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SIXTH ORDER RADIATIVE CORRECTIONS TOTHE MUON MAGNETIC MOMENTToichiro Kinoshita^{*)}

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A B S T R A C T

It is shown that the α^3 corrections to $\frac{1}{2}(g_\mu - g_e)$ can be expressed in the form

$$(\alpha/\pi)^3 \left[\frac{2}{9} (\ln(m_\mu/m_e))^2 - 1.114 \ln(m_\mu/m_e) + 1.344 + C \right]$$

where g_μ and g_e are the g factors of muon and electron, respectively, and C is a constant independent of the electron mass m_e yet to be calculated. This formula is derived by simple algebraic manipulations from the second and fourth order results of g_e and Z_3 (charge renormalization constant). Our result shows that the α^3 corrections to $\frac{1}{2}(g_\mu - g_e)$ will not be much larger than 2×10^{-8} , and thus will not seriously affect the experimental determination of the α^2 corrections. Furthermore it will provide a consistency check for any future calculation of the complete α^3 corrections to the muon magnetic moment.

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A preliminary measurement of the muon g factor g_μ recently carried out at the CERN muon storage ring gives the value ¹⁾

$$(1165 \pm 3) \times 10^{-6}$$

for $\frac{1}{2}(g_\mu - 2)$. This agrees with the theoretical value ²⁾

$$\frac{\alpha}{2\pi} + 0.766 \left(\frac{\alpha}{\pi} \right)^2 = 11655 \times 10^{-7}$$

within $\pm 0.6(\alpha/\pi)^2$. When the experimental accuracy is improved by an order of magnitude, it will thus be possible to put the α^2 corrections of the muon magnetic moment to an accurate experimental test. In order to deduce from such an experiment precise and useful information about the validity of quantum electrodynamics or the effect of strong interactions, however, it is necessary to know that higher order radiative corrections are not inordinately large and do not seriously disturb the theoretical result of order α^2 . As was emphasized by Feynman at the 12th Solvay Congress ³⁾, it is therefore desirable to calculate the radiative corrections to order α^3 , or at least to obtain a reliable estimate of their order of magnitude.

Recently, Drell and Pagels ⁴⁾ have estimated the magnitude of the α^3 corrections to the electron magnetic moment by a dispersion theoretical technique. The purpose of this paper is to point out that the leading $(\ln(m_\mu/m_e))^2$ and $\ln(m_\mu/m_e)$ terms of $g_\mu - g_e$, where g_e is the electron g factor, can be calculated exactly by simple algebraic manipulation (no integration is necessary) from the known results of the second and fourth order electron magnetic moment ⁵⁾ and the second and fourth order charge renormalization constant Z_3 ⁶⁾. Of course, the radiative corrections we have calculated are only a part of α^3 corrections to the muon magnetic moment. From the experimental point of view, however, this is probably the most interesting part because

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the rest is in a sense already known with a very high precision from the measurement of the electron magnetic moment ⁷⁾.

Unfortunately the ℓn and $(\ell n)^2$ terms almost cancel each other for the observed value of m_μ/m_e , and their contribution to $g_\mu - g_e$ is much smaller than the particular constant term which we have calculated by direct integration. Since we have left most of the constant terms uncalculated, we do not know whether their contribution is small or not. However, it is very unlikely that these terms give much larger contributions than what we have calculated. Hence, our result, if multiplied by a factor 2 or 3, may be safely regarded as an upper bound for the α^3 correction to $\frac{1}{2}(g_\mu - g_e)$. Of course nobody will be surprised if the total α^3 correction turns out to be much smaller than our result because of cancellation.

After this work was completed, I have learned that the α^3 correction to $\frac{1}{2}(g_\mu - g_e)$ has also been calculated independently by Drell and Trefil ⁸⁾ and by Peterman ⁹⁾. Although Drell and Trefil have not yet finished calculation of some diagrams (Fig. 5), the part that they have calculated agrees with our result. This provides us with an additional evidence for the validity of our rather unconventional approach which utilizes heavily some general properties of unrenormalized Feynman amplitudes regarded as functions of various masses involved ¹⁰⁾. In fact our method will enable us to obtain the logarithmic terms of $g_\mu - g_e$ in any order n once g_e and Z_3 are known up to order $n - 2$.

To illustrate how our method works, let us first show that the α^2 correction to $g_\mu - g_e$ can be calculated from g_e and Z_3 of the second order. In this order the only contribution to $g_\mu - g_e$ arises from the diagram of Fig. 1. If we separate out the divergent part of the closed electron loop in the conventional manner, this

diagram gives *)

$$Z_3^{(2)}(m_e, \Lambda) M_r^{(2)}(m_\mu) + \Delta M_r^{(4)}(m_e, m_\mu), \quad (1)$$

where $Z_3^{(2)}$ is the charge renormalization constant in the second order

$$Z_3^{(2)}(m_e, \Lambda) = -\frac{2\alpha}{3\pi} \left(\ln \frac{\Lambda}{m_e} - \frac{1}{12} \right), \quad (2)$$

Λ is the high energy cut-off, $M_r^{(2)}$ is the renormalized magnetic moment of muon in the second order ($=\frac{\alpha}{2\pi}$), and $\Delta M_r^{(4)}$ is the finite part of Fig. 1. On the other hand, we may also calculate the same matrix element without separating the Λ dependent terms in so far as Λ is finite though large. In this case we obtain the unrenormalized amplitude

$$\Delta M_{ur}^{(4)}(m_e, m_\mu, \Lambda), \quad (3)$$

which is a function of m_e , m_μ and Λ . Since (1) and (3) are the same quantity calculated differently, we must have

$$\Delta M_{ur}^{(4)}(m_e, m_\mu, \Lambda) = Z_3^{(2)}(m_e, \Lambda) M_r^{(2)}(m_\mu) + \Delta M_r^{(4)}(m_e, m_\mu). \quad (4)$$

for very large Λ . Now, according to a theorem on the mass dependence of unrenormalized Feynman amplitudes ¹⁰⁾, $\Delta M_{ur}^{(4)}$ must tend to a finite limit as $m_e \rightarrow 0$. (Roughly speaking, this occurs when m_e appears, as in Fig. 1, in a purely internal line which does not touch any external line directly.) Thus the $\ln m_e$ terms on the right-hand side of (4)

*) In general we shall need two cut-off parameters in (2) and (3) because they contain integrations over two internal momenta. However, in so far as the part of the vertex diagram relevant to the magnetic moment is concerned, the second integration is convergent and the second cut-off parameter can be put equal to infinity. A similar remark applies to all sixth order diagrams considered in this paper.

must cancel each other. This determines the $\ln m_e$ term of $\Delta M_r^{(4)}$ completely. From a dimensional consideration, we can therefore write $\Delta M_r^{(4)}$ as

$$\Delta M_r^{(4)}(m_e, m_\mu) = \frac{2\alpha}{3\pi} \left(\ln \frac{m_\mu}{m_e} \right) M_r^{(2)}(m_\mu) + \dots \quad (5)$$

This is in agreement with the leading term of the direct perturbation result ²⁾

$$\Delta M_r^{(4)}(m_e, m_\mu) = \frac{2\alpha}{3\pi} \left\{ \ln \frac{m_\mu}{m_e} - \frac{25}{12} + \frac{3\pi^2}{4} \frac{m_e}{m_\mu} + 3 \left(3 - 4 \ln \frac{m_\mu}{m_e} \right) \left(\frac{m_e}{m_\mu} \right)^2 + O \left(\left(\frac{m_e}{m_\mu} \right)^3 \right) \right\} M_r^{(2)}(m_\mu). \quad (6)$$

It also serves as a consistency check of our method.

We shall now consider the α^3 correction to $g_\mu - g_e$. All diagrams contributing to this correction contain at least one closed electron loop and may be classified into four groups (Figs. 2, 3, 4, 5).

The graphs of Fig. 2 are obtained by inserting in all possible ways an electron loop of second order in the internal photon lines of all fourth order muon magnetic moment diagrams except the one shown in Fig. 1. If we separate out the cut-off dependent part of the electron loop as before, we obtain

$$2 Z_3^{(2)}(m_e, \Lambda) M_r^{(4)}(m_\mu) + \Delta_1 M_r^{(6)}(m_e, m_\mu), \quad (7)$$

for the sum of all diagrams of Fig. 2, where $M_r^{(4)}$ is the α^2 correction to the muon magnetic moment obtained by ignoring the contribution of Fig. 1, and $\Delta_1 M_r^{(6)}$ is the corresponding finite part. The factor 2 in front of $Z_3^{(2)}$ arises from the fact that an electron loop can be inserted

in either of the two internal photon lines of the fourth order diagrams. It should also be noted that individual diagrams of Fig. 2 may contribute infra-red divergences to $M_r^{(4)}$ or $\Delta_1 M_r^{(6)}$ as a consequence of separation of cut-off dependent terms from the vertex and self-energy parts. However, they cancel each other in the sum of all diagrams of Fig. 2 because of the Ward's identity. Thus possible dependence of (7) on the photon mass can be ignored.

If we calculate the same set of amplitudes without separating the cut-off dependent terms, then we obtain the unrenormalized amplitude $\Delta_1 M_{ur}^{(6)}(m_e, m_\mu, \Lambda)$ which satisfies the equation

$$\Delta_1 M_{ur}^{(6)}(m_e, m_\mu, \Lambda) = 2 Z_3^{(2)}(m_e, \Lambda) M_r^{(4)}(m_\mu) + \Delta_1 M_r^{(6)}(m_e, m_\mu). \quad (8)$$

By the same argument as above, we therefore obtain

$$\Delta_1 M_r^{(6)}(m_e, m_\mu) = 2 \left(\frac{2\alpha}{3\pi} \right) \left(\ln \frac{m_\mu}{m_e} \right) M_r^{(4)}(m_\mu) + \dots \quad (9)$$

It will by now be obvious that the diagrams of Fig. 3 lead us to the equation

$$\Delta_2 M_{ur}^{(6)}(m_e, m_\mu, \Lambda) = Z_3^{(4)}(m_e, \Lambda) M_r^{(2)}(m_\mu) + \Delta_2 M_r^{(6)}(m_e, m_\mu), \quad (10)$$

where $\Delta_2 M_r^{(6)}(m_e, m_\mu)$ is the finite part of all diagrams of Fig. 3, and $Z_3^{(4)}(m_e, \Lambda)$ is the α^2 term of Z_3 , which is of the form ⁶⁾

$$Z_3^{(4)}(m_e, \Lambda) = - \frac{\alpha^2}{2\pi^2} \left(\ln \frac{\Lambda}{m_e} + \text{const.} \right), \quad (11)$$

We therefore obtain

$$\Delta_2 M_r^{(6)}(m_e, m_\mu) = \frac{\alpha^2}{2\pi^2} \left(\ln \frac{m_\mu}{m_e} \right) M_r^{(2)}(m_\mu) + \dots \quad (12)$$

By a slight extension of the above consideration, it is easy to see that the graph of Fig. 4 leads us to the equation

$$\begin{aligned} \Delta_3 M_{ur}^{(6)}(m_e, m_\mu, \Lambda) \\ = \left[Z_3^{(2)}(m_e, \Lambda) \right]^2 M_r^{(2)}(m_\mu) + 2 Z_3^{(2)}(m_e, \Lambda) \Delta M_r^{(4)}(m_e, m_\mu) \\ + \Delta_3 M_r^{(6)}(m_e, m_\mu), \end{aligned} \quad (13)$$

where $\Delta_3 M_r^{(6)}(m_e, m_\mu)$ is the finite part of Fig. 4, and $\Delta M_r^{(4)}$ is defined in (1). In order to determine the coefficient of the term of $\Delta_3 M_r^{(6)}$ linear in $\ln(m_\mu/m_e)$, it is necessary to use the exact form (2) of $Z_3^{(2)}$ as well as the first two terms of (6) for $\Delta M_r^{(4)}$ instead of the approximate form (5). With this precaution, we find from (13) that

$$\Delta_3 M_r^{(6)}(m_e, m_\mu) = \left(\frac{2\alpha}{3\pi} \right)^2 \left[\left(\ln \frac{m_\mu}{m_e} \right)^2 - \frac{25}{6} \ln \frac{m_\mu}{m_e} \right] M_r^{(2)}(m_\mu) + \dots \quad (14)$$

To show that $\Delta_3 M_{ur}^{(6)}(m_e, m_\mu, \Lambda)$ is in fact well-behaved in the limit $m_e \rightarrow 0$, we shall also give the result of a direct perturbation calculation of $\Delta_3 M_r^{(6)}(m_e, m_\mu)$. This is most easily obtained starting from the formula (53) of Karplus and Kroll¹¹⁾ modified in an appropriate manner. We thus find that $\Delta_3 M_r^{(6)}(m_e, m_\mu)$ is equal to

$$\begin{aligned} \left(\frac{\alpha}{\pi} \right)^3 \int_0^1 u^4 (1-u) du \left[\int_0^1 dv \frac{v^2 (1 - \frac{1}{3} v^2)}{u^2 (1-v^2) + 4(1-u)(m_e/m_\mu)^2} \right]^2 \\ = \frac{1}{9} \left(\frac{\alpha}{\pi} \right)^3 \int_0^1 (1-u) du \left[-\frac{5}{3} - \frac{m_\mu u}{\Delta} \ln \frac{\Delta - m_\mu u}{\Delta + m_\mu u} \right. \\ \left. - \frac{2m_e^2(1-u)}{m_\mu u \Delta} \ln \frac{\Delta - m_\mu u}{\Delta + m_\mu u} + \frac{4m_e^2(1-u)}{m_\mu^2 u^2} \left(1 + \frac{2m_e^2(1-u)}{m_\mu u \Delta} \ln \frac{\Delta - m_\mu u}{\Delta + m_\mu u} \right) \right]^2, \end{aligned} \quad (15)$$

where

$$\Delta = \left(m_\mu^2 u^2 + 4m_e^2 (1-u) \right)^{1/2}. \quad (16)$$

Noting that the last two terms in the square bracket of (15) give contributions of order m_e/m_μ to the integral, we can simplify (15) to

$$\frac{1}{9} \left(\frac{\alpha}{\pi} \right)^3 \int_0^1 (1-u) du \left[-\frac{5}{3} + 2 \ln \frac{m_\mu}{m_e} - \frac{1}{2} (1-u) + 2 \ln u \right]^2 + O\left(\frac{m_e}{m_\mu}\right), \quad (17)$$

Carrying out the integration, we obtain

$$\frac{2}{9} \left(\frac{\alpha}{\pi} \right)^3 \left[\left(\ln \frac{m_\mu}{m_e} \right)^2 - \frac{25}{6} \ln \frac{m_\mu}{m_e} + \frac{317}{72} + \frac{\pi^2}{6} + O\left(\frac{m_e}{m_\mu}\right) \right], \quad (18)$$

which agrees with the result (14).

Substituting (2), (6) and (18) into the right-hand side of (13), we get

$$\Delta_3^{(6)} M_{ur}(m_e, m_\mu, \Lambda) = \frac{2\alpha^3}{9\pi^3} \left[\left(\ln \frac{\Lambda}{m_\mu} \right)^2 + 4 \ln \frac{\Lambda}{m_\mu} + \frac{65}{16} + \frac{\pi^2}{6} + O\left(\frac{m_e}{m_\mu}\right) \right], \quad (19)$$

which confirms our assertion that $\Delta_3^{(6)} M_{ur}$ has in fact no logarithmic dependence on the purely internal mass m_e .

Finally we shall consider the diagrams of Fig. 5. In these diagrams, some electron lines share a vertex with the external photon line. Because of this we cannot immediately rule out the existence of

$\ln m_e$ terms on the basis of general consideration of Ref. ¹⁰⁾. We can nevertheless show that the diagrams of Fig. 5 give no contribution to the $\ln(m_\mu/m_e)$ term. The following is a sketch of our proof: First we note that $\ln m_\mu$ or $\ln m_e$ may arise only from the reduced diagram (in the sense defined in Ref. ¹⁰⁾) shown in Fig. 6, which is obtained by contracting two electron lines and one photon line in any given diagram of Fig. 5 in an appropriate manner. The fact that this reduced diagram consists of three irreducible loops meeting at only one point means that the $\ln m_e$ and $\ln m_\mu$ terms, if they exist at all, must appear as a product. If one notes that diagrams of Fig. 5 have no infra-red divergences, this is possible only if they contain terms of the form

$$c_i \left(\ln \frac{\Lambda}{m_\mu} + a_i \right)^n \left(\ln \frac{\Lambda'}{m_e} + b_i \right), \quad n = 1 \text{ or } 2, \\ i = 1, 2, \dots, 6,$$

where Λ, Λ' are the high-energy cut-offs. However, since the sum of six Feynman diagrams of Fig. 5 is gauge invariant and convergent in the limit $\Lambda, \Lambda' \rightarrow \infty$, these $\Lambda(\Lambda')$ dependent terms must cancel each other when these diagrams are combined. Noting that Λ and Λ' are independent, we obtain the desired result. Details of this proof will be given elsewhere.

Collecting the results (9), (12) and (18), we can express the sixth order radiative corrections to $\frac{1}{2}(g_\mu - g_e)$ as follows:

$$\begin{aligned} \frac{1}{2}(g_\mu - g_e)^{(6)} &= \frac{4}{3} \left(\frac{\alpha}{\pi} \right)^3 \left(\frac{197}{144} + \frac{\pi^2}{12} + \frac{3}{4} \zeta(3) - \frac{1}{2} \pi^2 \ln 2 \right) \ln \frac{m_\mu}{m_e} \\ &\quad + \frac{1}{4} \left(\frac{\alpha}{\pi} \right)^3 \ln \frac{m_\mu}{m_e} \\ &\quad + \frac{2}{9} \left(\frac{\alpha}{\pi} \right)^3 \left(\left(\ln \frac{m_\mu}{m_e} \right)^2 - \frac{25}{6} \ln \frac{m_\mu}{m_e} + \frac{317}{72} + \frac{\pi^2}{6} \right) \\ &\quad + \text{terms independent of } m_e. \end{aligned} \tag{20}$$

The first line uses the result of Ref. ⁵⁾. We have included in (20) the constant terms of (18) since they are explicitly known. Numerically we have

$$\begin{aligned} \frac{1}{2}(g_\mu - g_e)^{(6)} &= \left(\frac{\alpha}{\pi}\right)^3 \left(\frac{2}{9} \left(\ln \frac{m_\mu}{m_e} \right)^2 - 1.114 \ln \frac{m_\mu}{m_e} + 1.344 \right) + \dots \\ &= \left(\frac{\alpha}{\pi}\right)^3 (1.721) + \dots \\ &= 2.16 \times 10^{-8} + \dots \end{aligned} \quad (21)$$

As was mentioned already, the logarithmic terms almost cancel each other, and the major contribution arises from the constant term of (18).

Although we do not know the magnitude of uncalculated terms in (21), it is likely to be not larger than the calculated part of (21) if the result of the fourth order calculation is any indication. This feeling is also reinforced by the smallness of the value $+0.15(\alpha/\pi)^3$ obtained by Drell and Pagels ⁴⁾ for the α^3 corrections to the magnetic moment of an electron.

Thus we may conclude with reasonable confidence that the α^3 corrections to the muon magnetic moment are quite small and will not affect the comparison of experiment and theory in order α^2 . In fact it is so small that, if we want to measure it someday, we have to resolve several problems beforehand, such as the effect of strong interactions, weak interactions and the experimental uncertainty in the value of α . The strong interaction effect has been estimated by Durand and by Bouchiat and Michel ¹²⁾. According to Durand, the ρ meson contribution to $\frac{1}{2}(g_\mu - 2)$ can be expressed approximately as

$$\frac{\alpha^2}{36\pi} \left(m_\rho^2 - 4m_\pi^2 \right)^{3/2} m_\mu^2 m_\rho^{-4} \Gamma_\rho^{-1}, \quad (22)$$

where m_ρ is the ρ meson mass and Γ_ρ is the width of the resonance in pion-pion scattering. Together with the ω meson contribution, this gives a correction of order 1.1×10^{-7} which is about five times larger than (21). There is a clear need for a better estimate of strong interaction corrections. If the W boson actually exists in weak interactions, it will also contribute to $\frac{1}{2}(g_\mu - 2)$ a correction of the same order of magnitude as (21) ($\sim -1 \times 10^{-8}$)¹³⁾. This correction depends strongly on the assumption of the magnitude of the magnetic moment of the W boson.

Until recently the accepted value of α has been¹⁴⁾

$$\alpha^{-1} = 137.0388 \pm 0.0006. \quad (23)$$

This uncertainty in α produces an uncertainty of order $\pm 0.4(\alpha/\pi)^3$ in the magnitude of $\frac{1}{2}(g_\mu - 2)$. However, the recent measurements of the hyperfine splitting in muonium by the Hughes group¹⁵⁾, after correction by Ruderman¹⁶⁾, as well as the new measurement of the Lamb shift in hydrogen¹⁷⁾, indicate that (23) is in error. The most recent result obtained using the ac Josephson effect gives the value¹⁸⁾

$$\alpha^{-1} = 137.0359 \pm 0.0004. \quad (24)$$

If one accepts this value of α , the theoretical prediction 11655×10^{-7} (or, more precisely, 1165518×10^{-9}) for $\frac{1}{2}(g_\mu - 2)$ must be increased by $\sim 2.0(\alpha/\pi)^3$ ($\sim 25 \times 10^{-9}$) with an uncertainty of order $\pm 0.3(\alpha/\pi)^3$.

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FIGURE CAPTIONS

- Figure 1 The fourth order diagram that contributes to $g_\mu - g_e$. In this and the following figures, heavy, thin and dotted lines represent muon, electron and photon, respectively.
- Figure 2 Some diagrams obtained by inserting a second order electron loop in all possible ways in the internal photon lines of all fourth order muon magnetic moment diagrams except the one represented by Fig. 1. Several diagrams are not shown explicitly to save space.
- Figure 3 Diagrams containing fourth order vacuum polarization corrections.
- Figure 4 A diagram containing two second order vacuum polarization corrections.
- Figure 5 Diagrams containing subdiagrams of photon-photon scattering type. Each diagram corresponds to two conventional Feynman diagrams in which fermion lines have directions.
- Figure 6 A reduced diagram obtained by contracting two electron lines and one photon line of diagrams of Fig. 5.

FIG. 1

FIG. 2

FIG. 3

FIG. 4

FIG. 5

FIG. 6