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SOME ASPECTS OF
TRACE-SPACE COMPUTATION
FOR
HIGH-ENERGY CHARGED PARTICLE BEAMS

by

J. FRONTEAU

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As the article as given here was too long, the last part "III - Tertiary beams in vacuum" will be omitted from the book.

At the request of the publishers, American spelling has been used.

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J. FRONTEAU

European Organization for Nuclear Research

Geneva, Switzerland

*) Translated at CERN by Miss A.T. Sanders.

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INTRODUCTION

The majority of high-energy physics experiments are based on the same principle: particles are directed on to a target and a study is made of their behavior in that detector target. If the incident particles are those which were previously accelerated, the work is then shared by two categories of physicists, accelerator experts and experimenters in search of interactions in the detector target. However, the only particles which can be accelerated are stable charged particles, namely protons and electrons. In order to study others (and sometimes protons and electrons), there is no alternative but to use an additional target. The accelerator experts will direct their beam on to a production target, situated inside or outside the accelerator, which will emit the particles required by the experimenters on the detector target. The two targets are usually a few tens of meters apart; there is thus a large region, the inter-target region, which has only been explored with reluctance, as the accelerator theoreticians consider that it is outside their purview to explore it, and the experimenters regard its study as an unpleasant obligation; it is the so-called region of secondary beams. In fact these beams may be composed of either secondary particles produced in the first target by the primary beam of the accelerator, or tertiary particles emitted in flight from the particles of the secondary beams in the inter-target region; and the particles may move either in vacuum, or sometimes in solid absorbers which

slow them down. Although the method to be used has been extended to cover all these possibilities [Banaigs et al. (1964), Fronteau (1964)], the present study will be confined to secondary particles which move in vacuum.

The complete theoretical study of a secondary beam is usually carried out in successive steps which have to be repeated several times before the optimum solution is found. These different steps may be reduced in the first approximation to three stages, although in practice it is difficult to make such a clear distinction as has been made below.

1. The type of beam line¹⁾ is chosen which seems best adapted both to the requirements of the experimenters who are to use it and to the conditions imposed by the set-up in the experimental area, and from this a set of relative positions of the components may be estimated.
2. By successive approximations, calculations are made of the fields and the magnetic gradients which will be necessary in the magnets and the quadrupoles respectively, in order to create the focusing conditions of the type chosen. Nowadays, with the help of various computer programs [Baker (1961), Gardner and Whiteside (1961)(1963), Geibel (1964), King (1964)], this work can be entrusted to electronic computers. There will be no further reference to these aspects which are not the object of this article.

¹⁾ "Beam line" will generally be used to denote "beam transport system".

3. When a set of compatible distances and fields is known, an accurate plan of the beam line can be drawn up, together with the power requirements for the quadrupoles and magnets. The spectrum defined by this plan then has to be calculated. It is above all here that the "trace-space polygon method" may be used.

The reader is warned that it was decided after deliberation that in this paper the particle beams should not be represented by phase-space acceptance ellipses [King (1955), Citron and Øverås (1957), Hereward (1959), Citron et al. (1963-a), King (1964)] as is usually the case in the literature on accelerators and even secondary beams. Although these phase-space acceptance ellipses are well suited to the study of beams of particles which move in vacuum under the influence of periodic beam transport systems²⁾ they no longer have any *raison d'être* for beams in the inter-target region. Beam lines there are not necessarily periodic and, as has already been mentioned, the particles may be made to pass through solid absorbers in which they will dissipate part of their energy. It is then an advantage to work in "trace" space, for which, as will be seen later, it is easy to give a geometrical definition, and whose essential properties are independent of the energy loss of particles in matter. It is moreover important to

²⁾ So powered that the motion is stable [Sturrock (1955)].

represent the beams by surfaces more general than ellipses, polygons for example, which may then be applied to non-periodic transport systems.

I. GENERAL REMARKS

[TRACE SPACE AND ACCEPTANCE POLYGONS]

A. DEFINITION OF THE TRACE SPACE

THE TWO TYPES OF REPRESENTATION IN THIS SPACE

1. NOTATION

Let F be the axis of a beam line; this will be taken to be a horizontal plane curve, the arc length of which will be s (Fig. 1).

At any point S , Sz^1 is the vertical in the upward direction, St the tangent in the direction of motion and Sz^2 the normal in the plane of the curve, which is taken positive to obtain a right-hand set of axes (St, Sz^1, Sz^2) .

Let P be the plane $Sz^1 Sz^2$ normal to the trajectory.

The derivative with respect to the arc length will be represented by a dash $\left(x' = \frac{dx}{ds}\right)$ and the superscripts V or 1 , and H or 2 , will be used to designate vertical and horizontal conjugate quantities.

2. DEFINITION OF THE TRACE SPACE³⁾

In geometry, the trace of a straight line⁴⁾ in a plane P is its intersection with P . As this geometrical trace is not sufficient to determine a trajectory in the neighborhood of P , the direction of the

³⁾ Some authors call it "the characteristic co-ordinates space", [Hereward et al. (1956)].

⁴⁾ This term, which is current in French, is sometimes used in English, [Filon (1935)].

velocity vector must be introduced. Thus it will be agreed here not to call the trace of a particle⁵⁾ at s the geometrical trace of its trajectory in the plane Sz^1z^2 , but the pair $(M, \vec{dM}/ds)$ formed by this geometrical trace and the direction of the trajectory. The trace of a particle at s will therefore be the local trajectory element (Fig. 2).

The projections $[z^1, z'^1]$ and $[z^2, z'^2]$ of the trace $(M, \vec{dM}/ds)$ on to the two planes FSz^1 and FSz^2 will be called projected traces. It should be recalled that z'^1 and z'^2 , derivatives with respect to s , are the slopes of the projections of the trajectory element on to the vertical and horizontal planes.

The two planes in which two orthogonal reference axes, (z^1, z'^1) and (z^2, z'^2) are taken, and any point of which represents a projected trace (Fig. 3), will be defined as the two projected trace spaces. Thus all traces i.e. all trajectory elements, will be represented by a pair of points $(m^1, \bar{m}^2)$ each one belonging to one of the two symbolic planes (z^1, z'^1) , (z^2, z'^2) respectively. Each trace can also be associated with a point (z^1, z'^1, z^2, z'^2) of the four-dimensional space which is the trace space proper. In practice no use will be made of this four-dimensional space, but only of the two sub-spaces which have been called projected trace spaces (or planes).

⁵⁾ The trace of a particle must never be confused with the trace of a matrix.

3. THE TWO TYPES OF REPRESENTATION IN PROJECTED TRACE SPACE

Any projected particle trace Tr is a function of the particle M of whose trajectory element it is the projection, and of the point with arc length s at which it is considered $Tr = Tr(s, M)$.

Two essentially different representations are therefore possible in each of the two projected trace spaces.

a.) $Tr = Tr(s, M_0)$ (where M_0 is a given particle).

This is the representation of the trajectory of M_0 projected on to one of the two reference planes, vertical or horizontal. This will be called "the trajectory in the projected trace space".

b.) $Tr = Tr(s_0, M)$ (where s_0 is a given arc length).

This is the local representation of a set of trajectories.

If, for instance, all the particles of a beam are considered, the points $Tr(s_0, M)$ are spread over an area known as "the acceptance area".

Thus, there appear to be two ways of calculating beams in trace space: the behavior of a certain number of individual particles can be studied, or that of the whole beam. The second method will be described here, and the reader is referred to the appropriate literature [Lapostolle (1955), Blanc-Lapierre et al. (1959), Fronteau (1964)] for the first. In practice the first method is rarely used for secondary beams because if one wishes to follow the particles individually in their flight it is preferable to do this in real space [Knowles (1963)] rather than in a symbolic space, the latter being more difficult to imagine. On the other hand, it may

be advantageous to work in symbolic space for studying the behavior of a whole beam, because the acceptance areas in trace space have the advantage of a conservation theorem. This will now be recalled.

B. LIIOUVILLE TYPE INVARIANCE

Let $z_\alpha(s)$ and $z_\beta(s)$ be any linearly independent pair of solutions of the equation:

$$(1) \quad z'' + a(s)z' + b(s)z = 0 \quad \left(z' = \frac{dz}{ds} \right).$$

They constitute a fundamental set of solutions.

Let us introduce the determinant:

$$(2) \quad \Delta(s) = \begin{vmatrix} z_\alpha & z_\beta \\ z'_\alpha & z'_\beta \end{vmatrix}$$

and calculate its derivative, applying for instance the general rule for finding the derivative of determinants, i.e. adding the determinants obtained by deriving the initial determinant row after row or column after column:

$$(3) \quad \Delta' = \frac{d\Delta}{ds} = \begin{vmatrix} z'_\alpha & z'_\beta \\ z_\alpha & z_\beta \end{vmatrix} + \begin{vmatrix} z_\alpha & z_\beta \\ z''_\alpha & z''_\beta \end{vmatrix} = \begin{vmatrix} z_\alpha & z_\beta \\ z''_\alpha & z''_\beta \end{vmatrix}$$

which, by introducing the differential equation, gives:

$$(4) \quad \Delta' = \begin{vmatrix} z_{\alpha} & z_{\beta} \\ -a(s)z'_{\alpha} - b(s)z_{\alpha} & -a(s)z'_{\beta} - b(s)z_{\beta} \end{vmatrix} = \begin{vmatrix} z_{\alpha} & z_{\beta} \\ -az'_{\alpha} & -az'_{\beta} \end{vmatrix};$$

namely:

$$(5) \quad \frac{d\Delta}{ds} = -a\Delta.$$

Hence:

$$(6) \quad \Delta(s) = \Delta(0) \exp \left[- \int_0^s a(s) ds \right]$$

if $a(s)$ can be defined and integrated over the interval $(0, s)$.

Moreover, if $z(s)$ is the general solution of the differential equation, the value of z and z' at s depends linearly on the corresponding values at $s = 0$.

$$(7) \quad \begin{cases} z(s) = A(s)z(0) + B(s)z'(0) \\ z'(s) = C(s)z(0) + D(s)z'(0) \end{cases}.$$

These relations are valid whatever the value of the constants α and β which determine $z(s)$ in terms of an arbitrary set of solutions,

$$(8) \quad z(s) = \alpha z_{\alpha}(s) + \beta z_{\beta}(s).$$

This means that they are valid for $z_{\alpha}(s)$ and $z_{\beta}(s)$ independently,

hence:

$$(9) \quad \Delta(s) = \begin{vmatrix} A(s)z_{\alpha}(0) + B(s)z'_{\alpha}(0) & A(s)z_{\beta}(0) + B(s)z'_{\beta}(0) \\ C(s)z_{\alpha}(0) + D(s)z'_{\alpha}(0) & C(s)z_{\beta}(0) + D(s)z'_{\beta}(0) \end{vmatrix}$$

which, on expansion, is:

$$(10) \quad \Delta(s) = \begin{vmatrix} A(s) & B(s) \\ C(s) & D(s) \end{vmatrix} \Delta(0) .$$

When Eq. (10) is compared with Eq. (6), it follows that:

$$(11) \quad \begin{vmatrix} A(s) & B(s) \\ C(s) & D(s) \end{vmatrix} = \exp \left[- \int_0^s a(s) ds \right]$$

namely:

$$(12) \quad \frac{D[z(s), z'(s)]}{D[z(0), z'(0)]} = \exp \left[- \int_0^s a(s) ds \right] .$$

This functional determinant can only be constant if a is identically equal to zero. It will be noted that:

$$(13) \quad \boxed{\frac{D[z(s), z'(s)]}{D[z(0), z'(0)]} = 1 \iff a \equiv 0} .$$

In vacuum, the equations of motion of charged particles are well known [Penner (1961), King (1964), Miller (1964)] for the conventional transport components (drift spaces, magnetic quadrupoles, bending magnets) and all fulfil the conditions $a \equiv 0$. It is fairly easy to show [Fronteau (1964)] that the drift spaces and the magnetic quadrupoles also have this property when the particles are moving through a medium which slows them down. It is sufficient to choose $\underline{s}$ as a variable. The determinant $D[z(s), z'(s)] / D[z(0), z'(0)]$ is therefore always equal to 1.

This is the first form of Liouville type invariance. It will be seen later that this invariance shows itself in the projected trace plane by the invariance of the area of the acceptance polygon defined by a given set of diaphragms (see Section I, C, 2, a).

C. THE ACCEPTANCE POLYGONS

1. DEFINITION

Let s_i^D be the arc length which determines the position of the i -th member of a set of diaphragms limiting the lateral extension of a beam, and s_0 the arc length of any given point on the axis F (Fig. 4). For any particle of a para-axial beam ($\tan z' \approx z'$), the projected lateral extension at s_i^D is a linear and homogeneous function of the two elements of the projected trace at s_0 :

$$(14) \quad z(s_i^D) = A(s_i^D)z(s_0) + B(s_i^D)z'(s_0) \quad .$$

The diaphragm imposes a double limitation:

$$(15) \quad -r_{\alpha}(s_i^D) \leq z(s_i^D) \leq r_{\beta}(s_i^D)$$

namely:

$$(16) \quad -r_{\alpha}(s_i^D) \leq A(s_i^D)z(s_0) + B(s_i^D)z'(s_0) \leq r_{\beta}(s_i^D) \quad .$$

This double inequality defines a plane band, limited by two parallel straight lines, in the projected trace space related to s_0 (Fig. 5).

For every diaphragm with an index i there is a corresponding band [Hereward et al. (1956), Blanc-Lapierre et al. (1959), Braunersreuther et al. (1961), Fronteau (1961)(1962), Haddock (1963), Kirstein (1963)] in the projected trace plane at s_0 . For every finite set of diaphragms there is a corresponding polygon. This will be called the acceptance polygon at s_0 . A detailed drawing of such a polygon will be made later (Section II, B, 1).

It should be noted that if the limitations are symmetrical ($r_{\alpha} = r_{\beta}$), the origin of the axes $z(s_0), z'(s_0)$ is a center of symmetry for the polygon.

2. THE BEHAVIOR OF THE POLYGONS ALONG A BEAM LINE

a. Polygons defined by a given set of diaphragms

Let s_i^D continue to be the arc length which defines the position of the diaphragm with an index i . In addition, let us now take any two points with arc length s_0 and s_1 (Fig. 6).

Let us introduce the following two linear transformations, calculated for the beam momentum considered:

$$(17) \quad \begin{pmatrix} z(s_i^D) \\ z'(s_i^D) \end{pmatrix} = \begin{pmatrix} A_0 & B_0 \\ C_0 & D_0 \end{pmatrix} \begin{pmatrix} z(s_0) \\ z'(s_0) \end{pmatrix} = (m_0) \begin{pmatrix} z(s_0) \\ z'(s_0) \end{pmatrix}$$

$$(18) \quad \begin{pmatrix} z(s_i^D) \\ z'(s_i^D) \end{pmatrix} = \begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix} \begin{pmatrix} z(s_1) \\ z'(s_1) \end{pmatrix} = (m_1) \begin{pmatrix} z(s_1) \\ z'(s_1) \end{pmatrix} .$$

To the same diaphragm with an index i , therefore, two different bands of the projected trace plane correspond at s_0 and s_1 , namely:

$$(19) \quad -r_\alpha \leq A_0 z(s_0) + B_0 z'(s_0) \leq r_\beta \quad \left[\text{band 0} \right] \text{ (Fig. 7)}$$

and:

$$(20) \quad -r_\alpha \leq A_1 z(s_1) + B_1 z'(s_1) \leq r_\beta \quad \left[\text{band 1} \right] \text{ (Fig. 7).}$$

Further, if the projected trace at s_0 is expressed as a function of the projected trace at s_1 ,

$$(20) \quad \begin{pmatrix} z(s_0) \\ z'(s_0) \end{pmatrix} = (m_0)^{-1} (m_1) \begin{pmatrix} z(s_1) \\ z'(s_1) \end{pmatrix} ,$$

and if these relations are introduced into the analytical expression of band 0, this gives band 1. It should therefore be noted that band 1 can be constructed by two equivalent methods:

- by drawing it directly at s_1 , independently of s_0 ,
- by drawing it at s_0 , with subsequent transformation
from s_0 to s_1 .

This evidently applies to all acceptance polygons defined by a given set of diaphragms.

The polygons corresponding to a given set of diaphragms and drawn, for the same momentum, at different points along the beam line, may be deduced from each other by linear transformations whose matrix has its determinant equal to 1 (Liouville type invariance). It follows that these polygons all have the same area.

b. Polygons defined by a set of diaphragms which varies along the beam line

Let there be, along the axis F of a beam line, a set of diaphragms with arc length s_i^D , and a set of points with arc length s_j at which it is proposed to study the structure of the beam (Fig. 3). At every point s_j , the acceptance polygon defined by all the diaphragms preceeding s_j is called the "upstream" polygon, and the acceptance polygon defined by all the diaphragms following s_j is called the "downstream" polygon. As s_j progresses in the direction of motion of the particles, the two sub-sets of "upstream" and "downstream" diaphragms vary in such a way that the number of diaphragms in the "upstream" sub-set must increase, whereas the number of "downstream" diaphragms must decrease. It follows, in view of

the constancy of the area of any polygon defined by the same set of diaphragms, that for a beam of given momentum the area of the "upstream" polygon must decrease, whereas that of the "downstream" polygon must increase when s_j progresses in the same direction as the particles.

D. A FEW REMARKS CONCERNING VOCABULARY

[For further explanation of the different statements given in this section see, for example, Fronteau (1964)].

The reader who recognizes in trace space what is usually called the phase space, will no doubt be surprised that we insist on changing this expression which is in current use. At first sight this may even appear ridiculous.

However, this distinction draws attention first of all to the fact that even for particles moving in vacuum, trace space is **already** a doubly degenerate form of the phase space of analytical mechanics, as may be seen from the following explanation.

The conjugate momenta for the electromagnetic field [Swann (1933)], are:

$$(22) \quad P_z = \frac{m_0}{\sqrt{1-\beta^2}} \frac{dz}{dt} + eA_z ,$$

where A_z is the z component of the vector potential.

This may be rewritten:

$$(23) \quad P_z = \left(\frac{m_0}{\sqrt{1-\beta^2}} \frac{ds}{dt} \right) \frac{dz}{ds} + eA_z$$

and when neglecting the difference between the momentum p and its s component we get:

$$(24) \quad P_z = pz' + eA_z \quad .$$

Thus, when neglecting the term A_z ⁽⁶⁾ in two-dimensional phase space (z, P_z) one gets the singly degenerate form (z, pz') , and when one further disregards p the doubly degenerate form (z, z') is obtained, which is the projected trace space as previously defined.

But above all, the new expression "trace space" avoids the confusion inherent in speaking about phase space when studying particles moving in a medium which slows them down, in which case phase space does not exist.

The same distinction can be made with respect to Liouville's theorem. The expression "Liouville's theorem" is reserved for the invariance applicable to the phase space of analytical mechanics. It is considered preferable to speak about "Liouville type invariance" when the property

(6) For a deflecting magnet, for example, this term is not identically equal to zero in the horizontal plane [Fronteau (1964)].

applies to the trace space, i.e. when it remains valid for particles slowed down in matter⁷⁾, in which case phase space proper no longer exists.

One last point; the reader who is already accustomed to acceptance ellipses will easily accept the generalization which has been made in the form of polygons. The connection between the two ideas is very simple: for a periodic beam line in which the motion of the particles is stable, the polygon related to a finite number of periods is circumscribed about the conventional acceptance ellipse. When the number of periods becomes infinite, the polygon approaches the ellipse.

⁷⁾ It is clear that this may be true if only the loss of energy by ionization is taken into account. The rôle of multiple scattering is another part of the problem. [Banaigs et al. (1964), Fronteau (1964).]

II. SECONDARY BEAMS IN VACUUM

A. QUALITATIVE STUDY OF A BEAM SPECTRUM

1. THE CHOICE OF A BEAM TO BE STUDIED

The majority^{s)} of beam lines with a small momentum bite ($\Delta p/p = \pm 1$ or 2%) consist of a double analysing system (two magnets) and a double focusing system (two quadrupole doublets or triplets), which ensure a magnification of about 1. If in practice they appear to differ greatly, this is because lack of space or other difficulties in experimental areas have often led physicists when designing their beam line to abandon the simplicity (symmetry, periodicity ...) for which they had hoped. [Marshall (1962), Emmerson and Middlemas (1963), Benvenuti and Uccelli (1964).] Since there is no question here of setting up equipment, a master system can be chosen, consisting for example of two identical periods and giving a magnification of 1 at the level of the analysing slit and the detector target, i.e. the target associated with the detector. Figure 9 gives details of this "standard" beam line S.

(0) is the production target, (2) the detector target, and (1) the vertical slit for momentum separation in the horizontal plane; M_1 and M_2 are the magnets which ensure first momentum separation, then its compensation. Finally, it is assumed that the magnetic gradients of the quadrupoles have been determined by successive approximations

s) We have confined ourselves to beam lines without separators, whether electrostatic or radio frequency.

[Chamberlain (1960), Madsen (1964), Regenstreif (?)] so that the triplet T_1 forms in (1), in both planes and with the same unit magnification, an image of (0). On account of the periodicity of the system, the triplet T_2 will form the image of (1) on (2) and the magnification will again be equal to 1. The second diagram in Fig. 9 shows the optical system which served as a model for the focusing in our "standard" beam line S. Prisms corresponding to magnets are not represented as they only give secondary focusing effects.

2. THE ROLE OF THE DIFFERENT DIAPHRAGMS

Focusing will be studied first, and the analysis carried out by bending magnets will be introduced later.

Let $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$ be the optical matrix between (0) and (1):

$$(25) \quad \begin{pmatrix} z \\ z' \end{pmatrix}_1 = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} z \\ z' \end{pmatrix}_0.$$

On account of the object/image relation between (0) and (1) the term B is equal to zero. Furthermore, to get the same situation as in our light model, the linear magnification (term A) must be equal to -1, which gives:

$$(26) \quad \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ C & D \end{pmatrix}.$$

Finally, on account of the Liouville type invariance, $A^2 = 1$, and then:

$$(27) \quad \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ C & -1 \end{pmatrix} .$$

It should be noted that in Eq. (27) the terms A and D are equal, which means that the optical system [(0), (1)] is symmetrical with respect to the mid-point of [(0), (1)]. However, it will be seen later (Fig.13) that the system [(0), (1)] is not supplied strictly symmetrically. This is due to the fact that the magnetic gradients which ensure such conditions as Eqs. (26) or (27) are calculated by successive approximations with programs which are not drawn up only for symmetric systems. So, when the iterative process is stopped (when the results seem sufficiently precise) symmetry is not strictly achieved.

Let us now complete Eq. (27) by introducing the dispersion terms due to the bending magnets. For the horizontal optics we find:

$$(28) \quad \begin{pmatrix} z \\ z' \\ \delta p/p \end{pmatrix}_1 = \begin{pmatrix} -1 & 0 & \alpha \\ C & -1 & \beta \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} z \\ z' \\ \delta p/p \end{pmatrix}_0$$

and, since the inter-target region has two periods,

$$(29) \quad \begin{pmatrix} z \\ z' \\ \delta p/p \end{pmatrix}_2 = \begin{pmatrix} -1 & 0 & \alpha \\ C & -1 & \beta \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 & \alpha \\ C & -1 & \beta \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} z \\ z' \\ \delta p/p \end{pmatrix}_0$$

namely:

$$(30) \quad \begin{pmatrix} z \\ z' \\ \delta p/p \end{pmatrix}_2 = \begin{pmatrix} 1 & 0 & 0 \\ -2C & 1 & C\alpha \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} z \\ z' \\ \delta p/p \end{pmatrix}_0 .$$

From Eqs. (28) and (30), we will retain only the two relations:

$$(31) \quad \begin{cases} z_1 = -z_0 + \alpha \frac{\delta p}{p} \\ z_2 = z_0 \end{cases} .$$

These suffice to express the influence of the two diaphragms, the slit (1) and the detector (2), which are assumed to be equal and symmetrical with respect to the axis of the beam line (r will be their half-dimension). The half-dimension ρ of the production target (0) will also be introduced.

In order that a particle leaving the emitting target (0) shall pass through the slit (1) and reach the detector target (2), the following conditions must be fulfilled:

$$(32) \quad \begin{cases} -\rho \leq z_0 \leq \rho \\ -r \leq z_1 \leq r \\ -r \leq z_2 \leq r \end{cases}$$

or, on account of Eq. (31),

$$(33) \quad \begin{cases} -\rho \leq z_0 \leq \rho \\ -r \leq -z_0 + \alpha \frac{\delta p}{p} \leq r \\ -r \leq z_0 \leq r, \end{cases}$$

which will be written in the following form:

$$(34) \quad \boxed{\begin{aligned} &-\rho \leq z_0 \leq \rho \\ &-r + \alpha \frac{\delta p}{p} \leq z_0 \leq r + \alpha \frac{\delta p}{p} \\ &-r \leq z_0 \leq r \end{aligned}}$$

In the horizontal projected trace plane corresponding to the target (0), these three conditions define the three plane bands represented in Fig. 10.

Comparing the respective positions and dimensions of these three plane bands is a simple means of obtaining information concerning the shape of the beam spectrum. These will obviously be merely qualitative results, since neither the vertical optics nor the limitations resulting from the quadrupole lenses have been taken into account.

3. INFLUENCE OF THE DIMENSIONS OF THE PRODUCTION TARGET ON THE BEAM SPECTRUM

Let us return to Fig. 10.

At the level of the production target (0), the projected traces of the particles emitted are located in the hatched band. In order that these projected traces may correspond to particles likely to cross diaphragms (1) and (2), they must also be inside bands $\{(2)(2)\}$ and $\{(1)(1)\}$. For the dimensions chosen in the figure ($\rho < r$), the condition $\{(0)(0)\}$ inside $\{(2)(2)\}$ is always satisfied. With respect to band $\{(1)(1)\}$, everything depends on the value of the dispersion term $\alpha \delta p/p$.

For a relatively small dispersion term $[\alpha \delta p/p < r - \rho]$ all the traces of band $\{(0)(0)\}$ are accepted. The transmission is perfect; a transmission rate equal to unity ($\tau = 1$) will be introduced.

Thus, as long as the dispersion term remains less than $r - \rho$, the transmission will be perfect, but as soon as $\alpha \delta p/p$ exceeds $r - \rho$, only part of band $\{(0)(0)\}$ will remain inside band $\{(1)(1)\}$ and the narrowing of the accepted band is proportional to $\alpha \delta p/p$; the transmission decreases linearly with $\delta p/p$.

Finally, the part which is common to bands $\{(0)(0)\}$ and $\{(1)(1)\}$ is cancelled as soon as $\alpha \delta p/p = r + \rho$.

The spectrum shown in Fig. 11 sums up the situation.

The foregoing considerations, applied to different values of ρ/r , lead to the four spectra shown in Fig. 12. The influence of the horizontal dimension of the production target can be seen at first glance. It will be noted that the momentum bite at half height is a constant.

However, it should be recalled that the foregoing method has two defects:

- a. It does not take into account the chromatism of the quadrupole magnets. It is assumed that the transfer matrix from (0) to (1) and from (1) to (2) is always $\begin{pmatrix} -1 & 0 \\ C & -1 \end{pmatrix}$, whatever the value of $\delta p/p$. But this statement is not rigorous.
 - b. It does not give the absolute value of acceptance, since only the horizontal optics are taken into account and the quadrupole diaphragms are not introduced, although - as will be seen later - they in fact define the acceptance of the system.
- These two points will be dealt with in Section II, B.

B. QUANTITATIVE STUDY OF THE SPECTRUM

1. DRAWING OF AN ACCEPTANCE POLYGON; CALCULATION OF THE CORRESPONDING TRANSMISSION RATE

Let us now suppose that the type of beam to be built has been chosen, that this choice has been confirmed by a qualitative study of the spectrum, and that the fields and magnetic gradients have already been evaluated by successive approximations. Thus we come to the third stage of our study (cf. Introduction).

We therefore have a detailed plan of a beam line, of type S for instance, 50 meters long, whose momentum bite will be of the order of $\pm 2\%$ for 14 GeV/c particles. This project is shown in the fully dimensioned sketch in Fig. 13. It will be noticed that the different points along the axis are denoted by j , which assumes values of 0, 1, 2, 24. This numbering system will be retained. It will be used later to designate the different fields.

Before calculating the final beam spectrum, it is proposed to examine the importance of the chromatism of the triplets. Accordingly, the focusing system proper (T_1 and T_2) will first be isolated, replacing the magnets M_1 and M_2 by two drift spaces of the same length.

Now let us find out how to draw an acceptance polygon. Let us, for instance, draw the 14 GeV/c horizontal "downstream" polygon at the level of the production target (0). This polygon will result from the superposition of the limitations due to the eight diaphragms 2, 5, 8, 12, 14, 17, 20 and 24.

By applying formulae which have become conventional, [Chamberlain (1960), Grivet and Septier (1960), Penner (1961), Septier (1961), King (1964), Miller (1964), Regenstreif (?)], with the distances measured in meters and the angles in radians, 14 GeV/c horizontal matrices are first found for the various fields (see Table I).

The matrices between the point (0) and the different points where a diaphragm appears are then calculated by successive multiplications (see Table II).

The eight diaphragms therefore are represented by eight limitations (see Table III), which all have the form:

$$(35) \quad -r_j^H \leq z_j^H \leq r_j^H$$

namely:

$$(36) \quad -r_j^H \leq A_j^H z_0^H + B_j^H z_0'^H \leq r_j^H$$

Figure 14 represents the eight corresponding plane bands, drawn in the horizontal projected trace plane related to the emitting target (0).

The part common to these eight bands is the horizontal "downstream" acceptance polygon, at the level of the production target (0).

It will be noticed that, as mentioned in the previous chapter, the angular acceptance is defined by the quadrupoles and not by diaphragms (12) and (24).

In the same way as the "downstream" polygon represents the acceptance of the beam line, the "upstream" polygon must represent the emittance of the source. The limitation due to the horizontal dimension of the production target ($-0.005 \leq z_0^H \leq 0.005$ m, for instance) defines only a band of the projected trace plane and not a closed polygon; if the target is regarded as a small emitting plate, nothing limits the angle of emission (the limitation is introduced by the acceptance and not by the emittance). If therefore, in order to form a closed "upstream" polygon, an a priori angular limitation is imposed on the emittance ($-0.006 \leq z_0'^H \leq 0.006$ rad, for instance) this choice is to a large extent arbitrary. Further mention will be made of this point when the final beam spectrum is calculated (Section II, B, 3, a.). For the present, it will be decided to limit the emission to 6 mrad, simply to obtain a horizontal spectrum practically normalized to 1, as will be shown below.

The rectangle $\{-0.005 \leq z_0^H \leq 0.005$ m; $-0.006 \leq z_0'^H \leq 0.006$ rad}, representing the emittance, is the horizontal "upstream" polygon at the level of the emitting target (0). If this is compared with the corresponding

"downstream" polygon, the horizontal transmission rate at the level of (0) is defined as:

$$\tau = \frac{\text{Area common to "upstream" and "downstream" polygons}}{\text{Area of the "upstream" polygon}} = 0.87$$

(see Fig. 15).

If it is assumed that the "upstream" polygon is uniformly populated (homogeneous and isotropic emission), the transmission rate defined above represents the proportion of particles which - if they are stable - may horizontally reach the end of the beam line, in spite of the diaphragms forming obstacles along their path. It is clear that this horizontal transmission rate will have to be completed by its vertical conjugate (Section II, B, 3, b.).

2. INFLUENCE OF THE CHROMATISM OF THE QUADRUPOLES

a. Acceptance polygons of the beam line S without bending magnets

It has already been seen from the previous paragraph (II, B, 1) that drawing an acceptance polygon is a relatively long operation. No further details of the calculations will therefore be given, but only the results. In order to re-assure the reader, who may already be alarmed at the amount of numerical calculation called for by this method, it should be mentioned that in addition to early French programs [Fronteau (1961)], there are Fortran programs⁹⁾ at CERN, [Fronteau and Hornsby (1962),

⁹⁾ These programs were written at CERN by J. Hornsby, at present Director of the Computing Laboratory, University College of North Wales, Bangor, Caernarvonshire, United Kingdom. The author has just been informed that a similar program now exists at Brookhaven [Sacharidis (1964)].

Citron et al. (1963 - b)], which make it possible for the computer to supply the physicists directly with the acceptance polygons defined by their vertices, for any beam line studied.

Figure 16 shows four horizontal "downstream" polygons, drawn at the level of the production target for four different momenta (13.86, 14, 14.14, 14.28 GeV/c). It will be noticed that the defective matching due to the chromatism of the quadrupoles shows itself essentially by a quasi-rotation of the polygons round their center. Only the 14 GeV/c polygon, whose sides are parallel to the axis z' , recalls the bands parallel to the axis which were drawn during the qualitative study of the beam, when neglecting the chromatism (Fig. 10).

In Fig. 16 are given the four horizontal transmission rates calculated, for the four momenta chosen, as indicated in Section (II, B, 1). The lowering of the transmission rate as one moves away from 14 GeV/c measures the defective horizontal matching due to the chromatism. It will be understood that this phenomenon may deform the spectra which were found during the qualitative study of the beam. This point will be dealt with later. (Section II, B, 2, c and Fig. 18.)

b. Acceptance polygons of the complete beam line S

Crossing a bending magnet causes all particles with a momentum different from the nominal to undergo lateral displacement and angular deflection with respect to the nominal trajectory. This effect shows

itself by the apparition of constants (E,F) in the linear relations between projected traces [Luckey (1961), Penner (1961), Knowles (1963), Fronteau (1964)].

$$(37) \quad \begin{cases} z = Az_0 + Bz'_0 + E \\ z' = Cz_0 + Dz'_0 + F \end{cases} .$$

Any limitation by a diaphragm is then written as:

$$(38) \quad -r \leq Az_0 + Bz'_0 + E \leq r .$$

This double inequality produces a plane band in the trace space which is no longer symmetrical with respect to the origin: passing through a bending magnet results in a translation of the projected trace space bands. (This property has already been used for the qualitative study of the beam line S.)

Figure 17 shows the horizontal "downstream" polygons drawn at the level of the production target for the four momenta 13.86, 14, 14.14 and 14.28 GeV/c and for the complete beam line S.

The rather surprising shape of these polygons will be easily understood if it is remembered that the acceptance of the system takes into account both the chromatism of the quadrupoles and the bending in the magnets. This double phenomenon shows itself by a double displacement (quasi-rotation, and translation) of the different bands drawn in the projected trace plane, which, varying according to the band concerned, completely deforms the figures.

c. Influence of chromatism on the beam spectrum

The few transmission rates indicated in Fig. 17 are several points of the horizontal transmission spectrum, taking into account the chromatism of the quadrupoles and the bending in the magnets.

Since the Fortran program already mentioned [Fronteau and Hornsby (1962), Citron et al. (1963-b)], can not only draw the polygons but also calculate the transmission rates, it is sufficient to ask the computer for a greater number of points in order to obtain a spectrum . Then, by varying the horizontal half-dimension ρ of the production target, several spectra (Fig. 18-b) can be obtained, which can then be compared with those found when neglecting the chromatism (Figs. 12 and 18-a). At present let us simply compare the shape of the different spectra. In the following paragraph the numerical value of the acceptance will be found (Section II, B, 3).

Note

The method of trace space polygons is obviously not the only one possible [Uccelli (1963-a)(1963-b), Miller (1964)]. For instance, it is interesting to compare Fig. 18-b with Fig. 16 of Miller (1964).

3. NUMERICAL CALCULATION OF BEAM INTENSITY

a. Interpretation of transmission rate

The emission is represented in the projected trace plane corresponding to the production target by a rectangle P_{up} , one of whose dimensions

is that of the target ($\Delta z_0 = h$) and the other an angle $\Delta z'_0$ offered to the system (Fig. 19). Further mention will be made at the end of this paragraph of the choice of the angle $\Delta z'_0$. For the moment, it is sufficient to assume that it is greater than the actual acceptance.

In order to calculate the transmission rate, the "upstream" polygon P_{up} is compared with the "downstream" polygon P_{down} , which represents all the diaphragms of the beam line, and one writes:

$$(39) \quad \tau = \frac{\text{Area common to } P_{up} \text{ and } P_{down}}{\text{Area of } P_{up}}$$

namely:

$$(40) \quad \tau = \frac{1}{h\Delta z'_0} \int_{-h/2}^{h/2} \Delta z'(z) dz$$

which gives:

$$(41) \quad \tau\Delta z'_0 = \frac{1}{h} \int_{-h/2}^{h/2} \Delta z'(z) dz = \overline{\Delta z'}$$

This is the mean angular acceptance of the beam line, taking into account the size of the target. It is usual, when evaluating the number of particles in a secondary beam, not to take into direct account the projected trace space acceptance, which is measured in meters $\times$ radians, but only the mean [Haddock (1963)] angular acceptance $\overline{\Delta z'}$ (radians) which derives from it, as has just been shown.

The production density of secondary particles is usually given for a point target. If it is assumed that all interactions occur at the center point of the actual target and if the angular acceptance is calculated from this point, the number of accepted particles will be overestimated. The correct result can only be obtained by averaging the angular acceptance over the whole size of the target.

It will be remembered that the mean angular acceptance is expressed for each plane, vertical or horizontal, by a product of the following form:

$$(42) \quad \overline{\Delta z'} = \Delta z'_0 \times \tau$$

Only the value $\overline{\Delta z'}$ will appear in the final result. It alone has an absolute physical meaning. This means that the factor τ varies inversely as $\Delta z'_0$ or that $\Delta z'_0$ may be chosen arbitrarily; the only limitation to this arbitrary choice is that $\Delta z'_0$ must be greater than the actual acceptance.

b. Calculation of the final spectrum

The above interpretation applies to the vertical and horizontal planes. The space acceptance obtained is as follows:

$$(43) \quad \overline{\Delta \Omega} = \overline{\Delta z'}^V \overline{\Delta z'}^H = \{(\Delta z'_0)^V \tau^V\} \times \{(\Delta z'_0)^H \tau^H\}$$

$$(44) \quad \overline{\Delta \Omega} = (\Delta z'_0)^V (\Delta z'_0)^H \tau^V \tau^H$$

$$(45) \quad \overline{\Delta \Omega} = \Delta \Omega_0 \times T$$

calling the product of the two vertical and horizontal conjugate rates the space rate T .

Let us return to the beam line S and consider the case of a square production target with total dimensions of $0.01 \times 0.01 \text{ m}^2$. In the horizontal plane, for which the polygons have been drawn (Fig. 17), the acceptance is known to be less than $\pm 0.006 \text{ rad}$. For the vertical plane, it can be checked that the acceptance is greater; for instance let us take a maximum of $\pm 0.02 \text{ rad}$. The two rectangles representing emission will be found in Fig. 20.

For both planes the drawing of polygons and the calculation of transmission rates is as explained in Section II, B, 1; the results for different momenta are found in Table IV.

The corresponding spectrum (Fig. 21) has an area of $4.32 \times 10^{-5} (\text{GeV}/c) \times \text{steradian}$. This area A measures the intensity of the beam [Chamberlain (1960)]. If N_0 is the number of interactions giving rise in the production target to the secondary particles studied, and if $d^2N/d\Omega dp$ is the production density of secondary particles (per interacting primary particle per GeV/c and per steradian), the intensity of the beam can be written as:

$$(46) \quad N = N_0 \frac{d^2N}{d\Omega dp} A$$

For 10^{12} interactions in the target and a density of production equal to 1 (which is approximately the case for 14 GeV/c pions produced by 30 GeV/c protons), the beam S would contain 4.32×10^7 particles of the kind considered.

If in addition, decay in flight is taken into account (flight of 50 meters, whereas the mean free path of 14 GeV/c pions is 770 meters), in fact only $4.32 \times 10^7 \times \exp[-50/770] = 4.04 \times 10^7$ particles reach the end of the beam line.

Rounding off, this gives:

$$(47) \quad N_{\pi} = 4 \times 10^7$$

at the output of the beam line S.

Note: The beam S was studied independently of the accelerator providing the primary proton beam. However, the problem is often complicated by the field of the accelerator magnets which has to be taken into account if the production target is placed inside the machine, or by the field of a magnetic distribution system for use with an external production target [De Raad (1963), Morpurgo and Petrucci (1963), Smith (1963), Hoffmann and Weber (1964), Zembery et al. (1964)]. Furthermore, certain beam lines are designed for a large momentum bite [Citron et al. (1963-a)], whereas the beam line S, for instance, accepts only a narrow momentum range. However, the trace space polygon method can also solve these different problems [Fronteau (1964)] and the CERN Fortran programs [A. Citron et al. (1963-b)] permit detailed evaluation by a computer.

III. TERTIARY BEAMS IN VACUUM

A. GENERAL REMARKS; THE EXAMPLES GIVEN

The only tertiary charged particle beams in which the experimenters have any interest are muon beams.

This does not mean, however, that the only muon beams possible are tertiary beams. On the contrary, the work and suggestions of Ballam (1960), Drell (1960), Masek (1962), Tinlot (1962) and Panofsky (1963) at Stanford, and of Clément, Goldzahl and Kessler in Paris [Kessler (1962), Clément (1964), Clément and Kessler (1964)], give the impression that in future muon beams will be secondary beams produced by electrons striking a lead target. The methods of calculation described in Section II could be used for such beams.

However, before setting up secondary muon beams near large electron accelerators, it should be known that the trace space polygon method can be applied to tertiary muon beams produced by the decay in flight of the pions of a secondary beam guided by a conventional magnetic system¹⁰⁾. All the past or present muon beams, the low intensity Masek line at Berkeley [Heggie et al. (1960), Masek et al. (1961), Davis et al. (1963)],

10) "Conventional" is here meant to denote guiding systems with the two conjugate optics, vertical and horizontal, linear and decoupled. Some of the focusing systems proposed [Krienen (1957), Van der Meer (1962), Burgov and Beda (1964), Regenstreif (1964), Van der Meer (1964)] unfortunately do not satisfy this condition.

the Citron channel [Citron and Øverås (1957), Citron et al. (1960)(1961) (1962)(1963-a)] or the Hyams low intensity line [Backenstoss et al. (1963)] in Geneva, the Lederman channel [Marshall and Yamanouchi (1962), Pepper (1962), Lederman and Melissinos (1964)] at Brookhaven, the Telegdi channel [Culligan et al. (1964-a)(1964-b)] in Chicago, and the Dumas channel at Saclay, are or were set up near proton accelerators.

The two examples which we have chosen to discuss here are shown, in the form of fully dimensioned sketches, in Fig. 22. It will be noticed that the beam line S' has the same sequence, except for the deflecting magnets, as the beam line S, and that the two systems S' and Σ have the same length, the same number of lenses, and end with the same square detector $0.2 \times 0.2 \text{ m}^2$. It will also be noticed that the lenses of Σ are all supplied with the same power, the gradient having been calculated so as to ensure maximum acceptance ($k = 3.35 \text{ Teslas/m}$) [Citron and Øverås (1957), Citron et al. (1963-a)], whereas the lenses of S' are supplied with the same power as those of S. It is also assumed that the 4×10^7 pions of the beam S are injected at the input of the beam lines S' and Σ , and that these particles in the projected trace space uniformly populate the surface of the two parallelograms (defined by their vertices) given in Table V.

It can be checked that these parallelograms are very close to the polygons of the beam line S drawn at the level of what has so far been called the detector target (2) (see Fig. 9). In fact, this "detector target" will now be an analysing slit ($0.01 \times 0.01 \text{ m}^2$) similar to slit (1). The

beam injected at the input of S' and Σ may thus have the spectrum shown in Fig. 21. Subsequently, in view of the enormous range of the momentum spectrum of muon emission, the injected spectrum can be considered to be monochromatic.

The transmission rates calculated under these conditions for 14 GeV/c at the input of the beam lines S' and Σ are 0.80 and 0.51 respectively. Thus, at the output of beam lines S' and Σ the total pion flux will be 3.2×10^7 for S' and 2×10^7 for Σ . It may seem surprising that there should be such a great loss of intensity as compared to the preliminary beam S . A few words of explanation are therefore necessary. A loss of intensity is inevitable for the beam S' ; it is not due to bad matching of S' to S but to the fact that the motion in the beam line S' is not stable (see Appendix); each time S' is extended by increasing the number of periods, particles are lost; if it was extended indefinitely, all the particles would be lost, independently of decay. The phenomenon is quite different for beam line Σ and is more trivial; no attempt was made to match Σ to S . However, and this is the important point, it will subsequently be seen (Section III, D, 2 and Fig. 35) that in spite of the poor matching of Σ to S , due simply to laziness, the muon flux at the output of Σ is greater than the muon flux at the output of S' .

B. EXTENSION OF THE CONCEPT OF THE TRANSMISSION RATE OF SECONDARY BEAM PARTICLES

1. DEFINITION OF THE LOCAL TRANSMISSION RATE

So far, the concept of a transmission rate has been introduced only at the input of a beam line. However, the "upstream" and "downstream" acceptance polygons have been defined at any given point with arc length s (Section I, C, 2, b). It is therefore possible to extend the definition of the transmission rate to these points:

$$(48) \quad \tau^{V,H}(s) = \frac{\text{Area of the part common to the "upstream and"} \\ \text{"downstream" polygons at s}}{\text{Area of the "upstream" polygon at s}}$$

$$(49) \quad T = \tau^V \times \tau^H$$

Since the pair of "upstream" polygons (vertical and horizontal) represents all the particles present at s, and the pair of "downstream" polygons represents all the particles which will reach the end of the beam line, the transmission rate at s, assuming uniform population of the polygons, will be the fraction of the particles present at s which will reach the end of the beam line. It will be seen later that, in this local form, the transmission rate can be extended to the tertiary particles emitted by decay in flight (Section III, C, 2).

It will be noticed that since the common part of the "upstream" and "downstream" polygons is the acceptance polygon defined by all the diaphragms of the beam line, its area is constant (Section I, C, 2, a). This means that each of the two conjugate rates τ^V and τ^H varies inversely

as the area of the corresponding "upstream" polygon. However, it has been established that, for a beam of given momentum, the area of the "upstream" polygons can only decrease as one moves along the beam line in the direction of motion (Section I, C, 2, b). For a secondary beam, τ^H , τ^V and T are thus increasing functions of s; their extreme values are obviously 0 and 1.

2. COMPLETE NUMERICAL CALCULATION OF A LOCAL TRANSMISSION RATE

Let us choose, for instance, to calculate the 14 GeV/c vertical transmission rate at point 9 of the beam line Σ (see Fig. 22 for the numbering of the points along the axis).

First, the vertical matrices for 14 GeV/c (see Table VI) are found for the different fields, distances being measured in meters and angles in radians.

By successive multiplication of these matrices or their inverse, the matrices from point 9 to the different points where a limitation appears are calculated (see Table VII).

The "upstream" polygon at 9 is then obtained by superimposing the vertical polygon at the output of S (see Section III, A) transformed by the matrix (0,9), i.e. the parallelogram given in Table VIII, and the three pairs of parallel lines corresponding to the three diaphragms 1, 4 and 7, that is:

$$(50) \quad \left\{ \begin{array}{l} 0.920z_9^V - 10.825z_9'^V = \pm 0.1 \\ 2.228z_9^V - 17.183z_9'^V = \pm 0.1 \\ 1.040z_9^V - 4.293z_9'^V = \pm 0.1 \end{array} \right. .$$

These different limitations and the polygon to which they give rise are represented in Fig. 23.

With respect to the "downstream" polygon at 9, it is the result only of the limitations due to the diaphragms consisting of the "downstream" lenses (see Table IX). It can be checked that the $0.2 \times 0.2 \text{ m}^2$ detector target is large enough not to entail additional conditions.

These various pairs of straight lines and the resulting polygon will be found in Fig. 24. Finally, the two "upstream" and "downstream" 14 GeV/c vertical polygons at point 9 are superimposed in Fig. 25. The corresponding transmission rate, $\tau_9^V = 0.7157$, can then be deduced.

3. THE BEHAVIOR OF PION TRANSMISSION ALONG BEAM LINES S' AND Σ

The CERN Fortran programs [Citron et al. (1963-b)], which have already been mentioned, calculate in this way the conjugate rates τ^V and τ^H and the space rate $T = \tau^V \tau^H$ at any points of a given beam line. Since the computer has taken over the mass of calculations which this work involves, the transmission rate at a great number of points can be calculated and the relative rôle of the different diaphragms thus studied. In Figs. 31 and 32, along with similar results relating to muons, which will be dealt with later (Section III, C, 4), will be found the curve showing the variation of

the space transmission of 14 Gev/c pions along the beam lines S' and Σ supplied by the preliminary beam line S.

C. STUDY OF THE LOCAL TRANSMISSION OF TERTIARY PARTICLES EMITTED IN FLIGHT

1. Notes on Kinematics [Rossi (1952)]

We will simply recall a few well-known properties of $\pi \rightarrow \mu + \nu$ decay.

- a. The differential energy spectrum of the muons emitted by pions of a given energy is flat between its two limits $(E_\mu)_{\min}$ and $(E_\mu)_{\max}$ (Fig. 26a). It follows that, for high energy muons ($\beta \simeq 1$), the same applies approximately to the differential momentum spectrum.

- b. The two limits $(E_\mu)_{\min}$ and $(E_\mu)_{\max}$ are expressed as:

$$(51) \quad \begin{cases} (E_\mu)_{\min} = \frac{E}{mc^2} E_\mu^* - \frac{p}{m} p_\mu^* \\ (E_\mu)_{\max} = \frac{E}{mc^2} E_\mu^* + \frac{p}{m} p_\mu^* \end{cases},$$

where E_μ^* and p_μ^* are the total energy and the momentum of the muon in the center of mass system, m the rest mass of the pion, E its total energy and p its momentum in the laboratory system.

In the case of high-energy particles, $\beta \simeq 1$, $E_\mu \simeq (pc)_\mu$, $E \simeq pc$, which with more explicit notation gives:

$$(52) \quad \left\{ \begin{array}{l} (pc)_{\mu, \min} \simeq (pc)_\pi \frac{E_\mu^* - (pc)_\mu^*}{m_\pi c^2} \\ (pc)_{\mu, \max} \simeq (pc)_\pi \frac{E_\mu^* + (pc)_\mu^*}{m_\pi c^2} \end{array} \right. .$$

The values relating to the muon in the center of mass system are constants which are simply expressed as a function of the rest mass of the pion and the muon:

$$(53) \quad \left\{ \begin{array}{l} E_\mu^* = \frac{1}{2} m_\pi c^2 \left[1 + \left(\frac{m_\mu}{m_\pi} \right)^2 \right] \\ (pc)_\mu^* = \frac{1}{2} m_\pi c^2 \left[1 - \left(\frac{m_\mu}{m_\pi} \right)^2 \right] \end{array} \right.$$

which gives, still for high-energy ($\beta \simeq 1$) pions:

$$(54) \quad \left\{ \begin{array}{l} (pc)_{\mu, \min} \simeq \left(\frac{m_\mu}{m_\pi} \right)^2 (pc)_\pi \simeq 0.575 (pc)_\pi \\ (pc)_{\mu, \max} \simeq (pc)_\pi \end{array} \right. .$$

In the case under consideration, $p_\pi = 14 \text{ GeV}/c$:

$$(55) \quad (p_\mu)_{\min} \simeq 8 \text{ GeV}/c \quad (p_\mu)_{\max} = 14 \text{ GeV}/c .$$

- c. The angle of emission of the muons in the laboratory system varies, as a function of the muon momentum, as shown in Fig. 26b for 14 GeV/c pions. It may be useful to remember, in order to keep in mind the order of magnitude of the angles of emission, that for 1 GeV/c pions the maximum of φ_μ is of the order of 40 milliradians, and that for higher energies this maximum varies approximately as $1/p_\pi$.

2. DEFINITION¹¹⁾ OF THE LOCAL TRANSMISSION RATE OF THE MUONS EMITTED IN FLIGHT

The particles of a secondary beam are emitted by a target which is perfectly defined in space. On the contrary, the particles of a tertiary beam come into being all the way along the beam line; the source is spread out. In order to overcome the difficulty, the decay path will be broken up into elements of a length Δs , which will be considered as so many individual sources. Each of these local sources will have a corresponding mean rate of muon transmission, which will now be defined.

a. Introduction of the parameters of $\pi \rightarrow \mu + \nu$ decay into the two conjugate optics

Supposing that a pion with a momentum p_π decays at M , and that $(M, d\vec{M}/ds)_\pi$ is the trace of the pion immediately before the decay. What will be, immediately afterwards, the trace of the muon with a momentum p_μ , which has appeared in this decay? (See Fig. 27.)

¹¹⁾ A variant of this definition has recently been proposed [Abrosimov et al. (1963)].

Let us define the traces by their projections on to the horizontal and vertical planes and introduce the half-angle of the decay cone in the laboratory $\varphi(p_\mu)$, and the decay azimuth ψ . Rapid geometrical reasoning leads, for para-axial pions, to the transformation:

$$(56) \quad \left(M, \frac{d\vec{M}}{ds} \right)_\pi \begin{Bmatrix} z^1, z'^1 \\ z^2, z'^2 \end{Bmatrix} \rightarrow \left(M, \frac{d\vec{M}}{ds} \right)_\mu \begin{Bmatrix} z^1, z'^1 + \varphi(p_\mu) \sin \psi \\ z^2, z'^2 + \varphi(p_\mu) \cos \psi \end{Bmatrix} .$$

It should be noted that this result supposes that the angle of emission $\varphi(p_\mu)$ is small ($\tan \varphi \approx \varphi$)

b. The local muon transmission rate

The following method has been introduced by Citron and Øverås (1957) [see also Citron et al. (1963-a)] but the acceptance areas were always ellipses. Here polygons will be used.

In order to have a clear idea, without being committed to a particular case, let us represent symbolically by circles the acceptance polygons at the point with arc length s (in Fig. 28 the radii of the circles are chosen arbitrarily).

The properties which have just been recalled (Section III, C, 2, a) lead us to introduce three polygons at s for each of the two optics, vertical and horizontal.

(i) P_π , "upstream" polygon calculated for the pion momentum

This polygon contains all the projected traces of the pions with a momentum p_π arriving at s .

- (ii) $P_{\mu P}$, polygon deduced from P_{π} by the translation $\phi(p_{\mu}) \sin \psi$ or $\phi(p_{\mu}) \cos \psi$ parallel to the axis z'

This polygon contains all the projected traces of the muons with a momentum p_{μ} , produced at s, at the azimuth ψ , by pions with a momentum p_{π} .

- (iii) $P_{\mu A}$, "downstream" polygon calculated for muon momentum
 $P_{\mu A}$ contains all the projected traces of the muons which will be accepted by the system from s onwards.

If it is supposed - and this is the first assumption of the method - that the "upstream" polygons are uniformly populated, this leads, for muons with a momentum p_{μ} produced at s, at the azimuth ψ , by pions with a momentum p_{π} , to defining two conjugate transmission rates for the two conjugate planes¹²⁾:

$$(57) \quad \left\{ \begin{array}{l} \tau^V(\psi) = \frac{\text{Area common to } P_{\mu P}^V \text{ and } P_{\mu A}^V}{\text{Area } P_{\mu P}^V} \\ \tau^H(\psi) = \frac{\text{Area common to } P_{\mu P}^H \text{ and } P_{\mu A}^H}{\text{Area } P_{\mu P}^H} \end{array} \right.$$

Supposing also - and this is the second assumption of the method - that the diaphragms are rectangular, a muon will be transmitted if, and only if, its two geometrical projections satisfy the transmission conditions

¹²⁾ It is recalled that the superscripts V and 1, and H and 2 have respectively the same meaning.

in the two conjugate planes, which gives the transmission rate in space as:

$$(59) \quad T(\psi) = \tau^V(\psi) \times \tau^H(\psi) \quad .$$

Since the decay azimuth ψ is not determined by any kinematic condition, an average will have to be established over ψ , in order to reach a "mean" transmission rate for muons with a momentum p_μ , produced in the neighborhood of s , by the p_π pions considered.

Since all azimuths ψ are a priori equally probable, the following mean will be chosen:

$$(59) \quad \overline{T(\psi)} = \frac{1}{2N} \left[T_0 + 2 \sum_{n=1}^{N-1} T_n + T_N \right] \quad .$$

In fact, if extended to the closed interval $0 \leq \psi \leq 2\pi$, this "trapezoid" mean amounts to the arithmetic mean over the half-open interval $0 \leq \psi < 2\pi$; furthermore, it will be seen from an example (see Table X) that when $N = 4k$ and the acceptance polygons are symmetrical with respect to the origin of axes z and z' , the trapezoid mean over a half or a quarter of the $(0, 2\pi)$ interval will give the same result as the arithmetic mean over the whole interval.

Finally, it should be pointed out that the mean rate $\overline{\tau(\psi)}$ introduced above is a function of s , of p_μ , of p_π and of the polygons P_0 which, at the beginning of the system, contain the projected traces of the emitting pions, since the "upstream" polygons at every point s result from

4. THE VARIATION OF MUON TRANSMISSION ALONG BEAM LINES S' AND Σ

By means of the programs written at CERN by J. Hornsby [Citron et al. (1963-b)] which calculate both the muon and the pion transmission rate, the variation of the muon transmission can be followed along lines S' and Σ , for different values of the momentum of the muons emitted. The results for a few characteristic momenta are grouped in Figs. 31 and 32.

At first sight, these results appear rather incoherent, and the reader is likely to think that the machine is out of order or that the programmer has slipped up. Accordingly, it is a good idea to present them in another form.

First, let us recall another well-known example of coherence hidden under apparent disorder. Take a periodic beam line, of Σ type for instance, and particles of such momentum that their motion along it will be stable. Let T_s be the period of the structure of the system. If the trajectory of a particle is now drawn in one of the two projected trace planes, this will give a broken line¹³⁾ consisting of a set of segments of straight lines and of arcs of ellipses and hyperbolas [Hereward et al. (1956)]. This curve is very irregular; it is not even a closed curve, since the motion of the particles is only quasi-periodic (the quasi-period of the motion, T_M , is not generally an exact multiple [Citron and Øverås (1957), Septier (1961), Citron et al. (1963-a)] of the period of the structure, T_s).

¹³⁾ When the fringing fields are neglected.

However, if from the continuous set of points composing this curve, only those representing the trace at the same point of each period T_s of the structure are considered, they are all on an ellipse [King (1955), Sturrock (1955), Hereward et al. (1956), Fronteau (1964)]. To each point of the period, considered as the leading point of the same points in the other periods, there corresponds a different ellipse. Thus, a family of ellipses can be associated with a curve which appears to be extremely irregular; order is discovered beneath the disorder.

The different curves in Fig. 32 can be dealt with in the same way. However, in order to keep the new figure (Fig. 33) as simple as possible, the method will be applied only to the curve $p_\mu = 10.20 \text{ GeV/c}$, which is where the phenomenon is perhaps the most clearly defined. Furthermore, the transmission of the muons will be normalised to that of the pions; the ratio $\bar{T}_\mu/T_\pi$ will be plotted. This method has already been used several times [Fronteau and Hornsby (1962), Keil(1963)]. Finally, for greater clarity, only half the 96 points of the curve $p_\mu = 10.20 \text{ GeV/c}$ in Fig. 32 will be retained. Figure 33 then clearly shows that these points generate a family of pseudo-periodic curves.

It will be noticed that these curves all accept approximately the same quasi-period $T_c \simeq 43.5$ meters. If, on the other hand, the quasi-period T_M of the motion for $p_\pi = 14 \text{ GeV/c}$ is calculated [Citron et al. (1963-a)] one finds 86 meters, namely $T_c \simeq T_M/2$. This confirms the following interpretation of the curves of Fig. 33. At a given point of a period,

and at all the similar points of the other periods T_s , the "downstream" acceptance polygon has the same orientation and a form which is always very similar (for an infinitely long system, one would say that the acceptance ellipse is always the same) [Citron et al. (1963-a)]. Thus, considering only the points deduced from a leading point by arithmetic progression with the step T_s along the axis, amounts to considering only very similar acceptance conditions or, to some extent, immobilizing the acceptance. The band representing the injected beam (see Figs. 25 and 30, for instance) turns with respect to this immobilized polygon, with the quasi-period T_M of the pion motion [Citron et al. (1963-a)]. This means that there are extremes of transmission when the band, in its rotation, is situated according to the largest **or** the smallest dimension of the "downstream" polygon.

This reasoning is obviously very qualitative (even the conjugate rôle of the two planes, vertical and horizontal, is not taken into account). However, it justifies the **shape** of the curves in Fig. 33 which in turn inspire confidence in those of Figs. 31 and 32. Finally, Fig. 33 shows that it is often possible to reduce the amount of numerical calculation. It is often sufficient, **in order** to find a "mean" transmission rate for the whole extent of the beam line, to consider for each momentum nothing but the curve $\bar{T}_\mu/T_\pi$ drawn for the centers of the inter-lens intervals, for instance.

D. CALCULATION OF THE SPECTRUM OF MUONS TRANSMITTED

1. THE METHOD OF INTEGRATION ALONG THE BEAM LINE

Let us imagine the decay path, from one end to the other of the beam line KJ, dotted with a set of points j at which the respective transmission rates of the pions and the muons are known. Figure 34 shows the path KJ divided into elements $(\Delta s)_j$ whose extremities are the two points K and J and the centers of the segments $\{j, j+1\}$. A segment $(\Delta s)_j$ is thus associated with each point j , at a distance ℓ_j from J.

Let n be the number of 14 GeV/c pions found at the output of the beam line KJ. The intensity of this pion beam has decreased all the way along the path, owing to decay in flight and progressive narrowing down of the beam by the successive diaphragms. Consequently, the pions of the beam studied were more numerous at any point j than at the end of the beam line. At j there were:

$$(62) \quad \frac{n e^{\frac{\ell_j}{L}}}{[T_\pi]_j}$$

where L is the mean free path of the pions considered. These pions have necessarily emitted on the path element $(\Delta s)_j$ a total number of muons which can be expressed by the product:

$$(63) \quad \frac{n e^{\frac{\ell_j}{L}}}{[T_\pi]_j} \times \frac{(\Delta s)_j}{L}$$

It is moreover known (see Fig. 26) that the differential energy spectrum of the muons is flat; this means that the number of muons emitted in the energy band ΔE_μ was:

$$(64) \quad \frac{\frac{\ell_j}{L}}{\left[\frac{T_\pi}{\pi}\right]_j} \times \frac{(\Delta s)_j}{L} \times \frac{\Delta E_\mu}{(E_\mu)_{\max} - (E_\mu)_{\min}}$$

and the beam line will only have transmitted a fraction equal to:

$$(65) \quad \delta N_\mu = \frac{\frac{\ell_j}{L}}{\left[\frac{T_\pi}{\pi}\right]_j} \times \frac{(\Delta s)_j}{L} \times \frac{\beta(p_\mu) \Delta p_\mu}{(E_\mu)_{\max} - (E_\mu)_{\min}} \times \left[\bar{T}_\mu(p_\mu) \right]_j$$

where $\bar{T}_\mu$ is the mean rate $\overline{T_\mu(\psi)}$.

The momentum spectrum of the muons at the end of the beam line is found by summing over j (sum along the beam line):

$$(66) \quad \frac{\Delta N_\mu}{\Delta p_\mu} = \frac{n\beta(p_\mu)}{L[(E_\mu)_{\max} - (E_\mu)_{\min}]} \sum_j e^{\frac{\ell_j}{L}(\Delta s)_j} \left[\frac{\bar{T}_\mu(p_\mu)}{T_\pi} \right]_j$$

Note: for a high-energy beam, $\beta(p_\mu) \simeq 1$ and

$$(67) \quad (E_\mu)_{\max} - (E_\mu)_{\min} \simeq (p_\mu)_{\max} - (p_\mu)_{\min}$$

2. THE MUON SPECTRUM OF BEAM LINES S' AND Σ

The foregoing method of integration was applied for 96 quasi-equidistant points j (see Figs. 31, 32) and for 15 momenta p_μ , chosen so

that the corresponding angles of emission form an arithmetical progression between 0 and $(\varphi_\mu)_{\max}$. For 14 GeV/c pions, see Table XI.

This choice of momenta may not be considered very judicious at high energy, since lower down the momentum spectrum the points are too close together. The choice is in fact due to the form of the existing CERN programs [Citron et al. (1963-b)] which were drawn up so as to remain valid for low energy beams. For such beams the angle of emission φ_μ is so important, as has already been pointed out, that it is necessary to sort the momenta studied according to the corresponding angle of emission. It then appears most reasonable to choose angles in arithmetical progression.

Nevertheless, the momenta chosen gave the two spectra in Fig.

35. The area enclosed by these curves denotes the intensity of the muon beams corresponding to S' and Σ .

$$(68) \quad (N_\mu)_{S'} = 1.50 \times 10^6; \quad (N_\mu)_\Sigma = 2.14 \times 10^6.$$

Thus, as stated in Section III, A, the beam line Σ is a better muon guide than the beam line S' , in spite of the small amount of trouble taken to match the beam line Σ to the preliminary beam line S .

3. REMARKS

a. In the foregoing calculation, only the muons produced along beam lines S' and Σ themselves were taken into account; the muons from the preliminary beam S were neglected.

b. The reader who is familiar with the concept of the acceptance ellipse will realize that in the beam line S' the use of polygons is obligatory. The motion in S' is unstable [Sturrock (1955)], for all the momenta of the muon emission spectrum (the envelope of the pairs of straight lines representing the diaphragms is then a hyperbola [Fronteau (1964)], in at least one of the two planes, vertical and horizontal), and even for the pion momentum (the envelope is then reduced to two points; see Appendix). Under these conditions it is difficult to see how any acceptance ellipse could be introduced.

c. Further, the reader will not forget that the beams coming out of beam lines S' and Σ are mixed beams of pions and muons, with the muons definitely in the minority:

$$(69) \quad \left(\frac{N_{\mu}}{N_{\pi}} \right)_{S'} = \frac{1.5 \times 10^6}{3.2 \times 10^7} = 0.047; \quad \left(\frac{N_{\mu}}{N_{\pi}} \right)_{\Sigma} = \frac{2.14 \times 10^6}{2.00 \times 10^7} = 0.107 \quad .$$

In order to transform this mixed pion and muon beam into a muon beam, the pion content must be eliminated. Different ways of doing this have been considered. Most of them have been, or will be, used. For instance, simple magnetic separation [Heggie et al. (1960), Citron et al. (1963-a), Culligan et al. (1964)] can be used, if it is decided to keep only a band of muons which is clearly distinct from the narrow pion band. The beam line may also be lengthened by multiplying the number of periods [Melissinos and Pepper (1962)], (if the motion is stable in the line considered), or the system can even be bent into a circle by inserting magnets between the lenses [Farley (1964)] or replacing the lenses by magnets

(strong focusing [Tinlot (1962)(1964)] or weak focusing [Farley (1962/63) (1964)]). This lets the pion content disappear by decay in flight and thus not only increases the intensity of the muon beam but also ensures that it is completely free of pions. The trace space polygon method obviously applies to all these possibilities. The pion content can also be eliminated by means of an absorber [Lederman and Melissinos (1964), Fronteau (1964)] in which the pions are lost through strong interactions, whereas the muons, subjected only to electromagnetic multiple scattering, pass through the obstacle without too much difficulty. This system, however, reduces the muon content. In this case also, the trace space polygon method, slightly modified, can be used to study the phenomenon [Banaigs et al. (1964), Fronteau (1964)]. It may even be said that this is where the trace space and the corresponding Liouville-type invariance justify their separate existence, since the phase space of analytical mechanics no longer exists on account of the slowing down of the motion in matter.

d. Finally, it may be useful to specify the amount of calculation necessary to achieve the above results. For each of the beam lines S' and Σ , 9,000 elementary transmission rates had to be calculated, in other words, 9,000 comparisons of a given polygon with another. This for S' as for Σ , and with the assumptions made:

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TABLE I

The 14 GeV/c horizontal matrices of the different fields of
beam line S without its bending magnets

Fields	Matrix Terms			
	α^H	β^H	γ^H	δ^H
(0, 1) and (12,13)	1.0	8.82	0.0	1.0
(1, 2) " (13,14)	1.07746	1.07697	0.14942	1.07746
(2, 3) " (14,15)	1.07746	1.07697	0.14942	1.07746
(3, 4) " (15,16)	1.0	0.5	0.0	1.0
(4, 5) " (16,17)	0.87789	1.00690	-0.22773	0.87789
(5, 6) " (17,18)	0.87789	1.00690	-0.22773	0.87789
(6, 7) " (18,19)	1.0	0.5	0.0	1.0
(7, 8) " (19,20)	1.07776	1.07708	0.15001	1.07776
(8, 9) " (20,21)	1.07776	1.07708	0.15001	1.07776
(9,10) " (21,22)	1.0	0.6	0.0	1.0
(10,11) " (22,23)	1.0	2.18	0.0	1.0
(11,12) " (23,24)	1.0	6.04	0.0	1.0

TABLE II

The 14 GeV/c horizontal matrices from the production target to
to the different diaphragms of beam line S (without bending magnets)

Fields	Matrix Terms			
(0, j)	A ^H	B ^H	C ^H	D ^H
(0, 2)	1.0775	10.5802	0.1494	2.3953
(0, 5)	1.6260	18.2897	-0.0550	-0.0039
(0, 8)	0.8023	10.5644	-0.2767	-2.3974
(0,12)	-1.0024	-0.0079	-0.1779	-0.9991
(0,14)	-2.9621	-10.5787	-0.5759	-2.3943
(0,17)	-4.8833	-18.2853	0.0559	0.0043
(0,20)	-2.6835	-10.5608	0.7039	2.3973
(0,24)	1.0061	0.0158	0.3560	0.9995

TABLE III

The horizontal limitations due to the different diaphragms of beam line S, calculated for 14 GeV/c at the level of the production target.

Diaphragms (j)	Limitations
(2)	$-0.1 \leq 1.077z_0^H + 10.580z_0'^H \leq 0.1$
(5)	$-0.1 \leq 1.626z_0^H + 18.290z_0'^H \leq 0.1$
(8)	$-0.1 \leq 0.802z_0^H + 10.564z_0'^H \leq 0.1$
(12)	$-0.005 \leq -1.002z_0^H - 0.008z_0'^H \leq 0.005$
(14)	$-0.1 \leq -2.962z_0^H - 10.579z_0'^H \leq 0.1$
(17)	$-0.1 \leq -4.883z_0^H - 18.285z_0'^H \leq 0.1$
(20)	$-0.1 \leq -2.683z_0^H - 10.561z_0'^H \leq 0.1$
(24)	$-0.005 \leq 1.006z_0^H + 0.016z_0'^H \leq 0.005$

TABLE IV

The spectrum of beam S for a square production target -
($0.01 \times 0.01 \text{ m}^2$) and for complete beam line

Momentum (GeV/c)	τ^V	τ^H	$T = \tau^V \tau^H$	$\overline{\Delta\Omega} = \Delta\Omega_0 \times T$ $= 4.8 \times 10^{-4} T (\text{ster.})$
13.60		0.0	0.0	0.0 $\times 10^{-4}$
13.65	0.1692	0.0010	0.0002	0.001 "
13.70	0.1949	0.0144	0.0028	0.013 "
13.75	0.2292	0.0524	0.0120	0.058 "
13.80	0.2770	0.1363	0.0378	0.181 "
13.85	0.3303	0.2898	0.0957	0.459 "
13.90	0.3828	0.4703	0.1800	0.864 "
13.95	0.4346	0.6721	0.2921	1.402 "
14.00	0.4856	0.8699	0.4224	2.028 "
14.05	0.5269	0.6686	0.3629	1.742 "
14.10	0.4825	0.4794	0.2313	1.110 "
14.15	0.4348	0.2728	0.1186	0.569 "
14.20	0.3857	0.0987	0.0381	0.183 "
14.25	0.3364	0.0205	0.0069	0.033 "
14.30	0.2876	0.0	0.0	0.0 "

TABLE V

The vertices of input polygons of beam lines S' and Σ

Vertical Plane		Horizontal Plane	
z_0^V (meters)	$z_0'^V$ (radians)	z_0^H	$z_0'^H$
- 0.004942	- 0.010990	- 0.005000	- 0.005903
- 0.005000	0.009881	- 0.004972	0.004119
0.004942	0.010990	0.005000	0.005903
0.005000	- 0.009881	0.004972	- 0.004119

TABLE VI

The 14 GeV/c vertical matrices of the different
fields of beam line Σ

Fields	Matrix terms			
	α^V	β^V	γ^V	δ^V
(2,3) (5, 6)	1.0	6.21	0.0	1.0
(8,9) (9,10)	1.0	3.105	0.0	1.0
(0,1) (1, 2) (6, 7) (7, 8) ...	1.03981	1.06390	0.07632	1.03981
(3,4) (4, 5) (10,11) (11,12) ...	0.96072	1.03621	-0.07433	0.96072

TABLE VII

The 14 GeV/c vertical matrices from point 9 to the different
diaphragms of beam line Σ

Fields	Matrix Terms			
	A^V	B^V	C^V	D^V
(9, 0)	0.8524	-11.1807	0.0320	0.7529
(9, 1)	0.9204	-10.8247	0.0984	-0.0704
(9, 4)	2.2281	-17.1832	0.0072	0.3934
(9, 7)	1.0398	- 4.2925	-0.0763	1.2768
(9,11)	0.9607	4.0192	-0.0743	0.7299
(9,14)	-0.1946	7.8285	-0.1516	0.9617
(9,17)	-1.5575	19.9887	-0.0591	0.1161
(9,20)	-1.1756	9.7569	-0.0295	-0.6055
(9,23)	-2.0479	9.9333	0.0331	-0.6488
(9,26)	-0.6260	- 1.0180	0.1310	-1.3843
(9,29)	0.1280	-13.0551	0.0822	-0.5690
(9,32)	0.7387	-10.4675	0.1210	-0.3608
(9,35)	2.1373	-19.0460	0.0243	0.2516
(9,37)	1.2366	- 7.7437	-0.1356	1.6576

TABLE VIII

The vertical input polygon of beam line Σ seen at the
level of point 9 for 14 GeV/c

z_9^V (meters)	$z_9'^V$ (radians)
-0.12660	-0.00921
0.10671	0.00858
0.12660	0.00921
-0.10671	-0.00858

TABLE IX

The "downstream" vertical limitations calculated for 14 GeV/c
at point 9 of beam line Z

Diaphragms	Limitations
(11)	$-0.1 \leq 0.961z_9^V + 4.019z_9'^V \leq 0.1$
(14)	$-0.1 \leq -0.195z_9^V + 7.829z_9'^V \leq 0.1$
(17)	$-0.1 \leq -1.558z_9^V + 19.989z_9'^V \leq 0.1$
(20)	$-0.1 \leq -1.176z_9^V + 9.757z_9'^V \leq 0.1$
(23)	$-0.1 \leq -2.048z_9^V + 9.933z_9'^V \leq 0.1$
(26)	$-0.1 \leq -0.626z_9^V - 1.018z_9'^V \leq 0.1$
(29)	$-0.1 \leq 0.128z_9^V - 13.055z_9'^V \leq 0.1$
(32)	$-0.1 \leq 0.739z_9^V - 10.468z_9'^V \leq 0.1$
(35)	$-0.1 \leq 2.137z_9^V - 19.046z_9'^V \leq 0.1$

TABLE X

The 10 GeV/c muon transmission rates calculated at point 9 of
beam line Σ for different decay azimuths ψ

ψ	τ^V	τ^H	$T = \tau^V \tau^H$
0	0.4537	0.5822	0.2641
$\pi/4$	0.4280	0.6042	0.2586
$\pi/2$	0.4128	0.6449	0.2662
$3\pi/4$	0.4280	0.6042	0.2586
π	0.4537	0.5822	0.2641
$5\pi/4$	0.4280	0.6042	0.2586
$3\pi/2$	0.4128	0.6449	0.2662
$7\pi/4$	0.4280	0.6042	0.2586
2π	0.4537	0.5822	0.2641

TABLE XI

Angles of emission of muons produced by 14 GeV/c pions

φ_{μ} (rad.)	p_{μ} (GeV/c)
0.0	8.022
0.000403	8.040
0.000806	8.095
0.001209	8.192
0.001612	8.344
0.002015	8.575
0.002418	8.957
0.002821	10.200
0.002418	11.843
0.002015	12.583
0.001612	13.118
0.001209	13.511
0.000806	13.785
0.000403	13.947
0.0	14.000

A P P E N D I X

The instability of the beam line S' for 14 GeV/c

The study of the stability of the motion as a function of the structure of beam lines does not come within the scope of this paper. However, in order to clarify certain points the reader has been referred to various papers dealing with this problem. It may therefore be useful to give a simple example.

Let us take the case of beam line S' for 14 GeV/c. This is a borderline case (the trace of the transfer matrix is equal to -2) which is not difficult to study. Moreover it is so simple, and even trivial, that it is not usually studied in detail; this is no doubt why the solution of this problem is often overlooked.

Since the matrix from point 0 to point 10 of beam line S' (see Fig. 22) for 14 GeV/c takes on the following very special form:

$$(71) \quad \left(\bar{M}_{0,10} \right) = \begin{pmatrix} -1 & 0 \\ G & -1 \end{pmatrix},$$

the matrix between any pair of points j which are separated by a period T_S of the structure,

$$(72) \quad \left(\bar{M}_{j,j+10} \right) = \left(\bar{M}_{10,j+10} \right) \left(\bar{M}_{0,10} \right) \left(\bar{M}_{j,0} \right),$$

may be written as:

$$(73) \quad \begin{pmatrix} M_{j,j+10} \end{pmatrix} = \begin{pmatrix} t & u \\ v & w \end{pmatrix} \begin{pmatrix} -1 & 0 \\ c & -1 \end{pmatrix} \begin{pmatrix} w & -u \\ -v & t \end{pmatrix}; \quad (tw - uv = 1);$$

or, by working out the product, using Liouville type invariance and introducing more concise notation:

$$(74) \quad \begin{pmatrix} M_{j,j+10} \end{pmatrix} = \begin{pmatrix} -1 + \alpha & \beta \\ -\frac{\alpha^2}{\beta} & -1 - \alpha \end{pmatrix}; \quad \begin{aligned} \alpha &= Cuw \\ \beta &= -Cu^2 \end{aligned}$$

It is then easy to show by recurrence that:

$$(75) \quad \begin{pmatrix} M_{j,j+10n} \end{pmatrix} = \begin{pmatrix} -1 + \alpha & \beta \\ -\frac{\alpha^2}{\beta} & -1 - \alpha \end{pmatrix}^n = (-1)^n \begin{pmatrix} 1 - n\alpha & -n\beta \\ \frac{n\alpha^2}{\beta} & 1 + n\alpha \end{pmatrix}.$$

The matrix from point 0 to point $j + 10n$ is then deduced:

$$(76) \quad \begin{pmatrix} M_{0,j+10n} \end{pmatrix} = \begin{pmatrix} M_{j,j+10n} \end{pmatrix} \begin{pmatrix} M_{0,j} \end{pmatrix},$$

$$(77) \quad \begin{pmatrix} M_{0,j+10n} \end{pmatrix} = (-1)^n \begin{pmatrix} 1 - n\alpha & -n\beta \\ \frac{n\alpha^2}{\beta} & 1 + n\alpha \end{pmatrix} \begin{pmatrix} t & u \\ v & w \end{pmatrix},$$

or, by working out the product:

$$(78) \quad \begin{pmatrix} M_{0,j+10n} \end{pmatrix} = (-1)^n \begin{pmatrix} t - (\alpha t + \beta v)n & u - (\alpha u + \beta w)n \\ v + \alpha(\frac{\alpha}{\beta}t + v)n & w + \alpha(\frac{\alpha}{\beta}u + w)n \end{pmatrix}.$$

The relation is simplified by re-introducing explicit values of α and β ; there remains only:

$$(79) \quad \left(M_{0,j+10n} \right) = (-1)^n \left[\begin{pmatrix} t & u \\ v & w \end{pmatrix} - Cn \begin{pmatrix} u & 0 \\ w & 0 \end{pmatrix} \right],$$

or :

$$(80) \quad \begin{cases} z_{j+10n} = (-1)^n (tz_0 + uz'_0 - Cnuz_0) \\ z'_{j+10n} = (-1)^n (vz_0 + wz'_0 - Cnwz_0) \end{cases}.$$

Thus, for any given point j ($u \neq 0, w \neq 0$), the absolute value of z_{j+10n} and z'_{j+10n} increases indefinitely with n , except when $C = 0$ or when the production target is a point target ($z_0 = 0$). In other words, if the beam line S' is lengthened indefinitely by increasing the number n of periods, the particles which are not emitted on the axis will always end by encountering an obstacle on the side, which will block their path. This is what is meant by the instability of the beam line S' for 14 GeV/c.

To the diaphragms $\pm r$ at the different points $j+10n$ (a given j and a variable n) corresponds the following set of limitations:

$$(81) \quad -r < (-1)^n (tz_0 + uz'_0 - Cnuz_0) < r.$$

The pairs of parallel lines delimiting these bands of projected trace space are:

$$(82) \quad (tz_0 + uz'_0 \pm r) - Cnuz_0 = 0$$

One can recognize the equations of two pencils of concurrent straight lines, whose respective vertices are the following two points:

$$(83) \quad \boxed{\begin{array}{l} z_0 = 0 \\ z'_0 = \pm \frac{r}{u} \end{array}}$$

Figure 14 gives several examples of such straight lines (lines 5 and 17, 8 and 20, and 2 and 14).

The slope of the lines of the double pencil (Eq. 82) is $Cn - t/u$. It thus varies linearly with n and its absolute value increases indefinitely with n . In other words, when the beam line S' is lengthened by increasing the number of periods, the straight lines of the two pencils (Eq. 82) turn round their vertices, always in the same direction, and approach the z' axis. They thus progressively reduce the acceptance polygon and finally cancel it. This is the way in which the instability shows up in the trace space picture.

It should further be pointed out that this instability is a fundamental phenomenon connected with the particular form of the matrix $(M_{0,10})$ for 14 GeV/c. It is due neither to chromatic aberrations nor to aberrations resulting from the actual non-homogeneity of the various magnetic fields.

REFERENCES

- Baker, W.F. (1961). BNL internal report WFB-1.
- Banaigs, J., Citron, A., Fronteau, J., Hornsby, J. (1964). CERN 64-29.
- Benvenuti, P., Uccelli, F. (1964). Frascati INFN/TC-64/1.
- Blanc-Lapierre, A., Goudet, G., Lapostolle, P. (1959). "Electronique générale". Eyrolles, Paris.
- Braunersreuther, E., Chabaud V., Delorme, C., Morpurgo, M. (1961). CERN 61-12.
- Chamberlain, O. (1960). Annual Review of Nuclear Science 10, 161.
- Citron, A., Øverås, H. (1957). CERN SC/143.
- Citron, A., Morpurgo, M., Øverås, H. (1963-a). CERN 63-35.
- Citron, A., Fronteau, J., Hornsby, J. (1963-b). CERN 63-30.
- De Raad, B. (1963). Proceedings of the International Conference on High-Energy Accelerators, Dubna, p.730.
- Emmerson, J. McL., Middlemas, N. (1963). Nuclear Instruments and Methods 24, 93.
- Filon, L.N.G. (1935). "An introduction to projective geometry". Edward Arnold, London.
- Fronteau, J. (1961). Laboratoire de Physique Atomique et Moléculaire du Collège de France. Internal reports 611, 612.
- Fronteau, J. (1962). Nuclear Instruments and Methods 16, 233.
- Fronteau, J., Hornsby, J. (1962). CERN 62-36.
- Fronteau, J. (1964). CERN 64-11, Thesis, Paris.

Gardner, J.W., Whiteside, D. (1961). NIRL/M/21.

Gardner, J.W., Whiteside, D. (1963). NIRL/M/44.

Geibel, J. (1964). "Fortran matching and tracking program". CERN private communication.

Goursat, E. (1949). Cours d'Analyse Mathématique. Gauthier-Villars, Paris.

Grivet, P., Septier, A. (1960). Nuclear Instruments and Methods. 6, 126.

Haddock, R.P. (1963). Rev. Sci. Instr. 34, 745.

Hereward, H.G., Johnsen, K., Lapostolle, P. (1956). Proceedings of the CERN Symposium on High-Energy Accelerators and Pion Physics. Vol.1, p.179.

Hereward, H.G. (1959). CERN PS/Int. TH 59-5.

Hoffmann, L., Weber, G. (1964). "A three magnet distribution system for use with an external target". CERN private communication.

King, N.M. (1955). Harwell AERE T/M 118.

King, N.M. (1964). Progr. in Nuclear Phys. 9, 73.

Kirstein, P.T. (1963). Journal of Electronics and Control 14, 241.

Knowles, H.B. (1963). Nuclear Instruments and Methods 25, 29.

Lapostolle, P. (1955). CERN/PS-PL2.

Luckey, D. (1961). In "Techniques of high-energy physics". (D.M. Ritson, Ed.)

Chapter IX. Interscience publishers, New York.

Madsen, J.H.B. (1964). CERN/IPS/EP 64-5.

Marshall, L. (1962). Rev. Sci. Instr. 33, 919.

Miller, V.V. (1964). Dubna P-1590 (in Russian).

Morpurgo, M., Petrucci, G. (1963). CERN MSC/24/1439, M93.

Penner, S. (1961). Rev. Sci. Instr. 32, 150

Regenstreif, E. (?). In "Focusing of charged particles". (A. Septier, Ed.)
Chapter ? Academic Press, New York. rmb/g/2886

Sacharidis, E.J. (1964). BNL internal report EJS-1.

Septier, A. (1961). Advances in Electronics 14, 85.

Smith, L. (1963). Proceedings of the International Conference on High-Energy Accelerators, Dubna, p.80.

Sturrock, P.A. (1955). "Static and dynamic electron optics - an account of focusing in lens, deflector and accelerator". At the University Press, Cambridge.

Swann, W.F.G. (1933). Phys. Rev. 44, 224.

Uccelli, F. (1963-a). Frascati INFN/TC-63/2.

Uccelli, F. (1963-b). Frascati INFN/TC-63/15.

Zembery, J., Banner, P., Teiger, J., Van Rossum, L. (1964). "A pion beam set up at Saclay from an external target". Saclay private communication.

Abrosimov, N.K., Kaminker, D.M., Petrov, I.A., Sherman, S.G. (1963).

Proceedings of the International Conference on High-Energy
Accelerators, Dubna, p.821 (in Russian).

Backenstoss, G., Hyams, B.D., Knop, G., Marin, P.C., Stierlin, U. (1963).

Phys. Rev. 129, 2759.

Ballam, J. (1960). Stanford University Hansen Laboratories. Report M 200-8.

Purgov, N.A., Beda, A.G. (1964). Moscow ITEP N214 (in Russian).

Citron, A., Delorme, C., Farley, F.J.M. Goldzahl, L., Heintze, J.,

Michaelis, E.G., Morpurgo, M., Øverås, H. (1960). **Proceedings**
of the Conference on Instrumentation for High-Energy Physics,
Berkeley, p.286.

Citron, A., Delorme, C., Farley, F.J.M., Goldzahl, L., Heintze, J.,

Michaelis, E.G., Morpurgo, M., Øverås, H., Shtcherbakov Yu. (1961).

J. Phys. Radium 22, 152A.

Citron, A., Delorme, C., Fries, D., Heintze, J., Michaelis, E.G., Øverås, H.,

Shtcherbakov Yu. (1962). Nuclear Instruments and Methods 15, 121.

Clement, G. (1964). Thèse de 3eme cycle, Paris.

Clement, G., Kessler, P. (1964). "Electroproduction de muons de très haute
énergie". Submitted to Nuovo Cimento.

Culligan, G., Hinterberger, H., Øverås, H., Telegdi, V.L., Winston, R.

(1964-a). Proceedings of the Conference on High-Energy Cyclotron
Improvement, Williamsburg, Virginia, p.237.

Culligan, G., Lundy, R.A., Telegdi, V.L., Winston, R., Yovanovitch, D.D.

(1964-b). Proceedings of the Conference on High-Energy Cyclotron
Improvement, Williamsburg, Virginia, p.264.

Davis, H.F., Ewart, T.E., Hasek, G.E., Platner, E.D., Toutonghi, J.P.,

Williams, R.W. (1963). Phys. Rev. 131, 2192.

Drell, S.D. (1960). Phys. Rev. Letters, 5, 278.

- Farley, F.J.M. (1962/63). "Proposed high-precision g-2 experiment",
"Calculations on muon storage ring project" and MSR/7. CERN
limited distribution.
- Farley, F.J.M. (1964). In "Nucleon structure". (R. Hofstadter and L.I. Schiff,
Eds.) Chapter "The muon as a probe for nucleon structure".
Stanford University Press.
- Heggie, L.D., Kim, Y.B., Masek, G.E., Williams, R.W. (1960). Proceedings of the
International Conference on High-Energy Physics. Rochester, 1960.
- Keil, E. (1963). CERN/AR/Int. P Sep/63-5.
- Kessler, P. (1962). Cahiers de Physique, 143, 283.
- Krienen, F. (1957). CERN 57-28.
- Lederman, L.M., Melissinos, A.C. (1964). BNL 7957, 31.
- Marshall, L., Yamanouchi, T. (1962) BNL internal report PD-47.
- Masek, G.E., Heggie, L.D., Kim, Y.B., Williams, R.W. (1961). Phys. Rev.
122, 937.
- Masek, G.E. (1962). SLAC-5, 193.
- Melissinos, A.C., Pepper, S.V. (1962). University of Rochester, unpublished
report, and BNL 7957, p.40
- Panofsky, W.K.H. (1963). Am. J. Phys. 31, 409.
- Pepper, S.V. (1962). Rochester University report NYO 10251.
- Regenstreif, E. (1964). CERN 64-41.
- Rossi, B. (1952). "High-energy particles". Prentice Hall, New York.
- Tinlot, J. (1962). SLAC-5, 165.
- Tinlot, J. (1964). BNL 7957, 117.
- Van der Meer, S. (1962). CERN 62-16.
- Van der Meer, S. (1964). "Long-term programme - 500 m muon channel". CERN
internal memorandum.

FIGURE CAPTIONS

1. Notation used.
2. Definition of the trace of a particle and of the corresponding projected traces.
3. Definition of the two projected trace spaces.
4. Diaphragm seen in real space.
5. Diaphragm seen in projected trace space.
6. Diaphragm seen from two different points in real space.
7. Diaphragm seen from two different points in projected trace space.
8. Any set of diaphragms, seen in real space.
9. Block diagram of beam line S.
10. The respective rôle of diaphragms (0), (1) and (2) of beam line S, when the chromatism of the quadrupoles is neglected.
11. Spectrum of beam S in the case of a production target such that $\rho < r$, neglecting the chromatism of the quadrupoles.
12. Influence of the dimension of the production target on the spectrum of beam S, when the chromatism of the quadrupoles is neglected.
13. Fully-dimensioned drawing of beam line S.
14. Beam line S without its deflecting magnets.
Drawing of the 14 GeV/c horizontal "downstream" polygon at the level of the production target.

15. Beam line S without its deflecting magnets.
Superposition of the 14 GeV/c horizontal "upstream" and "downstream" polygons at the level of the production target.
16. Beam line S without its deflecting magnets.
Influence of the chromatism of the quadrupoles on the horizontal "downstream" polygon drawn at the level of the production target.
17. Complete beam line S.
Conjugate influence of the chromatism of the quadrupoles and the deflection in magnets on the horizontal "downstream" polygon drawn at the level of the production target.
18. Influence of the horizontal half-dimension ρ of the production target on the horizontal acceptance spectrum of beam line S (complete)
 - (a) neglecting the chromatism of the quadrupoles (see Fig. 12)
 - (b) taking into account the chromatism of the quadrupoles.
19. Interpretation of the transmission rate
20. Beam Line S. Representation of emission in projected trace space.
21. Final spectrum of beam S (complete beam line) for a square production target ($0.01 \times 0.01 \text{ m}^2$).

22. Fully-dimensioned drawing of beam lines S' and Σ .
23. Vertical "upstream" polygon at point 9 of beam line Σ , for 14 GeV/c.
24. Vertical "downstream" polygon at point 9 of beam line Σ for
14 GeV/c.
25. Graphical evaluation of the 14 GeV/c vertical transmission rate at
point 9 of beam line Σ .
26. (a) Emission spectrum of muons produced by monokinetic pions.
(b) Angles of emission of muons produced by 14 GeV/c pions.
27. Muon emission seen in real space.
28. Muon emission seen in the two conjugate projected trace spaces.
29. Vertical "downstream" polygon for 10 GeV/c, at point 9 of beam
line Σ .
30. Graphical evaluation of the vertical transmission rate of 10 GeV/c
muons, at point 9 of beam line Σ .
31. Behavior of the pion and muon transmission rate along beam line S' .
32. Behavior of the pion and muon transmission rate along beam line Σ .
33. Beam line Σ . Quasi-periodic transmission curves for $p_\mu = 10.20$ GeV/c.
34. Method of integrating partial results along a beam line.
35. Muon spectrum at output of beam lines S' and Σ .

FIG. 1

FIG. 2

FIG. 3

FIG. 4

FIG. 5

FIG. 6

FIG. 7

FIG. 8

FIG. 9

FIG. 10

FIG. 11

FIG. 12

FIG. 13

FIG. 14

FIG. 15

FIG. 16

P	Z^H
13.86	0.32
14.00	0.87
14.14	0.31
14.28	0.00

FIG. 17

FIG. 19

FIG. 20

FIG. 21

V⁺ FIG. 22

FIG. 23

$Z_q^V = 0.72$

FIG. 25

FIG. 26^a

FIG. 26^b

FIG. 27

FIG. 28

FIG. 29

FIG. 30

FIG. 31

FIG. 32

FIG 33

FIG. 34

FIG. 35

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