REGGE POLES AND THE SCATTERING OF UNEQUAL MASS PARTICLESR.J. Oakes ^{*)}

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ABSTRACT

It is suggested that a Regge trajectory $\alpha(t)$ generally will also have angular momentum components $\alpha_\lambda(t) = \alpha(t) - \lambda$ ($\lambda = 1, 2, \dots$) associated with it for all t which become redundant whenever $\alpha(t)$ makes a particle.

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Recently, there has been a great deal of interest in establishing the Regge asymptotic behaviour of the scattering amplitude, compatible with the Mandelstam representation, in cases where the scattering particles do not have equal masses ¹⁾⁻⁶⁾. For example, in backward elastic scattering or in forward inelastic scattering of unequal mass particles, the kinematics are such that $z = \cos\theta$ in the crossed channel does not uniformly become large with increasing energy in the direct channel and the usual derivation of the Regge asymptotic formula breaks down. In trying to establish the Regge asymptotic behaviour in backward πN scattering Freedman and Wang ²⁾ have introduced a family of "daughter" Regge trajectories $\alpha_\lambda(t)$ for each "parent" $\alpha(t)$ such that at $t=0$ $\alpha_\lambda(0) = \alpha(0) - \lambda$ ($\lambda = 1, 2, \dots$). The residues $\beta_\lambda(t)$ of these "daughter" trajectories are all singular at $t=0$ like $t^{-\lambda}$ but they are related to each other in such a way that the singularity cancels and does not appear in the scattering amplitude as required by the Mandelstam representation. On the other hand, Omnès and Leader ⁶⁾ have recently disputed the existence of such trajectories, asserting that they are unnecessary.

In this note we propose that while these "daughter" trajectories do indeed exist they are not actually Regge trajectories in the usual sense but that they are simply a reflection of the extra spin components that can be present in the tensor field describing an elementary particle whenever the particle is off its mass shell. That is, each Regge trajectory $\alpha(t)$ should be thought of as being accompanied by a set of parallel component trajectories displaced by integers at $\alpha(t)-1$, $\alpha(t)-2, \dots$ for all values of t . However, just as for the free field where the extra components become redundant and are eliminated by appropriate subsidiary conditions, the component trajectories should not contribute at the poles of the scattering amplitude where the usual trajectory makes a particle of physical spin. Consequently, there should be no free particles or resonances lying on the "daughter" trajectories, contrary to the speculation of Freedman and Wang ²⁾. At energies where the spin is not physical, they can contribute to the scattering amplitude just as, for example, the scalar component of a vector field can contribute to the amplitude away from the pole.

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To put the above physical ideas on a mathematical foundation, we introduce a representation of the scattering amplitude which is free of the kinematical difficulties associated with the usual partial wave expansion in the case of unequal masses. In this representation the analyticity implied by the Mandelstam representation is preserved while parallel families of Regge trajectories emerge naturally and the expected Regge asymptotic behaviour $s^{\alpha(t)}$ obtains. At energies where the usual Regge trajectory makes a physical particle and $\alpha(m^2) = J$, one recovers the field theory result, i.e., the Feynman diagram for the exchange of an elementary particle of mass m and spin J , by imposing subsidiary conditions to eliminate the redundant angular momentum components. At such a pole of the amplitude there are no contributions from the component trajectories, the angular distribution being purely $P_J(z)$.

We begin with the fixed t dispersion relation ⁷⁾

$$A(s, t, u) = \frac{1}{\pi} \int_{s_0}^{\infty} ds' \frac{A_s(t, s')}{s' - s} + \frac{1}{\pi} \int_{u_0}^{\infty} du' \frac{A_u(t, u')}{u' - u} \quad (1)$$

describing the scattering of spinless particles of arbitrary masses $p_i^2 = m_i^2$. The Regge poles will be exchanged in the t channel and we will be concerned with the asymptotic behaviour in the s channel ($p_1 + p_2 \rightarrow p_3 + p_4$) as $s \rightarrow \infty$. It is convenient to introduce the four vectors $k = p_1 - p_3 = p_4 - p_2$, $p = (p_1 + p_3)/2$, and $p' = (p_2 + p_4)/2$ and to define the following quantities :

$$\begin{aligned} S &= (p_1 + p_2)^2 = (p + p')^2 \\ t &= (p_1 - p_3)^2 = k^2 \\ u &= (p_1 - p_4)^2 = (p - p')^2 \\ kp &= (m_1^2 - m_3^2)/2 \\ kp' &= (m_4^2 - m_2^2)/2 \\ pp' &= (s - u)/4 \end{aligned}$$

Now consider Eq. (1) in the region R defined by ⁸⁾ :

$$\text{Max}\{(m_1 - m_2)^2, (m_3 - m_4)^2\} < s < \text{Min}\{(m_1 + m_2)^2, (m_3 + m_4)^2\}$$

$$\text{Max}\{(m_1 - m_3)^2, (m_2 - m_4)^2\} < t < \text{Min}\{(m_1 + m_3)^2, (m_2 + m_4)^2\}$$

$$\text{Max}\{(m_1 - m_4)^2, (m_2 - m_3)^2\} < u < \text{Min}\{(m_1 + m_4)^2, (m_2 + m_3)^2\}$$

In this region the three momenta $\underline{p}$ and $\underline{p}'$ are imaginary, making the inner products $(\underline{p}|\underline{p}')^2$ Euclidean and the denominators in Eq. (1) do not vanish so that one can expand these denominators in Gegenbauer polynomials ⁹⁾ C_n^1 of argument $\Omega = \underline{p}\underline{p}'/(\underline{p}^2 \underline{p}'^2)^{1/2}$ obtaining

$$A(s, t, u) = \sum_{n=0}^{\infty} \{A_s^{(n)}(t) + (-)^n A_u^{(n)}(t)\} C_n^1(\Omega) \quad (2)$$

Here

$$A_s^{(n)}(t) = \frac{1}{\pi (\underline{p}^2 \underline{p}'^2)^{n/2}} \int_{s_0}^{\infty} ds A_s(t, s) \left\{ \frac{s - \underline{p}^2 - \underline{p}'^2}{2(\underline{p}^2 \underline{p}'^2)^{1/2}} - \sqrt{\left(\frac{s - \underline{p}^2 - \underline{p}'^2}{2(\underline{p}^2 \underline{p}'^2)^{1/2}} \right)^2 - 1} \right\}^{n+1} \quad (3)$$

with a similar definition for $A_u^{(n)}(t)$. Changing the summation into an integral and opening the contour after the method of Watson and Sommerfeld leads to the form

$$A(s, t, u) = \sum_i \xi_i(t) C_{n_i(t)}^1(\Omega) \left(\frac{1 \pm \exp[-i\pi n_i(t)]}{\sin \pi n_i(t)} \right) + \begin{matrix} \text{background} \\ \text{integral} \end{matrix} \quad (4)$$

since one encounters only moving poles $n_i(t)$ in the right-half n plane provided that there are only moving Regge poles $\alpha_i(t)$ in the right-half l plane as is usually supposed. In the following we shall assume that Eq. (4) can be continued out of the region R into the physical region of the crossed channels ¹⁰⁾.

The poles in the n plane and the ℓ plane can be related by using the Gegenbauer addition theorem ⁹⁾ in Eq. (2) to obtain

$$A(s, t, u) = \sum_{\ell=0}^{\infty} \sum_{\lambda=0}^{\infty} \left\{ A_s^{(\ell+\lambda)}(t) \pm (-)^\lambda A_u^{(\ell+\lambda)}(t) \right\} \frac{2^{2\ell-1} \Gamma(\lambda+1) \Gamma^2(\ell+1)}{\Gamma(2\ell+\lambda+2)} \quad (5)$$

$$\cdot (1-\omega^2)^{\ell/2} (1-\omega'^2)^{\ell/2} C_\lambda^{\ell+1}(\omega) C_\lambda^{\ell+1}(\omega') (2\ell+1) \{ P_\ell(-z) \pm P_\ell(z) \}$$

where $\omega = kp/(k^2 p^2)^{1/2}$, $\omega' = kp'/(k^2 p'^2)^{1/2}$, $z = \hat{p} \cdot \hat{p}'$, and $\Omega = \omega \omega' - (1-\omega^2)(1-\omega'^2)z$. The essential observation to make in Eq. (5) is that the amplitudes $A_s^{(\ell+\lambda)}(t)$ and $A_u^{(\ell+\lambda)}(t)$ depend only on the sum of the indices $n = \ell + \lambda$. Consequently, a single pole in the n plane at $\eta(t)$ will correspond to poles in the ℓ plane at $\alpha_\lambda(t) = \eta(t) - \lambda$ ($\lambda = 0, 1, 2, \dots$) of alternating signature: conversely, a single pole in the ℓ plane at $\alpha(t)$ will correspond to poles in the n plane at $\eta_\lambda(t) = \alpha(t) - \lambda$ ($\lambda = 0, 1, 2, \dots$).

According to our proposal that the usual Regge trajectories $\alpha(t)$ have components associated with them in the ℓ plane at $\alpha(t) - \lambda$ ($\lambda = 1, 2, \dots$) for all t the poles in the n plane generally will occur in corresponding families $\eta_\lambda(t) = \alpha(t) - \lambda$ ($\lambda = 0, 1, \dots$), also. Since the Gegenbauer functions $C_\lambda^1(\pm\Omega)$ are analytic at $t=0$ each residue function $\xi_\lambda(t)$ must be analytic here, which implies that the corresponding residues $\beta_\lambda(t)$ in the ℓ plane will behave as $t^{-\lambda}$ around $t=0$ in the unequal mass case. However, the usual Legendre functions $P_\ell(\pm z)$ are not analytic at $t=0$ in this case either but the cancellation postulated by Freedman and Wang ²⁾ yields a complete amplitude which remains analytic. This cancellation is implicit in the Gegenbauer expansion which is essentially a rearrangement of the Legendre series.

An important experimental consequence of our proposal is that there should be no particles lying on the component trajectories since these angular momentum components become redundant at energies where the usual trajectory $\alpha(t)$ makes a physical particle. If at $t=m^2$ there is a particle of spin $\alpha(m^2) = J$ then the subsidiary conditions one must impose on the residues β_λ in the ℓ plane to eliminate the redundant components are simply

$$\beta_{\lambda}(m^2) = 0 \quad (\lambda \geq 1) \quad (6)$$

In the n plane the corresponding constraints read

$$\sum_{\lambda=0}^J \xi_{\lambda}(m^2) C_{J-\lambda}^1(z) \propto P_J(z) \quad (7)$$

from which one can readily compute the residues $\xi_{\lambda}(m^2)$ up to a common factor. For example, one finds

$$\frac{\xi_{J-1}(m^2)}{\xi_J(m^2)} = \frac{2J}{m^2} \frac{(m_1^2 - m_3^2)(m_2^2 - m_4^2)}{\sqrt{(2m_1^2 + 2m_3^2 - m^2)(2m_4^2 + 2m_2^2 - m^2)}} \quad (8)$$

In the general case it seems that one cannot say much about the relations between the residues away from the poles of the amplitude. This reflects the well-known ambiguities in the propagator for an arbitrary spin particle off the mass shell. Presumably in the equal mass case the component trajectories do not contribute at all, Eq. (6) holding for all t , which accounts for their only recently being discovered.

Finally, note that the Regge asymptotic behaviour $s^{\alpha(t)}$, which was originally the motivating question, follows immediately from Eq. (4) even in the case of unequal masses, since ¹¹⁾

$$\Omega = (s-u) \left\{ (s+u)^2 - (m_1^2 + m_3^2 - m_2^2 - m_4^2)^2 \right\}^{-1/2}$$

and $\alpha_{\alpha}^1(\Omega) \sim \Omega^{\alpha}$ asymptotically.

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- 2) D.Z. Freedman and J.M. Wang, Phys.Rev.Letters 17, 569 (1966) and Lawrence Radiation Laboratory report UCRL-17014 (unpublished).
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- 4) L. Durand, III, University of Wisconsin preprints (unpublished).
- 5) E.A. Paschos, Cornell University preprint (unpublished).
- 6) R. Omnès and E. Leader, Cambridge University preprint (unpublished).
- 7) Subtraction terms are an inessential complication and we shall ignore them.
- 8) Such a region always exists provided all the external particles are stable.
- 9) Higher Transcendental Functions, A. Erdélyi, Editor, McGraw-Hill (New York, 1953). The Gegenbauer functions are particularly convenient but are not the only possibility.
- 10) The corresponding assumption usually made in the equal mass case is that the Legendre expansion can be analytically continued into the crossed channel after performing the Watson-Sommerfeld transformation. In both cases one is supposing similar convergence properties of the background integral which have never been rigorously proved.
- 11) For equal external masses $\Omega = z$.