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CURRENT ALGEBRA AND K_{l3} FORM FACTORS

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A B S T R A C T

Using current algebra
and a "smoothness" hypothesis
we predict a small value of f_-
in K_{l3} decay.

The leptonic decays of kaons have been calculated by several authors ^{1),2),3)}, using the assumptions of current algebra and PDDAC. In this note we extend the above calculations showing that the second form factor in $K_{\ell 3}$ decay, f_- , is small, unless a strong dependence in momentum transfer is assumed.

Let us start with the matrix element of $K_{\ell 4}$ decay. We shall use the notation of J.S. Bell ⁴⁾

$$\begin{aligned} \langle \pi_1(p), \pi_2(q) | \bar{S} A_\mu | K(k) \rangle &= F_1(p+q)_\mu + F_2(p-q)_\mu + F_3(k-p-q)_\mu \\ &= (\bar{S} K)(\vec{\pi}_1^* \cdot \vec{\pi}_2^*) [F_1(p+q)_\mu + F_3^0(k-p-q)_\mu] \\ &\quad - i(\bar{S} \vec{e} K)(\vec{\pi}_1^* \times \vec{\pi}_2^*) [F_2(p-q)_\mu + F_3^1(k-p-q)_\mu] \end{aligned} \quad (1)$$

where $\bar{S}$ represents an isospin spurion and F_1 , F_2 , F_3^0 and F_3^1 are the $K_{\ell 4}$ form factors. In general, these form factors depend on three variables and we choose $k^2 = (k-p-q)^2$, kp and kq . To simplify the argument we examine first the limiting case of the so-called "maximum smoothness" hypothesis, i.e., we assume that if we subtract out all the possible pole terms, the above form factors are slowly varying functions of all the variables. The only pole that contributed in $K_{\ell 4}$ is represented by the diagram on the Figure. It contributed only to F_3^0 and F_3^1 . Therefore, we assume that F_1 and F_2 are approximately constants.

Using the standard current algebra techniques one finds ⁴⁾:

$$\begin{aligned} \langle \pi_1(p), \pi_2(q) | \bar{S} A_\mu | K(k) \rangle_{q=0} &= \\ &= - \left[(\bar{S} K)(\vec{\pi}_1^* \cdot \vec{\pi}_2^*) - i(\bar{S} \vec{e} K)(\vec{\pi}_1^* \times \vec{\pi}_2^*) \right] \frac{1}{f_\pi} \left[2f_+ p_\mu + (f_+ + f_-)(k-p)_\mu \right] \end{aligned} \quad (2)$$

2.

where f_+ and f_- are the two form factors of K_{l3} decay and f_π is the pion decay coupling constant.

By direct comparison of (1) and (2) one obtains:

$$F_1 = F_2 = -2 f_\pi^{-1} f_+ \quad (3)$$

$$F_3^0(q=0) = F_3^1(q=0) = -f_\pi^{-1} (f_+ + f_-) \quad (3')$$

We see already here that the "maximum smoothness" assumption for K_{l4} implies the constancy of f_+ and f_- .

As we have already mentioned f_3 contains the pole given on the Figure. Therefore, we write

$$f_3 = f_K \frac{R(k_p, k_q)}{q^2 + m_K^2} + C_1 + \dots \quad (4)$$

where f_K is the K_{l2} coupling constant. Equation (4) is the Laurent expansion of f_3 in $q^2 + m_K^2$ and our hypothesis requires that C_1 etc., are constants. The residue R is the $K \rightarrow K + 2\pi$ amplitude and it is given by current algebra in first order in the pion momenta by ⁵⁾

$$R = M^{(0)} + R_1' (k_p - k_q) + O(p^2, q^2, p \cdot q) \quad (5)$$

$M^{(0)}$ is a constant term related to the commutator of an axial current with an axial divergence and it is symmetric in isospin. R_1' is anti-symmetric in isospin and can be written as:

$$R_1' = -i (\bar{5} \vec{\pi} \kappa) (\vec{\pi}_1^* \times \vec{\pi}_2^*) R_1 \quad (6)$$

In first order in pion momenta $\ell^2 + m_K^2 \approx -2kp - 2kq$. Therefore, Eq. (4) gives:

$$F_3^0 = - \frac{f_K}{2} \frac{M^{(0)}}{kp + kq} + C_1 \quad (7)$$

$$F_3^1 = - \frac{f_K}{2} \frac{R_1 (kp - kq)}{kp + kq} \quad (7')$$

At $q=0$, combining with (3'), we find:

$$\frac{f_K}{2} R_1 = - C_1 = f_{\pi}^{-1} (f_+ + f_-) \quad (8)$$

$$M^{(0)} = 0 \quad (8')$$

Let us now consider the matrix element of $\partial_\mu A_\mu$. Using (1) we can write:

$$\begin{aligned} \langle \pi_1 \pi_2 | \bar{5} \partial_\mu A_\mu | \kappa \rangle = & (\bar{5} \kappa) (\vec{\pi}_1^* \cdot \vec{\pi}_2^*) [-F_1 (m_K^2 + kp + kq + \ell^2) + F_3^0 \ell^2] \\ & - i (\bar{5} \vec{\pi} \kappa) (\vec{\pi}_1^* \times \vec{\pi}_2^*) [F_2 (kp - kq) + F_3^1 \ell^2] \end{aligned} \quad (9)$$

4.

This matrix element contains the same pole represented by the Figure and, according to our hypothesis, we can write:

$$\begin{aligned} \langle \pi_1 \pi_2 | \bar{S} \partial_\mu A_\mu | K \rangle = & \\ & (\bar{S} K) (\vec{\pi}_1^* \cdot \vec{\pi}_2^*) \left[- \frac{m_K^2 f_K R^{I=0}}{\ell^2 + m_K^2} + D_1 + D_2 (\ell^2 + m_K^2) + \dots \right] \\ & - i (\bar{S} \vec{\pi} K) (\vec{\pi}_1^* \times \vec{\pi}_2^*) \left[- \frac{m_K^2 f_K R^{I=1}}{\ell^2 + m_K^2} \right] \end{aligned} \quad (10)$$

where $R^{I=0}$ is proportional to $M^{(0)}$ and $R^{I=1} = R_1(kp-kq)$.

Comparing the Eqs. (9) and (10) and using (7) and (7'), we find :

$$F_2 + f_K R_1 = 0 \quad (11)$$

Combined with (3) and (8), Eq. (11) gives

$$f_- = 0 \quad (12)$$

This result depends essentially on the hypothesis that all the dependence on kp and kq except from the ℓ^2 terms comes from the residues of the poles. This has allowed us to write C_1 , C_2 in (4), D_1 , D_2 in (10) as constants. It is clear that if this assumption fails badly, i.e., if the non-pole terms in (4) and (10) are rapidly varying functions of kp and kq , then the same should also be true for F_1 and F_2 . Therefore, Weinberg's successful prediction of the $K_{\ell 4}$ branching ratio should be regarded as supporting our assumption.

We shall now show that our result is stable against small variations of the above constants considered as functions of kp and kq . Let us introduce further terms in Eq. (7), (7') and in the last term of (10):

$$F_3^0 = - \frac{f_K}{2} \frac{M^{(0)}}{kp+kq} + C_1 + C_2 (kp+kq) \quad (13)$$

$$F_3^1 = - \frac{f_K}{2} \frac{R_1 (kp-kq)}{kp+kq} + G_1 (kp-kq) \quad (13')$$

at $q=0$ we find:

$$\frac{f_K}{2} R_1 - G_1 kp = \int_{\pi}^{-1} (f_+ + f_-) = -C_1 - C_2 kp \quad (14)$$

We notice that $G_1 \neq 0$ and/or $C_2 \neq 0$ imply that f_+ and f_- are functions of the momentum transfer. In the same way, the last term of Eq. (10) will be

$$-i(\vec{s} \cdot \vec{\pi} \times \vec{\pi})(\vec{\pi}_1^* \times \vec{\pi}_2^*) \left[\frac{-m_K^2 f_K R^{I=1}}{l^2 + m_K^2} + G_1' (kp - kq) \right]$$

It is clear that these are the only terms that will modify Eq. (11) ⁶⁾

$$2 \int_{\pi}^{-1} f_- = G_1' + m_K^2 G_1 - G_1 kp \quad (15)$$

We assume that the terms C_2 , G_1 and G_1' do not introduce a very strong variation on either the form factors, or the matrix element $\langle \pi_1 \pi_2 | \partial_{\mu} A_{\mu} | K \rangle$. As an illustration, let us assume that C_2 , G_1 and G_1' are of the order of the corresponding constant terms C_1 and D_1 in a system where the nucleon (or the K^*) mass is taken equal to unity. This will introduce a dependence on kp of the order of 10% - 15%. It is worth while to notice that (7) and (3) show that, if the so-called σ term commutator, which is proportional to $M^{(0)}$, is

different from zero, then the $K_{\ell 3}$ form factors will show a dependence of the form $1/kp$ and this corresponds to a pole in the momentum transfer of the order of the kaon mass. It is clear that if there exists such a strong dependence, our results are meaningless, since they are based on a power series expansion in the variables kp and kq .

Equation (14) gives:

$$-f_{\pi}^{-1}(f_{+} + f_{-}) = C_1 + C_2 kp = C_1(1 + kp) \quad (16)$$

From the equality of the constant terms in (9) and (10) we find:

$$D_1 = -m_K^2 C_1 \quad (17)$$

Combining (15), (16) and (17) we find ⁷⁾:

$$\frac{f_{-}}{f_{+}} = -\frac{m_K^2 - \frac{kp}{2}}{1 + m_K^2 + \frac{kp}{2}} \approx -0.25 \quad (18)$$

The experimental situation is still quite confusing but it seems to support our qualitative conclusion, namely that, unless one is willing to accept a very strong dependence of the form factors on the momentum transfer, f_{-} is probably quite small.

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R E F E R E N C E S

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- 6) We still consider that Eq. (3) is valid since Weinberg's calculations seem to confirm it.
- 7) Our assumption about the magnitude of C_2 , G_1 and G_1' does not fix their signs. Equation (18) has been calculated in the most defavourable case, i.e., in the case when $\left| \frac{f_-}{f_+} \right|$ is maximum. However, the relative sign of f_- and f_+ is unambiguously predicted negative.

Pole term to $K l_4$ decay