 ± 0.06	0.42 ± 0.11
	ρ_{1-1}	0	0.10 ± 0.05	-0.04 ± 0.09

- Table VII -

Data from Ref. ²⁰⁾ for 3 GeV/c, and report of private communication in Ref. ²¹⁾ for the data at 5 GeV/c .

The results are surprisingly bad, especially in the light of the success encountered in vector meson production previously. We must conclude that the amplitudes which involve λ behave in a different fashion from those that do not. This is not particularly surprising, however, since even in elastic scattering at asymptotic energies, the requirements of charge invariance and the Pomeranchuk theorem reduce the number of independent amplitudes to 3: the qq amplitude, the $q\lambda$ and the $\lambda\lambda$, and there is no particular reason why similarities should exist between them.

CONCLUSION

The most decisive test of the quark model as applied to inelastic scattering is contained in Table II. The theoretical results involve no statements about quark dynamics, and thus are exact predictions of the model against which future improved experimental results can be compared. The model compares favourably to experiment at present except for reactions in which ρ and Δ are both produced in the final state. One possible explanation of this discrepancy might be that in this type of reaction, where two broad resonances are being produced, the decay of particles in the final states begins so soon after the collision that the representation of the final states in terms of the quark wave functions is not valid. This would explain, for example, the fact that ω production is much better fitted than ρ production. If this explanation is correct, then at higher energies, when the time during which the interacting particles are in contact with each other is smaller, we would expect the density matrices to approach the theoretical values given in Table II.

The problem of estimating corrections to the model due to multiple scattering is quite difficult in general, since a good calculation would require a knowledge of the quark propagator.

However, we can find some experimental justification for the assumption that such corrections must be small by considering the reaction

$$\pi^- \rho \rightarrow \pi^+ \Delta^-$$

which can only go by double quark scattering. This amplitude is found ²²⁾ to be quite small compared to the amplitudes which the quark model does not forbid, so that the model seems to be justified in this case.

The results of Tables V, VI and VII, which involve assumptions about quark interactions are roughly those which would be obtained by a simple one particle exchange model ²¹⁾, and have the same good and bad points as that model. However, in all of these results, better fits would be possible if the individual quark amplitudes could be calculated.

It seems, then, that the quark model for inelastic scattering gives quite good results for spin correlations. The exact results for cross-sections are not so sure, since they involve only inequalities. However, the results in Eq. (15) do give a clean test of the theory in the asymptotic region. When high energy data on the charge-exchange reactions involved become available, one will be able to say more about the model, and also, from a knowledge of β , about the basic quark interaction.

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- APPENDIX I -

In this Appendix, we present the wave functions of the elementary particles in terms of quark assignments. We use the notation

$$S(i, j) = \xi_i \bar{\xi}_j + \xi_j \bar{\xi}_i$$

$$A(i, j) = \xi_i \bar{\xi}_j - \xi_j \bar{\xi}_i$$

and

$$\begin{aligned} S(i, j, k) = & \xi_i \xi_j \bar{\xi}_k + \xi_i \xi_k \bar{\xi}_j + \xi_k \xi_i \bar{\xi}_j \\ & + \xi_j \xi_i \bar{\xi}_k + \xi_j \xi_k \bar{\xi}_i + \xi_k \xi_j \bar{\xi}_i \end{aligned}$$

The mesons are given by ^{*)}

$$\rho_{+1}^+ = (1\bar{1})$$

$$\rho_0^+ = \frac{1}{\sqrt{2}} S(13)$$

$$\rho_{-1}^+ = (3\bar{3})$$

$$\rho_{+1}^- = (2\bar{2})$$

$$\rho_0^- = \frac{1}{\sqrt{2}} S(42)$$

$$\rho_{-1}^- = (4\bar{4})$$

$$\rho_{+1}^0 = \frac{1}{\sqrt{2}} S(21)$$

$$\rho_0^0 = \frac{1}{\sqrt{2}} [S(41) + S(23)]$$

$$\rho_{-1}^0 = \frac{1}{\sqrt{2}} S(43)$$

^{*)}

the upper index refers to charge, the lower to helicity.

$$\bar{K}^{*-}_{+1} = (\bar{2}5)$$

$$\bar{K}^{*0}_{+1} = (15)$$

$$\bar{K}^{*-}_0 = \frac{1}{\sqrt{2}} [(\bar{2}6) + (\bar{4}5)]$$

$$\bar{K}^{*0}_0 = \frac{1}{\sqrt{2}} [(16) + (\bar{3}5)]$$

$$\bar{K}^{*-}_{-1} = (\bar{4}6)$$

$$\bar{K}^{*0}_{-1} = (\bar{3}6)$$

While the baryons are

$$P_{\frac{1}{2}} = \frac{1}{3\sqrt{2}} [5(123) - 25(114)]$$

$$P_{-\frac{1}{2}} = \frac{1}{3\sqrt{2}} [25(233) - 5(134)]$$

$$N_{\frac{1}{2}} = \frac{1}{3\sqrt{2}} [25(223) - 5(124)]$$

$$N_{-\frac{1}{2}} = \frac{1}{3\sqrt{2}} [5(243) - 25(144)]$$

$$\Delta^{++}_{\frac{3}{2}} = (111)$$

$$\Delta^{+}_{\frac{3}{2}} = \frac{1}{\sqrt{3}} 5(112)$$

$$\Delta^{++}_{\frac{1}{2}} = \frac{1}{\sqrt{3}} 5(113)$$

$$\Delta^{+}_{\frac{1}{2}} = \frac{1}{3} [5(114) + 5(123)]$$

$$\Delta^{++}_{-\frac{1}{2}} = \frac{1}{\sqrt{3}} 5(133)$$

$$\Delta^{+}_{-\frac{1}{2}} = \frac{1}{3} [5(134) + 5(233)]$$

$$\Delta^{++}_{-\frac{3}{2}} = (333)$$

$$\Delta^{+}_{-\frac{3}{2}} = \frac{1}{\sqrt{3}} 5(334)$$

$$\pi^+ = \frac{1}{\sqrt{2}} A(13)$$

$$\pi^0 = \frac{1}{2} [A(23) + A(14)]$$

$$\pi^- = \frac{1}{\sqrt{2}} A(24)$$

$$\omega_{+1}^0 = \frac{1}{\sqrt{2}} A(21)$$

$$\omega_0^0 = \frac{1}{2} [A(23) + A(41)]$$

$$\omega_{-1}^0 = \frac{1}{\sqrt{2}} A(43)$$

$$\eta_0^0 = \frac{1}{2} [S(23) - S(14)]$$

$$\phi_{+1}^0 = (\bar{5}5)$$

$$\phi_0^0 = \frac{1}{\sqrt{2}} S(56)$$

$$\phi_{-1}^0 = (\bar{6}6)$$

$$K^+ = \frac{1}{\sqrt{2}} [(\bar{5}3) - (\bar{6}1)]$$

$$K^0 = \frac{1}{\sqrt{2}} [(\bar{5}4) - (\bar{6}2)]$$

$$K^{*+}_{+1} = (\bar{5}1)$$

$$K^{*+}_0 = \frac{1}{\sqrt{2}} [(\bar{5}3) + (\bar{6}1)]$$

$$K^{*+}_{-1} = (\bar{6}3)$$

$$K^{*0}_{+1} = (2\bar{5})$$

$$K^{*0}_0 = \frac{1}{\sqrt{2}} [(4\bar{5}) + (2\bar{6})]$$

$$K^{*0}_{-1} = (4\bar{6})$$

$$\bar{K}^- = \frac{1}{\sqrt{2}} [(5\bar{4}) - (6\bar{2})]$$

$$\bar{K}^0 = \frac{1}{\sqrt{2}} [(5\bar{3}) - (6\bar{1})]$$

$$\Delta_{\frac{3}{2}}^0 = \frac{1}{\sqrt{3}} S(122)$$

$$\Delta_{\frac{3}{2}}^- = (222)$$

$$\Delta_{\frac{1}{2}}^0 = \frac{1}{3} [S(223) + S(124)]$$

$$\Delta_{\frac{1}{2}}^- = \frac{1}{\sqrt{3}} S(224)$$

$$\Delta_{-\frac{1}{2}}^0 = \frac{1}{3} [S(234) + S(144)]$$

$$\Delta_{-\frac{1}{2}}^- = \frac{1}{\sqrt{3}} S(244)$$

$$\Delta_{-\frac{3}{2}}^0 = \frac{1}{\sqrt{3}} S(344)$$

$$\Delta_{-\frac{3}{2}}^- = (444)$$

and

$$\Lambda_{\frac{1}{2}}^0 = \frac{1}{6} \{ 2S(126) - S(145) - S(235) \}$$

- APPENDIX II -

In this appendix, we catalogue the W and Z matrices.
For pseudoscalar-pseudoscalar transitions we find

final initial	π^0	η^0
π^-	$W_{21} = W_{43} = \frac{1}{2\sqrt{2}}$	$W_{21} = W_{43} = -\frac{1}{2\sqrt{2}}$
π^+	$W_{12} = W_{34} = \frac{1}{2\sqrt{2}}$	$W_{12} = W_{34} = \frac{1}{2\sqrt{2}}$

while for pseudoscalar-vector transitions we find

final initial	ρ_{+1}^-	ρ_0^-	ρ_{-1}^-	ω_{+1}^0	ω_0^0	ω_{-1}^0
π^-	$W_{42} = -\frac{1}{\sqrt{2}}$	$W_{22} = -W_{44} = \frac{1}{2}$	$W_{24} = \frac{1}{\sqrt{2}}$	$W_{41} = \frac{1}{2}$	$W_{21} = -W_{43} = -\frac{1}{2\sqrt{2}}$	$W_{23} = -\frac{1}{2}$
	ρ_{+1}^+	ρ_0^+	ρ_{-1}^+	ω_{+1}^0	ω_0^0	ω_{-1}^0
π^+	$W_{31} = -\frac{1}{\sqrt{2}}$	$W_{11} = -W_{33} = \frac{1}{2}$	$W_{13} = \frac{1}{\sqrt{2}}$	$W_{32} = \frac{1}{2}$	$W_{12} = -W_{34} = \frac{1}{2\sqrt{2}}$	$W_{14} = \frac{1}{2}$

40.

For the strange particles, we have

	K^0	$\bar{K}^0$	ω_{+1}^0	ω_0^0	ω_{-1}^0
K^+	$W_{12}=W_{34}=\frac{1}{2}$				
$\bar{K}^-$		$\bar{W}_{21}=\bar{W}_{43}=-\frac{1}{2}$	$\bar{W}_{61}=\frac{1}{2}$	$\bar{W}_{63}=-\bar{W}_{51}$ $=\frac{1}{2\sqrt{2}}$	$\bar{W}_{53}=-\frac{1}{2}$
			ϕ_{+1}^0	ϕ_0^0	ϕ_{-1}^0
$\bar{K}^-$			$\bar{W}_{45}=\frac{1}{\sqrt{2}}$	$\bar{W}_{25}=-\bar{W}_{46}$ $=-\frac{1}{2}$	$\bar{W}_{26}=-\frac{1}{\sqrt{2}}$

	K_{+1}^{*0}	K_0^{*0}	K_{-1}^{*0}
K^+	$W_{32} = \frac{1}{\sqrt{2}}$	$W_{34} = -W_{12} = \frac{1}{2}$	$W_{14} = -\frac{1}{\sqrt{2}}$
	K_{+1}^{*+}	K_0^{*+}	K_{-1}^{*+}
K^+	$W_{31} = -\bar{W}_{65}$ $= \frac{1}{\sqrt{2}}$	$W_{33} = -W_{11}$ $= \bar{W}_{55} = -\bar{W}_{66}$ $= \frac{1}{2}$	$W_{13} = -\bar{W}_{56}$ $= -\frac{1}{\sqrt{2}}$

The non-zero elements of the Z matrices are given below, where each matrix goes from a $P_{\frac{1}{2}}$ state to the state indicated in the table.

$P_{\frac{1}{2}}$	$P_{-\frac{1}{2}}$	$N_{\frac{1}{2}}$	$N_{-\frac{1}{2}}$
$Z_{11} = 10$ $Z_{22} = 2$ $Z_{33} = 2$ $Z_{44} = 4$	$Z_{13} = 8$ $Z_{24} = -2$	$Z_{12} = 8$ $Z_{34} = -2$	$Z_{14} = 10$
$\Delta_{\frac{3}{2}}^{++}$	$\Delta_{\frac{1}{2}}^{++}$	$\Delta_{-\frac{1}{2}}^{++}$	$\Delta_{-\frac{3}{2}}^{++}$
$Z_{41} = -2\sqrt{3}$	$Z_{21} = 2$ $Z_{43} = -2$	$Z_{23} = 2$	$Z = 0$
$\Delta_{\frac{3}{2}}^{+}$	$\Delta_{\frac{1}{2}}^{+}$	$\Delta_{-\frac{1}{2}}^{+}$	$\Delta_{-\frac{3}{2}}^{+}$
$Z_{31} = 2\sqrt{3}$ $Z_{42} = -2\sqrt{3}$	$Z_{11} = -2$ $Z_{22} = 2$ $Z_{33} =$ $Z_{44} = -2$	$Z_{24} = 2$ $Z_{13} = -2$	$Z = 0$
$\Delta_{\frac{3}{2}}^0$	$\Delta_{\frac{1}{2}}^0$	$\Delta_{-\frac{1}{2}}^0$	$\Delta_{-\frac{3}{2}}^0$
$Z_{32} = 2\sqrt{3}$	$Z_{12} = -2$ $Z_{34} = 2$	$Z_{14} = -2$	$Z = 0$

It must be emphasized that the over-all normalization of the baryon wave functions has not been included in these numbers, but must be included in the amplitudes.

	$\Lambda \frac{1}{2}$
$P_{\frac{1}{2}}$	$Z_{36} = 4$ $Z_{15} = 2$
$P_{-\frac{1}{2}}$	$Z_{35} = -2$

- APPENDIX III -

In this Appendix, we catalogue the amplitudes for the processes discussed in the text. The notation is as follows: the quark amplitudes are written A_T^λ , where λ refers to the helicity index from Table I, and T refers to the total isotopic spin. The notation A_{CE} is given by

$$A_{CE} = 2 \langle p n | n p \rangle$$

where the matrix element is understood to have helicities corresponding to λ , while

$$\bar{A}_{CE} = 2 \langle \bar{p} p | \bar{n} n \rangle$$

with the same understanding about helicities. The symbol $\bar{A}$ indicates that the preceding expression is to be recopied with $A_T \rightarrow \bar{A}_T$.

1) $\pi^- p \rightarrow \pi^0 n$

$$\langle \frac{1}{2} | \frac{1}{2} \rangle = \frac{3}{2\sqrt{2}} [A_{CE}^1 + A_{CE}^6 + \bar{A}]$$

$$\langle -\frac{1}{2} | \frac{1}{2} \rangle = -\frac{5}{\sqrt{2}} [A_{CE}^5 + \bar{A}]$$

2) $\pi^- p \rightarrow \pi^0 \Lambda^0$

$$\frac{1}{\sqrt{3}} \langle \frac{3}{2} | \frac{1}{2} \rangle = \langle -\frac{1}{2} | \frac{1}{2} \rangle = \frac{1}{\sqrt{2}} [A_{CE}^5 + \bar{A}]$$

$$\langle \frac{1}{2} | \frac{1}{2} \rangle = 0 = \langle -\frac{3}{2} | \frac{1}{2} \rangle$$

To get the corresponding reaction for η_0 production, let $\bar{A} \rightarrow -\bar{A}$. Also, because of the exclusion of double charge-exchange, we know that

$$\begin{aligned} A(\pi^- p \rightarrow \pi^0 \Delta^0) &= -\frac{1}{\sqrt{2}} A(\pi^- p \rightarrow \pi^- \Delta^+) \\ &= -\frac{1}{\sqrt{3}} A(\pi^+ p \rightarrow \pi^0 \Delta^{++}) = \frac{1}{\sqrt{2}} A(\pi^+ p \rightarrow \pi^+ \Delta^+) \end{aligned}$$

For vector meson production

$$3) \pi^- p \rightarrow \rho^- p$$

$$\langle 1 \frac{1}{2} | \frac{1}{2} \rangle = \langle -1 \frac{1}{2} | \frac{1}{2} \rangle = -\frac{1}{\sqrt{2}} [12 A_1^6 + 6 A_0^6 - \bar{A}]$$

$$\langle 1 -\frac{1}{2} | \frac{1}{2} \rangle = -\frac{1}{\sqrt{2}} [2 A_1^4 + 4 A_0^4 - \bar{A}]$$

$$\langle -1 -\frac{1}{2} | \frac{1}{2} \rangle = \frac{1}{\sqrt{2}} [2 A_1^2 + 4 A_0^2 - \bar{A}]$$

$$\langle 0 -\frac{1}{2} | \frac{1}{2} \rangle = 0$$

$$\langle 0 \frac{1}{2} | \frac{1}{2} \rangle = A_1^1 - A_1^3 + 2(A_0^1 - A_0^3) - \bar{A}$$

$$4) \pi^+ p \rightarrow \rho^+ p$$

$$\langle 1 \frac{1}{2} | \frac{1}{2} \rangle = \langle -1 \frac{1}{2} | \frac{1}{2} \rangle = -\frac{1}{\sqrt{2}} [15 A_1^6 + 3 A_0^6 - \bar{A}]$$

$$\langle 1 -\frac{1}{2} | \frac{1}{2} \rangle = -\frac{1}{\sqrt{2}} [7 A_1^4 - A_2^4 - \bar{A}]$$

$$\langle -1 -\frac{1}{2} | \frac{1}{2} \rangle = \frac{1}{\sqrt{2}} [7 A_1^2 - A_0^2 - \bar{A}]$$

$$\langle 0 \frac{1}{2} | \frac{1}{2} \rangle = \frac{1}{2} [7(A_1^1 - A_1^3) - (A_0^1 - A_0^3) - \bar{A}]$$

$$\langle 0 -\frac{1}{2} | \frac{1}{2} \rangle = 0$$

$$5) \pi^+ n \rightarrow \omega p$$

$$\langle 1 \frac{1}{2} | \frac{1}{2} \rangle = \langle -1 \frac{1}{2} | \frac{1}{2} \rangle = -\frac{1}{2} [3 A_{CE}^6 + \bar{A}]$$

$$\langle 1 -\frac{1}{2} | \frac{1}{2} \rangle = \langle -1 -\frac{1}{2} | \frac{1}{2} \rangle = -\frac{5}{2} [A_{CE}^4 + \bar{A}]$$

$$\langle 0 \frac{1}{2} | \frac{1}{2} \rangle = \frac{5}{2\sqrt{2}} [A_{CE}^1 - A_{CE}^3 + \bar{A}]$$

$$\langle 0 -\frac{1}{2} | \frac{1}{2} \rangle = 0$$

$$6) \pi^- p \rightarrow \rho^- \Delta$$

$$\langle 1 \frac{3}{2} | \frac{1}{2} \rangle = \sqrt{3} \langle -1 -\frac{1}{2} | \frac{1}{2} \rangle = \frac{1}{\sqrt{2}} \{ A_{CE}^2 - \bar{A} \}$$

$$\langle 1 -\frac{1}{2} | \frac{1}{2} \rangle = \sqrt{3} \langle -1 \frac{3}{2} | \frac{1}{2} \rangle = \frac{1}{\sqrt{2}} \{ A_{CE}^4 - \bar{A} \}$$

$$\langle 0 \frac{1}{2} | \frac{1}{2} \rangle = \frac{1}{\sqrt{2}} [A_{CE}^1 - A_{CE}^0 - \bar{A}]$$

all others = 0

where all amplitudes are multiplied by an over-all normalization factor of $(\frac{1}{\sqrt{18}} \cdot \frac{1}{3})$ for $p \rightarrow \Delta$ and $\frac{1}{\sqrt{18}}$ for $P \rightarrow P$.

The same remarks apply to reactions of the type $\pi N \rightarrow \rho \Delta$ as did to reactions $\pi N \rightarrow \pi \Delta$. Finally, turning to the strange particle interactions, we find

$$7) \quad K^+ p \rightarrow K^0 \Delta^{++}$$

$$\langle \frac{3}{2} \frac{1}{2} \rangle = 3 \quad A_{CE}^5$$

$$\langle -\frac{1}{2} \frac{1}{2} \rangle = -\sqrt{3} \quad A_{CE}^5$$

$$\langle \frac{1}{2} \frac{1}{2} \rangle = \langle -\frac{1}{2} \frac{1}{2} \rangle = 0$$

Since the two sides of the reaction can only communicate through the $T = 1$ channel, we find $A(K^+ p \rightarrow K^0 \Delta) = -\sqrt{3} A(K^+ p \rightarrow K^+ \Delta)$

$$8) \quad K^+ p \rightarrow K^{*+} p$$

$$\langle \frac{1}{2} \frac{1}{2} \rangle = \langle -1 \frac{1}{2} \frac{1}{2} \rangle = \frac{1}{\sqrt{2}} [15 A_1^6 + 3 A_0^6 - 18 A_{\frac{1}{2}}^6]$$

$$\langle 1 -\frac{1}{2} \frac{1}{2} \rangle = -\langle -1 -\frac{1}{2} \frac{1}{2} \rangle = \frac{1}{\sqrt{2}} [7 A_1^4 - A_0^4 - 6 A_{\frac{1}{2}}^4]$$

$$\langle 0 \frac{1}{2} \frac{1}{2} \rangle = \frac{1}{2} [7(A_1^3 - A_1') + (A_0' - A_0^3) + 6(A_{\frac{1}{2}}' - A_{\frac{1}{2}}^3)]$$

$$\langle 0 -\frac{1}{2} \frac{1}{2} \rangle = 2 [A_1^3 - A_1' + A_0^3 - A_0']$$

$$9) K^- p \rightarrow \omega \Lambda$$

$$\langle 1 \frac{1}{2} | \frac{1}{2} \rangle = \langle -1 \frac{1}{2} | \frac{1}{2} \rangle = 3 A_{\frac{1}{2}}^6$$

$$\langle 1 \frac{1}{2} | -\frac{1}{2} \rangle = -A_{\frac{1}{2}}^2 \quad \langle -1 \frac{1}{2} | -\frac{1}{2} \rangle = A_{\frac{1}{2}}^4$$

$$\langle 0 \frac{1}{2} | \frac{1}{2} \rangle = \frac{1}{\sqrt{2}} [A_{\frac{1}{2}}^1 - A_{\frac{1}{2}}^3]$$

$$\langle 0 \frac{1}{2} | -\frac{1}{2} \rangle = 0$$

$$10) K^- p \rightarrow \phi \Lambda$$

$$\langle 1 \frac{1}{2} | \frac{1}{2} \rangle = \langle -1 \frac{1}{2} | \frac{1}{2} \rangle = \frac{6}{\sqrt{2}} \tilde{A}_0^6$$

$$\langle 1 \frac{1}{2} | -\frac{1}{2} \rangle = -\sqrt{2} \tilde{A}_0^2$$

$$\langle -1 \frac{1}{2} | \frac{1}{2} \rangle = \sqrt{2} \tilde{A}_0^4$$

$$\langle 0 \frac{1}{2} | \frac{1}{2} \rangle = \langle 0 \frac{1}{2} | -\frac{1}{2} \rangle = 0$$

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