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INELASTICITY AND UNIQUENESS OF THE SCATTERING MATRIX

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ABSTRACT

It is shown, in the framework of the Mandelstam representation, that the $\pi\pi$ scattering amplitude is entirely fixed if the "overlap function", as defined by Van Hove, is known above a certain energy.

It is an appealing idea that high energy scattering is the shadow effect of inelastic reactions. This idea has been mathematically formulated by Van Hove ¹⁾ who showed how to construct the scattering amplitude, under the assumption that it is purely imaginary, from the "overlap function" defined as

$$A_s^{\text{inelastic}}(s, \cos \theta) = \frac{\sqrt{s}}{k} \sum (2\ell+1) \left(\operatorname{Im} f_\ell(s) - |f_\ell(s)|^2 \right) P_\ell(\cos \theta) \quad (1)$$

s square of the c.m. energy, k c.m. momentum, θ c.m. scattering angle, f_ℓ is the ℓ 's partial wave amplitude normalized in such a way that $A_s^{\text{inelastic}} = 0$ in the elastic region.

We want to show, in the special case of the $\pi\pi$ amplitude, that even if the scattering amplitude is not purely imaginary the knowledge of $A_s^{\text{inelastic}}$ for $s > s_0$ fixes completely the scattering amplitude. For simplicity, we shall consider a neutral pseudoscalar theory. However, there is no difficulty to extend the argument to real $\pi\pi$ scattering.

Our assumptions are :

- i) Mandelstam representation (this could be weakened. Analyticity in the strips $|t| < 16\mu^2$, $|u| < 16\mu^2$, $|s| < 16\mu^2$ would be enough.)
- ii) local boundedness of

$$\lim_{\varepsilon \rightarrow 0} F(s+i\varepsilon, \cos \theta)$$

for $s > 4\mu^2$, $\cos \theta$ inside the analyticity domain.

From Eq. (1), we see that

$$A_s^{\text{inelastic}} = A_s - \frac{\sqrt{s}}{k} \sum (2\ell+1) |f_\ell|^2 P_\ell(\cos \theta) \quad (2)$$

where A_s is the full absorptive part. From the fact that $F(s, \cos \theta)$ is analytic at least inside an ellipse with foci $\cos \theta = \pm 1$ and extremities $\pm \cos \theta_0 = 1 + (4\mu^2/2k^2)$, we see - using assumption ii) - that

2.

$$\frac{\sqrt{s}}{k} \sum (2l+1) |f_l|^2 P_l(\cos \theta)$$

is analytic inside a bigger ellipse, with extremities given by $\pm \cos \theta_1 = \pm (2(\cos \theta_0)^2 - 1)$, which corresponds, using the variable $t = -2k^2(1 - \cos \theta)$, to

$$t_1 = 16 + \frac{64}{s-4}$$

(putting the pion mass = 1). So the elastic contribution to the absorptive part is certainly analytic in

$$|t| < 16.$$

Therefore from Eq. (2), the discontinuity of $A_s^{\text{inelastic}}$ across the t cut coincides with the discontinuity of A_s , as long as t is less than 16. This discontinuity is just the double spectral function $\rho(s, t)$.

If we know $A_s^{\text{inelastic}}$ for $s > s_0$, we know therefore $\rho(s, t)$ for $s > s_0$, $4 \leq t < 16$. However, the double spectral function is zero in the region

$$t < 4 + \frac{64}{s-16} \quad \text{for } s > 21$$

$$t < 16 \quad \text{for } s < 21$$

Hence we know in fact the double spectral function in the strip

$$4 \leq t < 4 + \frac{64}{s_0 - 16}$$

Using crossing symmetry, we see that this is true not only for $s > 4$ but also for $u > 4$. This strip is contained in the elastic t strip. According to a theorem by the author ²⁾ this fixes entirely the scattering amplitude.

We are aware of the fact that this proof is not "constructive" in the sense that we have not given a recipe to obtain the amplitude. A constructive proof is complicated by the fact that not all overlap functions are acceptable. For instance, one could forget about the first L partial wave contributions to the overlap function and the uniqueness theorem would still hold. These are therefore entirely fixed by the higher partial waves.

Anyway we see once more in action the marvellously constraining power of analyticity, unitarity and crossing, used in a global form. It is a pity that to exploit them in practice, one has so often to look at individual partial waves for which so many spurious ambiguities appear.

REFERENCES

- 1) L. Van Hove, Nuovo Cimento 28, 798 (1963).
- 2) A. Martin, Phys.Rev.Letters 9, 410 (1962).