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CHARGE AND STRANGENESS EXCHANGE REACTIONS IN THE QUARK MODEL

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A B S T R A C T

Relations for cross-sections and density matrix elements for charge and strangeness exchange reactions are derived, using the quark model. The density matrix relations are shown to be more meaningful tests of the model than those for cross-sections. The effects of annihilation channels, not described by the quark model, are discussed.

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INTRODUCTION

In a series of recent papers ^{1),2),3)}, the simple quark model for two-body reactions has been extended to a wide class of inelastic interactions. In this note we present further results of the model developed in Ref. ²⁾. Our main concern shall be to derive relationships between measurable quantities from which all quark parameters have been eliminated, so that each relation provides a clean, unambiguous test of the model. As has been pointed out ⁴⁾, the number of quark amplitudes which appear in an expression for a particular inelastic amplitude can be reduced by choosing reactions in which many quantum numbers are exchanged. Thus we have a situation in which the best tests of the model involve those processes which are the hardest to measure.

As in Refs. ^{2),4)} we shall find two types of relations, those involving cross-sections, and those involving density matrices. As has been repeatedly stressed ^{2),3)}, the former do not provide particularly good tests of the model, since, as we shall see, it is not possible to compare the theoretical and experimental results without making assumptions about the dynamical properties of the quarks. The density matrix relations, on the other hand, are much easier to compare with experiment, since they involve amplitudes which must all be evaluated at a common s and t . We shall also argue that in density matrix elements, effects which are manifestly not included in the simple model, such as multiple scattering, resonance formation, and annihilation ⁵⁾ should tend to cancel. With these points in mind, we can turn to a discussion of the results of the quark model for charge and strangeness exchange reactions.

2.

1. RESULTS OF THE MODEL

The amplitude for a quasi two-body interaction with scattering angle θ is given in the quark model by

$$A_h(\theta) = F(t_{\min}) D(\theta) \sum_{h'} c_{h'}^{h'}(\theta) Q_{h'}(\theta) \quad (1.1)$$

where h, h' represent the helicity configuration of the particles and quarks, respectively, $F(t_{\min})$ is the factor which takes into account the non-zero momentum transfer at $\theta=0$, $c_{h'}^h(\theta)$ is a coefficient which can be derived from the model which gives the relative weight of the quark amplitude $Q_{h'}(\theta)$, and $D(\theta)$ is a function which takes into account the fact that the spectator quarks must be dragged away from the collinear direction by the one quark which undergoes an interaction. By definition, $D(0)=1$. If we assume for the moment that $D(\theta)$ is the same for all reactions (this is one of the dynamical assumptions mentioned above), we find the following relations:

$$\frac{2}{3} \sigma(p\bar{p} \rightarrow \bar{\Lambda}\Lambda) = \sigma(\bar{p}p \rightarrow \bar{\Sigma}^0\Lambda + \bar{\Lambda}\Sigma^0) + \sigma(\bar{p}p \rightarrow \bar{Y}^{*0}\Lambda + \bar{\Lambda}Y^{*0}) \quad (1.2)$$

$$\sigma(\pi^-\bar{p} \rightarrow K^{*0}\Lambda) = \sigma(K^-\bar{p} \rightarrow \varphi\Lambda) \quad (1.3)$$

$$\sigma(\pi^-\bar{p} \rightarrow K^0\Lambda) = \frac{9}{8} \sigma(\bar{p}p \rightarrow \bar{\Sigma}^0\Lambda + \bar{\Lambda}\Sigma^0) - \frac{1}{12} \sigma(\bar{p}p \rightarrow \bar{\Lambda}\Lambda) \quad (1.4)$$

$$\sigma(\pi^-\bar{p} \rightarrow K^{*0}\Lambda) = \frac{9}{8} \sigma(\bar{p}p \rightarrow \bar{Y}^{*0}\Lambda + \bar{\Lambda}Y^{*0}) \quad (1.5)$$

$$\sigma(\bar{p}p \rightarrow \bar{\Delta}^{++} \Delta^{++}) = \frac{8}{9} \sigma(K^-p \rightarrow K^{*0} \Delta^0) \quad (1.6)$$

$$\sigma(\bar{p}p \rightarrow \bar{\Sigma}^+ \Sigma^+) = \sigma(\pi^+p \rightarrow K^+ \Sigma^+) + \frac{1}{9} \sigma(\pi^+p \rightarrow K^{*+} \Sigma^+) \quad (1.7)$$

$$\sigma(\pi^+p \rightarrow K^{*+} \Sigma^+) = \frac{9}{16} \sigma(\bar{p}p \rightarrow \bar{Y}^{*+} \Sigma^+ + \bar{\Sigma}^+ Y^{*+}) \quad (1.8)$$

$$\sigma(K^-p \rightarrow \varphi B) = \sigma(K^-p \rightarrow \omega B) \quad (1.9)$$

where σ represents either $d\sigma/dt$ for arbitrary s and t , but corrected for $F(t_{\min})$, or the total reaction cross-section for the process in parentheses corrected for $F(t_{\min})$ and phase space and B is any baryon allowed by the selection rule. A number of these relations have been presented in a slightly less general form elsewhere ⁶⁾. If we confine our attention to collinear processes, we obtain in addition the relations

$$\bar{\sigma}(K^-p \rightarrow \varphi \Lambda) = \bar{\sigma}(\pi^-p \rightarrow K^{*0} \Lambda) = 12 \bar{\sigma}(\pi^+p \rightarrow K^{*+} \Sigma^+) \quad (1.10)$$

$$\bar{\sigma}(pp \rightarrow \Delta^0 \Delta^{++}) = \frac{24}{25} \bar{\sigma}(pp \rightarrow \pi \Delta^{++}) = \frac{8}{3} \bar{\sigma}(K^+p \rightarrow K^{*0} \Delta^{++}) \quad (1.11)$$

$$\bar{\sigma}(\bar{p}p \rightarrow \bar{\Delta}^{++} \Delta^{++}) = \frac{24}{25} \bar{\sigma}(\bar{p}p \rightarrow \pi \bar{\Delta}^0) \quad (1.12)$$

$$\bar{\sigma}(\bar{p}p \rightarrow \bar{Y}^{*0} Y^{*0}) = \frac{4}{27} \bar{\sigma}(\bar{p}p \rightarrow \bar{Y}^{*0} \Lambda + \bar{\Lambda} Y^{*0}) = \frac{4}{9} \bar{\sigma}(\bar{p}p \rightarrow \bar{Y}^{*0} \Sigma^0 + \Sigma^0 Y^{*0}) \quad (1.13)$$

where $\bar{\sigma}$ represents $(d\sigma/dt) \big|_{\Theta=0}$

Turning to density matrices ²⁾ and confining our attention to collinear processes ($\theta = 0$), we find

$$D_{\Delta}(pp \rightarrow \Delta\Delta) = D_{\Delta}(pp \rightarrow N\Delta) = D_{\Delta}(Kp \rightarrow K^*\Delta) \quad (1.14)$$

$$D_{K^*}(\pi p \rightarrow K^*\Sigma) = D_{K^*}(\pi p \rightarrow K^*\Lambda) = D_{\varphi}(Kp \rightarrow \varphi\Lambda) \quad (1.15)$$

$$g_{11}(Kp \rightarrow K^*\Delta) = \frac{4}{3} g_{\frac{3}{2}\frac{3}{2}}(Kp \rightarrow K^*\Delta) \quad (1.16)$$

$$D(K^-p \rightarrow \varrho B) = D(K^-p \rightarrow \omega B) \quad (1.17)$$

$$\begin{aligned} D_{Y^*, \bar{Y}^*}(\bar{p}p \rightarrow \bar{Y}^* Y^*) &= D_{Y^*}(\bar{p}p \rightarrow \bar{\Lambda} Y^*) = D_{Y^*}(\bar{p}p \rightarrow \bar{\Sigma} Y^*) = \\ &= D_{\bar{Y}^*}(\bar{p}p \rightarrow \bar{Y}^* \Lambda) \end{aligned} \quad (1.18)$$

$$D_{\Delta}(\bar{p}p \rightarrow \bar{\Delta}\Delta) = D_{\bar{\Delta}}(\bar{p}p \rightarrow \bar{\Delta}\Delta) = D_{\bar{\Delta}}(\bar{p}p \rightarrow \bar{\Delta}N) = D_{\bar{\Delta}}(\bar{K}p \rightarrow K^*\bar{\Delta}) \quad (1.19)$$

$$g_{\frac{3}{2}\frac{3}{2}}(\bar{p}p \rightarrow \bar{Y}^* Y^*) = \frac{2}{3} g_{11}(\pi p \rightarrow K^*\Sigma) \quad (1.20)$$

where $D_C(A+B \rightarrow C+D)$ represents any density matrix element for the particle C in the reactions in parentheses, and N represents any nucleon. In all of the above we have used the fact that double charge exchange is forbidden in the quark model to make our results independent of the particular charge configuration of the reaction ¹⁾⁻³⁾. Finally, if we look at the Pomeranchuk limit ²⁾, we find

$$D_{\Delta}(pp \rightarrow \Delta\Delta) = D_{\Delta}(\bar{p}p \rightarrow \bar{\Delta}\Delta) \quad (1.21)$$

2. DISCUSSION

There are several difficulties which are encountered if one tries to make a comparison of the cross-section relations given above with experiment. First, there is a scarcity of data on the reactions in question, and the data which exist are generally at energies below 4 GeV/c, which is not really in the energy range where the model is supposed to be valid (although, for lack of data at higher energies, it is often applied in this energy range). More important, however, is the fact that we would not really know how to make a comparison even if data were available throughout the entire energy range.

To see this, we must recall that each cross-section in Part 1 actually represents the sum of squares of quark amplitudes. Each quark amplitude is evaluated at a particular s_q and t_q , where s_q and t_q refer to the centre of mass energy and momentum transfer in the quark-quark system. Our data, on the other hand, involve energies and momentum transfers in the particle-particle system. To give meaning to our results, we must therefore make some statement about the relation between s_q and s , and t_q and t . There are a variety of prescriptions which have been suggested for doing this ^{5),6)}, and so long as one looks only at elastic scattering, where cross-sections are roughly constant, the differences between them are not terribly important. In inelastic events, however, where the cross-sections may fall off like s^{-2} , Ref. ⁷⁾, small changes in the relation of s to s_q can have catastrophic effects on the cross-section. Thus the cross-section relations turn out to be extremely sensitive to the details of the dynamical nature of the quarks, and are not unambiguous tests of the quark model.

One systematic discrepancy between theory and experiment can be noticed, however. In those quark model predictions which relate a meson-baryon process to a baryon-antibaryon process, it is found experimentally that the cross-sections for the latter processes are always too small when compared to the former. Some relations for which data exist are given in Table I; these relations are derived from Eqs. (1.2), (1.4) and (1.7) by neglecting processes which are experimentally known to have very small cross-sections. This discrepancy can be understood as follows:

It has been pointed out that annihilation channels may play a large role in reactions initiated by a $P\bar{P}$ system ⁵⁾. In fact, if we write the amplitude of Eq. (1.1) as

$$A_h = A_h^q (1 - \alpha) \quad (2.1)$$

where A_h^q is the amplitude given by the quark additivity model, and α takes account of annihilation, and then assume that

$$\alpha = i\beta, \quad \beta \text{ real} \quad (2.2)$$

we find that β takes on the values given in Table I. The assumption of Eq. (2.2) allows a simple separation of peripheral and annihilation contributions to the cross-section, and assures that removing the annihilation contributions from the experimental cross-sections will increase the value of the cross-section which is to be inserted into the equations of Part 1.

Table I illustrates two points: first, that the dynamical assumption mentioned above plays a very important role in these particular reactions, and second, that annihilation contributions at low energies are comparable to or greater than the contributions from the quark additivity model.

When we turn to the density matrix predictions, however, the situation is much better. In the first place, density matrix elements do not vary greatly with energy, so the sensitivity to the relation between s_q and s is much less than was the case with cross-sections. In addition, there are relations [(1.16) for example], where this consideration does not even enter, since they involve relations between density matrix elements for a single process. Finally, annihilation is a factor, only in Eqs. (1.18), (1.19) and (1.20).

The basic advantages of the density matrices from the point of view of the quark model consist of the fact that they involve ratios of amplitudes. Thus, any effect which is independent of helicity, will cancel exactly in expressions for density matrix elements. Thus, whenever an effect occurs which can influence a given amplitude, but which may reasonably be assumed to be spin independent, it can simply be ignored in calculating density matrices in the quark model. Multiple scattering is an example of such an effect. If we could argue that annihilation was the same type of effect, the Eqs. (1.18)-(1.20) would still be valid results of the quark model, even though the relations involving cross-sections would not be. Let us examine the reasons why it seems reasonable to assume that the factor $(1-\alpha)$ in Eq. (2.1) is helicity independent.

On grounds of simplicity, such an assumption seems to be in order. In addition, there are two pieces of experimental evidence which support it, both involving the reaction

$$\bar{p} p \rightarrow \bar{\Delta}^{++} \Delta^{++} \quad (2.3)$$

In Ref. ³⁾, the joint decay correlations for this reaction were calculated in the quark model, ignoring possible effects of annihilation. As we have pointed out, this is equivalent to assuming that annihilation is independent of spin. Good agreement between the model and experiment was found in this case.

We can go further than this and note that, if we make the assumption of Ref. ²⁾ that quark spin flip amplitudes are small, compared to non-spin flip, we can get numerical predictions for reaction (2.3), and these are compared to experiment in Table II. The agreement is reasonably good.

Thus we see that in, at least one case, the assumption that annihilation does not affect the density matrix predictions of the quark model seems to be borne out. Data on the other reactions in Eq. (1.18) will soon be available ¹⁰⁾, so that further verification will be possible.

In keeping with the purposes set out in the Introduction, however, it should be noted that the $p\bar{p}$ initiated reactions do not give us clean tests of the quark model because even for the density matrix relations, an additional assumption is necessary, and no matter how reasonable the assumption may be, one can never be sure that a failure of Eq. (1.18) would signal a failure of the quark model itself.

We conclude, then, that the best tests of the quark model lie in density matrix relations for reactions in which no annihilation channels are present, such as Eqs. (1.14)-(1.17), and the relations given in Refs. ^{2),3)}. If these relations turn out to be incorrect, it would seem that the quark model in its present form would be in deep difficulty. Conversely, if they are correct, it would lend a great deal of support to the idea of the composite structure of elementary particles.

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TABLE I

Relation	l.h.s. at 3 GeV/c (μb)	r.h.s. at 3 GeV/c (μb)	β_1^2	β_2^2
$\sigma(\pi^+p \rightarrow K^+\Sigma^+) = (\bar{p}p \rightarrow \bar{\Sigma}^+\Sigma^+)$	100 ± 10	40 ± 10	1.5	4.5
$\sigma(\pi^-p \rightarrow K^0\Lambda) = 2/3 \sigma(\bar{p}p \rightarrow \bar{\Lambda}\Lambda)$	230^{+100}_{-20}	$2/3(120 \pm 20)$	0.5	3.5
$\sigma(\pi^-p \rightarrow K^+\Sigma^+) < 4/3 \sigma(\bar{p}p \rightarrow \bar{\Lambda}\Sigma^0)$	100 ± 10	$4/3(45 \pm 5)$	0.7	6.5

Values of β [Eq. (2.2)]. β_1 represents the value of β if the amplitudes are compared at the same Q value ⁶⁾, while for β_2 they are compared at the same quark centre of mass energy, Ref. ⁵⁾. Energy extrapolations are made according to the $p_{\perp}^{-2}$ prescription of Ref. ⁷⁾, while data is taken from Ref. ⁷⁾, and Ref. ⁸⁾.

TABLE II

Matrix element	Theory	Experiment at 5.7 GeV/c
$\rho_{\frac{1}{2} \frac{1}{2}}$	0.5	0.41 ± 0.002
$\rho_{\frac{3}{2} - \frac{1}{2}}$	0	0.0 ± 0.01
$\rho_{\frac{3}{2} \frac{1}{2}}$	0	-0.05 ± 0.02

Density matrix elements for reaction $\bar{p}p \rightarrow \bar{\Delta}\Delta$
 with the experimental results of Ref. 9).

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