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YET ANOTHER PROPOSAL FOR DETERMINING THE
INDUCED PSEUDOSCALAR COUPLING CONSTANT

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A B S T R A C T

It has been known for some time that μ capture from the 0^+ ground state of ^{16}O to the first excited 0^- level of ^{16}N could, in principle, provide an accurate measurement of the induced pseudoscalar coupling constant in weak interactions. However, the calculation of the rate of this reaction has been hampered by the lack of an accurate wave function for the $^{16}\text{N}(0^-)$ state. We propose that this difficulty could be circumvented by a measurement of the rate of the β decay from the $^{16}\text{N}(0^-)$ state back to the $^{16}\text{O}(0^+)$ ground state. The ratio of the μ capture to β decay rates turns out to be almost independent of the details of the nuclear wave function and thus should allow a reasonably accurate determination of the induced pseudoscalar coupling constant to be made.

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In this paper we discuss a proposal for measuring the induced pseudoscalar coupling constant, g_p , in μ capture with a minimum of uncertainty due to the details of the nuclear wave functions of the states involved in the reaction. We consider the reaction

where ${}^{16}\text{O}(0^+)$ is the ground state of ${}^{16}\text{O}$ and ${}^{16}\text{N}(0^-)$ is the first excited state of ${}^{16}\text{N}$ at 120 keV. The suitability of this process for the determination of g_p has been discussed by various authors ¹⁾⁻³⁾; it is due to two reasons. In the first place, in a $0^+ \leftrightarrow 0^-$ transition only the axial and pseudoscalar couplings contribute to the nuclear part of the interaction Hamiltonian ⁴⁾. Secondly, if one takes the state ${}^{16}\text{O}(0^+)$ as the effective vacuum state, it is expected that the shell model will provide a rather accurate and simple description of the state ${}^{16}\text{N}(0^-)$ as resulting from the creation of a hole in the 1p proton shell and a particle in the (2s, 1d) neutron shell. Due to the requirement that the hole and the particle couple to zero total angular momentum, only two such hole-particle configurations can contribute, namely $(1\bar{p}_{\frac{1}{2}}, 2s_{\frac{1}{2}})$ and $(1\bar{p}_{\frac{3}{2}}, 1d_{\frac{3}{2}})$, giving the wave function

$$|{}^{16}\text{N}(0^-)\rangle = X_1 |1\bar{p}_{\frac{1}{2}}, 2s_{\frac{1}{2}}\rangle + X_2 |1\bar{p}_{\frac{3}{2}}, 1d_{\frac{3}{2}}\rangle \quad (2)$$

Since the admixture coefficients X_1 and X_2 satisfy the normalization condition

$$X_1^2 + X_2^2 = 1, \quad (3)$$

the wave function for $^{16}\text{N}(0^-)$ depends on only one unknown parameter, say X_2 . Previous calculations ^{2),3),5)} of the rate of reaction (1) have shown that it is quite sensitive to the values of both g_p and X_2 . Thus, to obtain an accurate measurement of g_p it would be necessary either to determine X_2 from an independent experiment or to find some way of eliminating its influence.

It has been suggested ⁶⁾ that this could be accomplished by measuring the rate of the β decay

Since the nuclear states involved in reaction (4) are the same as in reaction (1), and since the contribution of the pseudoscalar coupling is negligible due to the small value of the electron mass compared to that of the muon, the rate of reaction (4) depends on only one unknown parameter, namely, X_2 . We shall show that the ratio $\mathcal{M}_\mu/\mathcal{M}_\beta$ of the rate of reaction (1) to that of reaction (4) is quite sensitive to g_p and very insensitive to X_2 for any reasonable values of these two parameters. Hence an experimental determination of this ratio should provide a fairly accurate value of g_p reasonably independent of the details of nuclear wave function calculations.

Experimentally the only decay mode of the $^{16}\text{N}(0^-)$ state so far observed has been the radiative decay to the $^{16}\text{N}(2^-)$ ground state with a half-life of 5.7×10^{-6} seconds ⁷⁾. However, there is every reason to suppose that the β decay (4) also occurs, although its detection will be rather difficult. This is due first of all to the low value of the expected branching ratio of the β decay mode relative to the radiative one, which our calculations indicate to be of the order of 10^{-6} . Second, and perhaps more important, is the problem of disentangling the β decay reaction (4) from the β decay $^{16}\text{N}(2^-) \rightarrow ^{16}\text{O}(0^+) + e^- + \bar{\nu}_e$ of the ^{16}N ground state, since both decays have (ft)

values of the same order of magnitude and end-point energies differing by only 120 keV. Nevertheless, in spite of the above complications the measurement of the rate of reaction (4) is probably within the realm of experimental feasibility.

In Section 2 we present our treatment of the μ capture and β decay interactions. Section 3 is devoted to a discussion of the nuclear wave function for the state $^{16}\text{O}(0^-)$. We present our numerical results in Section 4 and our conclusions in Section 5.

2. TRANSITION RATES

Our treatment of nuclear μ capture follows that of Ericson, Sens, and Rood ²⁾. From the interaction Hamiltonian density ⁸⁾

$$\sqrt{2} H = \bar{\Psi}_\nu \gamma_\lambda (1 + \gamma_5) \Psi_\mu \left\{ g_A \bar{\Psi}_n \gamma_\lambda \gamma_5 \Psi_p + \frac{g_p}{m_\mu} \frac{\partial}{\partial x^\lambda} (\bar{\Psi}_n \gamma_5 \Psi_p) \right\} \quad (5)$$

for the elementary μ capture process

$$\mu^- + p \rightarrow n + \nu_\mu \quad (6)$$

we obtain an effective Hamiltonian for nuclear μ capture in the form

$$\begin{aligned} \sqrt{2} H_{\text{eff}} = & (1 - \vec{\sigma}_\ell \cdot \hat{p}_\nu) \sum_{j=1}^A \tau_j^{(-)} \left\{ g_A \vec{\sigma}_\ell \cdot \vec{\sigma}_j \right. \\ & \left. + g_A \vec{\sigma}_j \cdot \vec{p}_j / M + (g_p - g_A) \frac{p_\nu}{2M} \hat{p}_\nu \cdot \vec{\sigma}_j \right\} \end{aligned} \quad (7)$$

In Eqs. (5) and (7) g_A is the axial vector coupling constant for β decay, m_μ is the mass of the muon, M is the mass of the nucleon, p_ν is the neutrino energy, and $\hat{p}_\nu$ is a unit vector in the direction of the neutrino momentum. The operators with subscript ℓ act on the leptons, whereas those with subscript j act on the j^{th} nucleon in the initial nucleus. $\vec{\sigma}_j$ and $\vec{\tau}_j$ are 2x2 Pauli spin operators, $\vec{p}_j$ is the momentum operator and $\tau_j^{(-)}$ the isotopic spin lowering operator for the j^{th} nucleon.

To obtain Eq. (7) the impulse approximation, in which the interaction of the muon with the initial nucleus is viewed as the sum of all possible elementary interactions of type (6) with the constituent nucleons, has been used. Further, the nucleons have been treated non-relativistically by expanding the nucleon part of the Hamiltonian (5) in terms of two-component spinors to first order in the nucleon velocity ($\vec{p}/M$). The muon has also been treated non-relativistically, and its momentum and binding energy in the $1S$ atomic ground state from which the capture takes place have been neglected.

The capture rate is given by

$$\Lambda_\mu(\text{sec}^{-1}) = \frac{1}{\hbar} \frac{p_\nu^2}{2\pi} |\psi_\mu|_{\text{av}}^2 |g_A \langle \vec{\sigma} \cdot \vec{p}/M \rangle + g_A' \langle \tau_z \rangle|^2, \quad (8)$$

where $g_A' = (g_p - g_A)(p_\nu/2M) - g_A$. The quantity $|\psi_\mu|_{\text{av}}^2$ is the square of the muon wave function in the $1S$ state averaged over the nuclear volume; it is given by ⁹⁾

$$|\psi_\mu|_{\text{av}}^2 = \frac{K}{\pi} (m_\mu \alpha Z)^3, \quad (9)$$

where $K = 0.79$ and $Z = 8$ for ^{16}O , and α is the fine structure constant. The nuclear matrix elements are defined by

$$\langle \vec{\sigma} \cdot \vec{p} / M \rangle = \frac{-i}{M} \langle {}^{16}\text{N}(0^-) | \sum_{j=1}^{16} \tau_j^{(-)} e^{-i p_z z_j} \vec{\sigma}_j \cdot \vec{\nabla}_j | {}^{16}\text{O}(0^+) \rangle \quad (10)$$

and

$$\langle \sigma_z \rangle = \langle {}^{16}\text{N}(0^-) | \sum_{j=1}^{16} \tau_j^{(-)} e^{-i p_z z_j} (\sigma_j)_z | {}^{16}\text{O}(0^+) \rangle$$

where the z axis is along the direction of the neutrino momentum.

To obtain the β decay rate we use the results for first-forbidden β decay given by Lee-Whiting¹⁰⁾. However, the approximation used in that paper for the combinations of electron radial wave functions denoted by L_0 , M_0 , and N_0 is not sufficiently accurate for the decay under consideration, and we have used instead the more exact expressions for these quantities given by Greuling¹¹⁾. With Lee-Whiting's notation for the nuclear matrix elements, the decay rate is given by

$$\begin{aligned} \Lambda_\beta (\text{sec}^{-1}) &= \frac{1}{\hbar} \frac{Q^5}{30\pi^3} F_0(Z, E_e = p_e) \frac{g_A^2}{2} \\ &\times |\langle \chi_5; 00 \rangle + \langle \vec{\sigma}; 01 \rangle (\frac{1}{3}Q + \xi)|^2 \end{aligned} \quad (11)$$

In obtaining Eq. (11) we have set the electron mass equal to zero. This is an extremely good approximation in the present case due to the large value, $Q = 10.52$ MeV, of the mass difference between the initial and final nuclear states. In this limit of zero electron mass the Fermi function $F_0(Z, E_e)$ is given by

$$F_0(Z, E_e = p_e) = \frac{(\pi \alpha Z) \exp(\pi \alpha Z)}{\sinh(\pi \alpha Z)}, \quad (12)$$

6.

where $Z = 8$ for ^{16}O . The quantity ξ is defined by

$$\xi = \frac{\alpha Z}{2R} \quad , \quad (13)$$

where R is an appropriate "nuclear radius".

Using the impulse approximation and a non-relativistic treatment for the nucleons, as in the case of μ capture, we can express the nuclear matrix elements in the form

$$\langle \gamma_5; 00 \rangle = \frac{i}{M} \sum_{\lambda} (-1)^{\lambda} \langle ^{16}\text{O}(0^+) | \sum_{j=1}^{16} \tau_j^{(+)} (\sigma_j)_{-\lambda} (\vec{r}_j)_{\lambda} | ^{16}\text{N}(0^-) \rangle \quad (14)$$

and

$$\begin{aligned} \langle \vec{\sigma}; 01 \rangle &= i\sqrt{4\pi} \sum_{\lambda} \langle 1, \lambda; 1, -\lambda | 0, 0 \rangle \\ &\times \langle ^{16}\text{O}(0^+) | \sum_{j=1}^{16} \tau_j^{(+)} (\sigma_j)_{-\lambda} (r_j)_{\lambda} | ^{16}\text{N}(0^-) \rangle . \end{aligned}$$

Here $\tau_j^{(+)}$ is the isotopic spin raising operator and $\vec{r}_j$ the position operator for the j^{th} nucleon. The quantities $\sigma_{-\lambda}$, $\vec{V}_{\lambda}$, and r_{λ} are the spherical components of the vector operators $\vec{\sigma}$, $\vec{V}$, and $\vec{r}$, with $\lambda = 0, \pm 1$. For an arbitrary vector operator $\vec{A}$ the spherical components are defined in terms of the Cartesian components by

$$A_0 = A_z, \quad A_{\pm 1} = \mp \frac{1}{\sqrt{2}} (A_x \pm iA_y).$$

3. NUCLEAR WAVE FUNCTION FOR $^{16}\text{N}(0^-)$

In order to evaluate the nuclear matrix elements (10) and (14) we need an explicit expression for the wave function of the state $^{16}\text{N}(0^-)$. We use a wave function obtained from the shell model, which is expected to provide a good description of this state within the framework of a second-quantized formalism in which the ground state of ^{16}O is taken as the effective vacuum.

Details of calculating shell model wave functions for the low-lying odd parity states of ^{16}N within this scheme have been given by Elliott and Flowers ¹²⁾ and by Gillet and Vinh Mau ¹³⁾. The underlying idea is to assume that each nucleon in a nucleus moves in a static, spherically-symmetric potential due to the other nucleons, supplemented by a strong spin-orbit coupling for each individual nucleon. In this manner one reproduces the correct sequence of experimentally inferred single-particle energy levels together with their observed shell groupings. One then gets, for example, a single-particle energy level spectrum of the type shown in Fig. 1 for 1p and (2s, 1d) shell neutrons ¹³⁾. A popular choice for the central potential, which we also use here, is the one-parameter potential for a three-dimensional, isotropic harmonic oscillator,

$$V(r) = \frac{1}{2} \frac{1}{Mb^4} r^2, \quad (15)$$

where b is known as the oscillator length parameter.

For the 0^+ ground state of ^{16}O the single-particle configuration is $(1s)^2(1p_{\frac{3}{2}})^4(1p_{\frac{1}{2}})^2$ for both protons and neutrons. This ground state is usually chosen as an effective vacuum state, and it is assumed that the low-lying odd parity states of ^{16}N are obtained by creating a hole in the 1p proton shell and a particle in the (2s, 1d) neutron shell. Multiparticle excitations or excitations in other shells are neglected. In order to obtain an 0^- state for the ^{16}N nucleus the

spin of the resulting hole in the $1p$ proton shell must be appropriately coupled to the spin of the neutron in the $(2s, 1d)$ shell. The two possible hole-particle configurations for this purpose are $(1\bar{p}_{\frac{1}{2}}, 2s_{\frac{1}{2}})$ and $(1\bar{p}_{\frac{3}{2}}, 1d_{\frac{3}{2}})$. We see from Fig. 1 that the energy required to excite a particle from the $1p_{\frac{1}{2}}$ level to the $2s_{\frac{1}{2}}$ level is about 10 MeV less than that required to excite one from the $1p_{\frac{3}{2}}$ level to the $1d_{\frac{3}{2}}$ level; therefore we would expect the lowest-lying 0^- state of ^{16}N to correspond to the $(1\bar{p}_{\frac{1}{2}}, 2s_{\frac{1}{2}})$ configuration.

Although the average central potential and the spin-orbit coupling reproduce the correct sequence of single-particle energy levels and the correct shell groupings for these levels, they do not lead to correct energies for the various excited states. In order to obtain these energies one must also take into account the very strong forces between nucleons at small separation. This is done by assuming an effective residual two-body interaction between the nucleons involving several adjustable parameters. The effect of this residual interaction on the 0^- state of ^{16}N is to mix the two configurations given above, so that the actual wave function is given by the linear combination (2). On the basis of the previous argument we would still expect the $(1\bar{p}_{\frac{1}{2}}, 2s_{\frac{1}{2}})$ configuration to be dominant in (2), so that X_1 should be very nearly equal to one and X_2 very small. Energy-level fitting calculations for ^{16}N have been performed ^{12), 13)} for two different forms of the residual interaction and give $|X_2| \approx 0.05$, but with opposite signs for X_2 in the two cases.

As we shall see below, the μ capture rate for the $^{16}\text{O}(0^+) \rightarrow ^{16}\text{N}(0^-)$ transition is quite sensitive to both the magnitude and sign of X_2 . The present experimental rate for this transition ^{14), 15)} leads to the values $7 \lesssim g_p/g_A \lesssim 14$ and $11 \lesssim g_p/g_A \lesssim 17$ for $X_2 = +0.05$ and $X_2 = -0.05$, respectively ¹⁶⁾. Due to the availability of several adjustable parameters in the residual interaction potential, the level-fitting calculations are not very sensitive to the exact form of the wave functions, i.e., to the values of admixture coefficients. On the

other hand, for the calculation of transition rates accurate wave functions are of prime importance. We argue, therefore, that an appropriate value of X_2 for the calculation of the μ capture rate would be one determined from another transition between the same two nuclear states. In principle, the inverse β decay from $^{16}\text{N}(0^-)$ to $^{16}\text{O}(0^+)$ is just such a transition. We shall show, however, that even a very precise measurement of the rate of this transition could determine X_2 only within a relatively broad margin of error, due to the uncertainty in the value of the axial vector coupling constant, g_A . For example, one would not be able to distinguish between the two values $X_2 = \pm 0.05$ cited above. Fortunately, it turns out that the ratio $\mathcal{L}_\mu / \mathcal{L}_\beta$ is very insensitive to X_2 for reasonably small values of this parameter, say, $|X_2| \leq 0.25$; therefore this ratio can be used to determine g_p/g_A without an accurate knowledge of X_2 . Nevertheless, for the purpose of self-consistency one should still make a rough determination of X_2 to ensure that its value lies in the "reasonable" range mentioned above.

4. NUMERICAL RESULTS

Following standard procedures ²⁾ it can be shown that the nuclear matrix elements (10) for μ capture between the ground state of ^{16}O and the hole-particle configuration states $|1\bar{p}_{\frac{1}{2}}, 2s_{\frac{1}{2}}\rangle$ and $|\bar{p}_{\frac{3}{2}}, 1d_{\frac{3}{2}}\rangle$ for the state $^{16}\text{N}(0^-)$ reduce to the one-body matrix elements

$$\langle A \rangle = \delta_{j_p, j_n} \frac{1}{\sqrt{2j_p+1}} \sum_m \langle j_n, m; n_n, l_n | e^{-ip_n \cdot \tau} A | j_p, m; n_p, l_p \rangle, \quad (16)$$

where j_n, n_n, l_n are the quantum numbers of the excited neutron, j_p, n_p, l_p those of the proton hole, and the operator A is either $\vec{\sigma}_z$ or $\vec{\sigma} \cdot \vec{p}/M$. The matrix elements (14) for β decay are given in analogous fashion by

$$\langle \gamma_5; 00 \rangle = \delta_{j_p, j_n} \frac{1}{\sqrt{2j_p+1}} \frac{i}{M} \sum_{\lambda} (-1)^{\lambda} \sum_m \langle j_p, m; n_p, l_p | \nabla_{-\lambda} \nabla_{\lambda} | j_n, m; n_n, l_n \rangle \quad (17)$$

and

$$\begin{aligned} \langle \vec{\sigma}; 01 \rangle &= \delta_{j_p, j_n} \frac{1}{\sqrt{2j_p+1}} i\sqrt{4\pi} \sum_{\lambda} \langle 1, \lambda; 1, -\lambda | 0, 0 \rangle \\ &\times \sum_m \langle j_p, m; n_p, l_p | \nabla_{-\lambda} r_{\lambda} | j_n, m; n_n, l_n \rangle. \end{aligned}$$

For the single-particle wave functions we have

$$|j, m; n, l\rangle = R_{nl}(r) \sum_{\mu} \langle l, \mu; \frac{1}{2}, m-\mu | j, m \rangle Y_l^{\mu}(\Omega) X_{\frac{1}{2}}^{m-\mu} \quad (18)$$

where $Y_l^{\mu}(\Omega)$ is a spherical harmonic, $R_{nl}(r)$ is a radial wave function for the harmonic oscillator potential (15), and $X_{\frac{1}{2}}^{m-\mu}$ is a two-component spinor. The explicit values of the matrix elements (16) and (17) are given in the Table for the two hole-particle configurations of interest. The functions I_1, I_2, J_1 , and J_2 which appear in the Table are defined by

$$I_1(x) = -\frac{1}{3}x\left(1 - \frac{x^2}{4}\right)e^{-x^2/4}$$

$$I_2(x) = \frac{1}{3}\left(\frac{1}{Mb}\right)\left(1 - \frac{x^2}{12} + \frac{x^4}{24}\right)e^{-x^2/4}$$

$$J_1(x) = \frac{1}{3}\sqrt{\frac{5}{2}}x\left(1 - \frac{x^2}{10}\right)e^{-x^2/4}$$

$$J_2(x) = -\sqrt{\frac{5}{2}}\left(\frac{1}{Mb}\right)\left(1 - \frac{x^2}{3} + \frac{x^4}{60}\right)e^{-x^2/4}$$

(19)

T A B L E

MATRIX ELEMENT	$(1\bar{p}_{\frac{1}{2}}, 2s_{\frac{1}{2}})$	$(1\bar{p}_{\frac{3}{2}}, 1d_{\frac{3}{2}})$
$\langle \vec{\sigma} \cdot \vec{p}/M \rangle$	$i3\sqrt{2} I_2(p_\nu b)$	$i2 J_2(p_\nu b)$
$\langle \sigma_z \rangle$	$i\sqrt{2} I_1(p_\nu b)$	$i2 J_1(p_\nu b)$
$\langle \gamma_5; 00 \rangle$	$i\sqrt{2}\left(\frac{1}{Mb}\right)$	$-i\sqrt{10}\left(\frac{1}{Mb}\right)$
$\langle \vec{\sigma}; 01 \rangle$	$-i\sqrt{2}b$	$i\sqrt{10}b$

Nuclear matrix elements for μ capture and β decay for the hole-particle configurations $(1\bar{p}_{\frac{1}{2}}, 2s_{\frac{1}{2}})$ and $(1\bar{p}_{\frac{3}{2}}, 1d_{\frac{3}{2}})$.

The matrix elements in the Table all depend on the oscillator length parameter b . This is usually evaluated from a shell-model analysis of nuclear electron scattering experiments. Such an analysis has been made for electron scattering off ^{16}O , yielding a value $b = (1.76 \pm 0.02) \times 10^{-13} \text{ cm}$ (17). Since ^{16}N has a somewhat different charge distribution than ^{16}O , the value of b for ^{16}N should be slightly different from that for ^{16}O . However, since it turns out that the nuclear matrix elements listed in the Table are quite insensitive to a small difference in the value of b for the two nuclei, we have used the value given above for both nuclei. In any case, the actual value of b for ^{16}N is not known.

Using the Table and Eq. (8) we get the following expression for the μ capture rate :

$$\begin{aligned} \Lambda_{\mu} (\text{sec}^{-1}) = & \frac{1}{\hbar} \frac{P_{\nu}^2}{2\pi} |\varphi_{\mu}|_{\text{av}}^2 \left\{ g_A' \{ X_1 \sqrt{2} I_1(p_{\nu}b) + X_2 2 J_1(p_{\nu}b) \} \right. \\ & \left. + g_A \{ X_1 3\sqrt{2} I_2(p_{\nu}b) + X_2 2 J_2(p_{\nu}b) \} \right\}^2 \end{aligned} \quad (20)$$

where $|\varphi_{\mu}|_{\text{av}}^2$ is given by Eq. (9) and $p_{\nu} = 95 \text{ MeV}$.

The β decay rate is given by

$$\begin{aligned} \Lambda_{\beta} (\text{sec}^{-1}) = & (X_1 - \sqrt{5} X_2)^2 \frac{1}{\hbar} \frac{Q^5}{30\pi^3} F_0(Z, E_e = p_e) g_A^2 \\ & \times \left(\frac{1}{Mb} \right)^2 \left[1 - \frac{1}{3} \frac{Q}{M} (Mb)^2 - (Mb)(\xi b) \right]^2 \end{aligned} \quad (21)$$

where $Q = 10.52 \pm 0.05 \text{ MeV}$ (18), (19), $F_0(Z, E_e = p_e)$ is given by Eq. (12) and ξ is defined by Eq. (13). For the "nuclear radius" R which occurs in the expression for ξ we take the value $(2.65 \pm 0.04) \times 10^{-13} \text{ cm}$ of the rms radius of the charge distribution for ^{16}O obtained from electron scattering experiments (20).

We have plotted the μ capture rate as a function of $y_p = g_p/g_A$ for various values of X_2 in Fig. 2. Each curve is labelled by the value of X_2 to which it corresponds. The two shaded bands correspond to the experimental values of the rate obtained at Columbia ¹⁴⁾, $\Lambda_\mu = (1.1 \pm 0.2) \times 10^3 \text{ sec}^{-1}$, and at Berkeley ¹⁵⁾, $\Lambda_\mu = (1.6 \pm 0.2) \times 10^3 \text{ sec}^{-1}$. Since these two values fall outside each other's error limits, a more accurate measurement of this rate would be highly desirable for the determination of g_p . In Fig. 3 we have plotted the β decay rate as a function of X_2 and have indicated an uncertainty in the rate of $\pm 15\%$ due to the uncertainties in the measured values of g_A , Q , and b . [We have used the value $g_A = -(1.18 \pm 0.04)g_V$ ²¹⁾, with $g_V = (1.01 \pm 0.01) \times 10^{-5}/M_p^2$, where M_p is the mass of the proton.] Due to this uncertainty, it will be possible to obtain only a rather rough determination of X_2 even if the experimental statistics in the measurement of the β decay rate are very good.

Figure 4 gives the ratio $\Lambda_\mu/\Lambda_\beta$ as a function of X_2 for various values of y_p . It can be seen from this figure that $\Lambda_\mu/\Lambda_\beta$ is almost independent of X_2 for reasonable values of y_p and X_2 ; hence this ratio should allow one to establish reasonable limits on the value of y_p . We have also indicated in Fig. 4 by means of the shaded bands the range of y_p and X_2 values allowed by the present experimental limits on the μ capture rate. In this connection it should be pointed out that in using Fig. 4 to set limits on the value of y_p one must take into account not only the experimental error in $\Lambda_\mu/\Lambda_\beta$ but also a theoretical error due to the uncertainties in the quantities Q and b . This uncertainty depends on the value of y_p itself and ranges from about 7% for $y_p = 0$ down to about 4% for $y_p = 20$. If one also ascribes an additional uncertainty of a few percent to the square of the average muon wave function, $|\psi_\mu|_{av}^2$, and to the difference between the actual values of b for ^{16}N and ^{16}O , one gets an over-all theoretical error of about 10%.

We would also like to point out that Eq. (20) actually gives two values of g_p for each value of the capture rate $\mathcal{A}_\mu$ due to the quadratic dependence of $\mathcal{A}_\mu$ on g_p . One of these values can always be excluded, however, by requiring that y_p lie within the present experimental range of about $0 < y_p < 20$ (²²). Similarly, Eq. (21) gives two values of X_2 for each value of the decay rate $\mathcal{A}_\beta$. Here again we can exclude one of these values by requiring X_2 to be reasonably small, say in the range $|X_2| < 0.25$.

5. CONCLUSIONS

We have shown that accurate measurements of the rates of the μ capture reaction $\mu^- + {}^{16}\text{O}(0^+) \rightarrow {}^{16}\text{N}(0^-) + \nu_\mu$ and of the inverse β decay ${}^{16}\text{N}(0^-) \rightarrow {}^{16}\text{O}(0^+) + e^- + \bar{\nu}_e$ could lead to a good determination of the induced pseudoscalar coupling constant g_p for weak interactions. In general the determination of this constant from reactions involving nuclei has been hampered by the lack of accurate nuclear wave functions, chiefly as a result of configuration mixing. In the present instance the ratio $\mathcal{A}_\mu/\mathcal{A}_\beta$, which is almost independent of the configuration mixing in the wave function for the state ${}^{16}\text{N}(0^-)$, can be used by itself to set reasonable limits on the value of g_p .

It would be unfair to conclude this paper without mentioning the rather large discrepancies between experiment and the existing calculations (²), (³) of the rates for μ capture in ${}^{16}\text{O}$. These discrepancies occur not only for capture to the 0^- state of ${}^{16}\text{N}$, but also for capture to the 2^- and 1^- states. In particular, a most puzzling problem is the factor of two difference between the experimental and theoretical results for the ratio of the rate of μ capture to the 2^- state to that of the inverse β decay to the ${}^{16}\text{O}$

ground state ²). Since this particular ratio is almost independent of the details of the nuclear wave functions, one is tempted to ascribe this discrepancy to an incorrect measurement of the experimental rate of the μ capture reaction. So far only a single measurement exists for the rate of μ capture from the ground state of $^{16}_0\text{O}$ to the 2^- ground state of $^{16}_{14}\text{N}$ ¹⁴⁾; another measurement would be highly desirable for resolving the above discrepancy, since it has a direct bearing on the general validity of the type of calculations outlined in the present paper. In this connection it should be mentioned that an experimental arrangement of sufficient accuracy to measure the rate of reaction (4) would probably also be able to provide very accurate measurements of the rate of μ capture from $^{16}_0\text{O}$ to the various low-lying states of $^{16}_{14}\text{N}$.

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FOOTNOTES AND REFERENCES

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FIGURE CAPTIONS ---

- Figure 1 Single-particle neutron energy levels for the 1p and (2s, 1d) shells. The energy separations for proton levels are slightly different. The numerical values for the level separations were obtained from Ref. 13).
- Figure 2 The μ capture rate $\mathcal{L}_\mu$ as a function of $y_p = g_p/g_A$. Each curve is labelled by the value of X_2 to which it corresponds. The two shaded areas indicate the experimental results of Refs. 14) and 15).
- Figure 3 The β decay rate $\mathcal{L}_\beta$ as a function of X_2 . The shaded band indicates the uncertainty in $\mathcal{L}_\beta$ due to the uncertainties in the measured values of g_A , Q , and b .
- Figure 4 The ratio of the μ capture to β decay rates, $\mathcal{L}_\mu/\mathcal{L}_\beta$, as a function of X_2 for various values of $y_p = g_p/g_A$. The two shaded areas indicate the range of y_p and X_2 values allowed by the experimental μ capture rates of Refs. 14) and 15).

FIG. 1

FIG. 2

FIG. 3

FIG.4