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FORWARD DISPERSION RELATIONS AND LOW ENERGY $\bar{K}N$ SCATTERING

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ABSTRACT

$\bar{K}N$ dispersion relations, evaluated with the correct Y_0^* effective range formula, give $g^2_{\bar{K}N} \approx 7.4$ as against the value 4.8 of Lusignoli et al. Some $T=0$, P wave is also needed.

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1. INTRODUCTION

Recently Lusignoli et al. ^{1) *)} computed $g^2_{\bar{K}N}$, using the forward dispersion relations for the $\bar{K}N$ system. Using the same method as LRSV, we obtain a quite different value for $g^2_{\bar{K}N}$. The main reason is that we use a more correct description of the low energy $\bar{K}N$, $T=0$ scattering which takes into account certain kinematical factors. We show further that a full application of the forward dispersion relations may be helpful and complementary to the analysis of low energy $\bar{K}N$ scattering.

Section 2 contains the formulae for the $T=0$, S wave amplitude and discusses the influence of the $Y_0^*(1405)$. The correct value of $g^2_{\bar{K}N}$ is calculated in Section 3. Section 4 shows that there is definite evidence for $T=0$, P wave scattering. Section 5 summarizes our conclusions and gives a limited discussion of remaining problems.

2. LOW ENERGY $T=0$, $\bar{K}N$ SCATTERING AND THE $Y_0^*(1405)$

The formalism for the low energy $\bar{K}N$ scattering is extensively treated by Dalitz and Tuan ²⁾, cf., also Ross and Shaw ³⁾. We assume in this Section that we have only S wave in the $T=0$ channel.

The imaginary part of the $T=0$ amplitude F below the $\bar{K}N$ threshold, is given by

$$\text{Im } F = \frac{b}{(1 + |p|a)^2 + |p|b|^2} \quad (2.1)$$

*) We quote Ref. 1) in the following as LRSV.

2.

where

$$a = \alpha - \gamma \frac{\beta^2 q}{1 + (q\gamma)^2} \quad (2.2)$$

$$b = \frac{\beta^2 q}{1 + (q\gamma)^2} \quad (2.3)$$

p and q are the c.m. momentum in the $\bar{K}N$ and $\pi \Sigma$ system, respectively. (p is imaginary below the $\bar{K}N$ threshold.) α , β and γ are the (real) elements of the two channel reaction matrix given by Dalitz^{2),4)}. The zero range approximation amounts to taking α , β and γ energy independent³⁾. However, in the analysis of the low energy experiments as yet⁵⁾, one always takes a and b energy independent. This is also done by LRSV. It is clearly not correct, especially if one uses (2.1) over a large energy region down to the $Kp \rightarrow \Sigma \pi$ threshold. It is also seen that in the effective range approximation [i.e., $\alpha = \alpha(E_0) + r_\alpha p^2$, $\gamma = \gamma(E_0) + r_\gamma p^2$] the parameter R in the formula for the scattering length A

$$\frac{1}{A} = \frac{1}{A(E_0)} + R p^2 \quad (2.4)$$

should be taken as essentially complex [cf., Ref. 6)]. Kittel and Otter⁶⁾ take in their wrong effective range approximation the width and place of the $Y_0^*(1405)$, measured in the final state interaction of $\Sigma \pi$ in production processes, to be the same for the $\bar{K}p \rightarrow \Sigma \pi$ transition. As pointed out by Dalitz et al.⁴⁾, these may be quite different. To show this in a more quantitative way, we assume that the experimentally determined width and place are the width and place of the resonance in the $\Sigma \pi \rightarrow \Sigma \pi$ channel. The cross-section for this channel (only $T=0$) is given by¹⁾

$$\sigma_{\Sigma\pi} = \frac{4\pi A_{\Sigma\pi}^2}{1 + (q A_{\Sigma\pi})^2} \quad (2.5)$$

where, below the $\bar{K}N$ threshold

$$A_{\Sigma\pi} = \gamma - \frac{\beta^2 |p|}{1 + \alpha |p|} \quad (2.6)$$

Table I compares the dependence of resonance energy and width of (2.5) and (2.1) on the parameters α , β , γ and r : r is defined by

$$\alpha(E) = \alpha + \kappa p^2 \quad (2.7)$$

We remark that :

- i) there is a shift of at most 6 MeV of the resonance energy in the $\bar{K}N \rightarrow \Sigma\pi$ channel with respect to the $\Sigma\pi \rightarrow \Sigma\pi$ channel : experimentally $E^{\text{res}} = 1405 \text{ MeV}$ ⁷⁾. The parameter α is almost completely fixed by the place of the resonance, which depends but very little on β , γ and r ;
- ii) the width for the two channels is quite different : it grows rapidly with increasing β^2 in the $\Sigma\pi$ cross-section. Thus the experimental width, $\Gamma = 35 \pm 5 \text{ MeV}$ ⁷⁾, fixes β^2 to a large extent;
- iii) with γ increasing, β^2 for the right width has to be taken smaller. Both effects make b [cf., (2.3)] become rapidly too small with respect to its value derived from low energy K^-p scattering ⁵⁾ ($b = 0.5 - 0.7 \text{ fm}$). This provides evidence for a small γ . The same is to a lesser extent true for the effective range parameter r .

3. THE COUPLING CONSTANT $g_{\Lambda \bar{K} N}^2$

We follow the procedure of LRSV. They took a special combination of the forward dispersion relations for $K^\pm p$ and $K^\pm n$ scattering such that the $T=1$ part of the $\bar{K}N$ amplitude drops out. The consequence is that we have only one pole term left ($\Lambda \rightarrow \bar{K}N$). Generally, we may write this dispersion relation in the form

$$\begin{aligned}
 \left(\frac{1}{\omega_\Lambda - \omega_0} - \frac{1}{\omega_\Lambda - \omega_1} \right) \Gamma_\Lambda &= D^p(\omega_0) - \frac{1}{2} D^n(\omega_0) - \left\{ D^p(\omega_1) - \frac{1}{2} D^n(\omega_1) \right\} \\
 &+ \frac{1}{4\pi^2} \int_{M_K}^{\infty} p_L' \left(\sigma_p^+ - \frac{1}{2} \sigma_n^+ \right) \left(\frac{1}{\omega' + \omega_1} - \frac{1}{\omega' + \omega_0} \right) d\omega' + \\
 &+ \frac{1}{4\pi^2} \int_{\tilde{\omega}}^{\infty} p_L' \left(\sigma_p^- - \frac{1}{2} \sigma_n^- \right) \left(\frac{1}{\omega' - \omega_1} - \frac{1}{\omega' - \omega_0} \right) d\omega' \\
 &+ \frac{1}{\pi} \int_{\Sigma\pi}^{\infty} T^0 \left(\frac{1}{\omega' - \omega_1} - \frac{1}{\omega' - \omega_0} \right) d\omega'
 \end{aligned} \tag{3.1}$$

where

$$\omega_\Lambda = \frac{M_\Lambda^2 - M_N^2 - M_K^2}{2M_N}$$

$$\Gamma_\Lambda = \frac{\omega_\Lambda + M_N - M_\Lambda}{2M_\Lambda} g_{\Lambda \bar{K} N}^2$$

$\sigma_p^\pm$ and $\sigma_n^\pm$ are the total cross-sections for $K^\pm$ proton and neutron scattering, respectively. p_L and ω are the momentum and energy of the K meson in the laboratory system, $\omega_{\Sigma\pi} = 332$ MeV and $\tilde{\omega} = 575$ MeV.

$$D(\omega) = \text{Re } F^-(\omega) \text{ if } \omega > 0 \\ = \text{Re } F^+(-\omega) \text{ if } \omega < 0$$

$F^+(\omega)$ is the K^+N amplitude, T^0 is the imaginary part of the $\bar{K}N$, $T=0$ amplitude.

As LRSV, we take in this Section $\omega_0 = M_K$ and $\omega_1 = -M_K$ and write (3.1) as

$$\frac{2M_K \sqrt{\lambda}}{\omega_\Lambda^2 - M_K^2} = \gamma_0 + R_0 \quad (3.2)$$

where I_0 is the $T=0$ contribution

$$\gamma_0 = \frac{1}{\pi} \int_{\omega_{\pi\pi}}^{\tilde{\omega}} T^0 \frac{2M_K}{M_K^2 - \omega'^2} d\omega' + D^0(M_K) \quad (3.3)$$

and R_0 is the remaining part of (3.1) with $\omega_0 = M_K$ and $\omega_1 = -M_K$. Using the results for the total cross-sections of Refs. 8), 9), 10) we obtain

$$R_0 = -400 \text{ Mev mb} \quad (3.4)$$

This is compatible with the result of LRSV,

$$R_{0L} = -463 \pm 80 \text{ Mev mb} \quad (3.5)$$

within the rather big error. This uncertainty is mainly due to the badly known low energy $K^\pm n$ cross-sections. The deviation from the value of LRSV comes mainly because we took a small $T=0$, S wave amplitude with scattering length $A_0 = 0.04^{10)}$, whereas LRSV probably neglected this contribution.

The contribution of the region above $p_L = 20 \text{ GeV}/c$ to R_0 is quite important. It amounts to about -150 MeV mb . We calculated it with a recent fit of the total cross-sections with a Regge pole model of Barger and Olsson ¹¹⁾ and obtained the same result as LRSV who used the fit of Rarita and Phillips. We remark that the contribution of the high energies does not necessarily change if one adopts a recent suggestion ¹²⁾ that the total cross-sections may go to zero, because this contribution depends on the differences $\sigma_{K^+N} - \sigma_{K^-N}$, for which one may maintain the same model ^{*)}.

Now we turn to the value of I_0 . We suppose that only S wave contributes and use the effective range approximation as described in Section 2. The last column of Table I lists I_0 for different sets of values for α , β , γ and r . The dependence on the resonance energy (i.e., α) is small. Also the dependence on β^2 (the width) is rather small. There is a somewhat larger dependence on γ but especially on r : a value of r of about -0.1 fm , which is possibly compatible with experiment ⁶⁾, changes I_0 with about $+60 \text{ MeV mb}$ (i.e., $g_{\Lambda KN}^2$ with $+0.7$).

A reasonable estimate for I_0 is

$$I_0 = 1000 \pm 50 \text{ MeV mb} \quad (3.6)$$

This deviates quite a lot from the value obtained by LRSV with the too simple zero range formulae (with Kim ⁵⁾ parameters),

$$I_{0L} = 044 \pm 15 \text{ MeV mb} \quad (3.7)$$

We combine our results for R_0 and I_0 , taking the error in R_0 the same as estimated by LRSV and obtain

$$g_{\Lambda \bar{K} N}^2 = 4.4 \pm 1.2 \quad (3.8)$$

*) I thank Dr. Kokkedee for a discussion on this point.

to be compared with the LRSV result

$$\left(g_{\Lambda \bar{K} N}^2 \right)_L = 4.4 \pm 1.0 \quad (3.9)$$

The error indicated in (3.8) should be considered with reservation, but our value is definitely larger than the value of LRSV.

The assumptions, made in the calculation of I_0 , are

- i) the S wave scattering is well represented by an effective range approximation in the energy region from the $\Sigma \pi$ threshold, $E = 1345$ MeV, to $E \approx 1500$ MeV;
- ii) the parameters for this approximation were determined from the width and place of the $Y_0^*(1405)$ in the $\Sigma \pi \rightarrow \Sigma \pi$ channel and are in agreement with the low energy $\bar{K} p$ data;
- iii) there are no other (P,D,...) waves which contribute.

We show in the next Section that there is definite indication for P wave even below the $\bar{K} N$ threshold in the $T=0$ amplitude. However, its contribution to the special combination of the forward dispersion relations (3.2) is probably small.

Finally, a remark on the $K \bar{K}_0$ mass difference. LRSV use the complete formulae given in Ref. 2), which take into account this mass difference. However, we think that one should not do this because the mass difference is of electromagnetic origin and its influence should be eliminated before applying dispersion relations (as one does generally for the Coulomb field of the proton). We then have two separate, $T=0$ and $T=1$, amplitudes. Luckily it appears that this has practically no effect on the result for I_0 and therefore $g_{\Lambda \bar{K} N}^2$.

4. P WAVE IN THE T=0 LOW ENERGY SCATTERING

We show that a more complete use of the forward dispersion relations provides definite evidence for P wave also in the unphysical region.

We use Eq. (3.1) for the following set of values of ω_1 and ω_0

$$\omega_1 = (345. + L\alpha) \text{ Mev}, \quad L = 1.16$$

$$\omega_0 = \omega_1 - \alpha \quad (4.1)$$

$$\alpha = .025 M_K$$

We write (3.1) in the form

$$\frac{\alpha \sqrt{\Lambda}}{(\omega_\Lambda - \omega_1 + \alpha)(\omega_\Lambda - \omega_1)} = R(\omega_1) + \mathcal{I}_S(\omega_1) + \mathcal{I}_K(\omega_1), \quad (4.2)$$

where

$$R(\omega_1) = -\frac{\alpha}{4\pi^2} \int_{M_K}^{\infty} \rho_L'(\sigma_\rho^+ - \frac{L}{2}\sigma_\pi^+) \frac{d\omega'}{(\omega' + \omega_1)(\omega' + \omega_1 - \alpha)} \\ + \frac{\alpha}{4\pi^2} \int_{\tilde{\omega}}^{\infty} \rho_L'(\sigma_\rho^- - \frac{1}{2}\sigma_\pi^-) \frac{d\omega'}{(\omega' - \omega_1)(\omega' - \omega_1 + \alpha)} \quad (4.3)$$

$\mathcal{I}_S(\omega_1)$ is the contribution of the T=0, S wave,

$$\mathcal{I}_S(\omega_1) = D_S^0(\omega_1 - \alpha) - D_S^0(\omega_1) + \frac{\alpha}{\pi} \int_{\omega_{\pi\pi}}^{\tilde{\omega}} T_S^0 \frac{d\omega'}{(\omega' - \omega_1)(\omega' - \omega_1 + \alpha)} \quad (4.4)$$

and $I_r(\omega_1)$ is an expression similar to (4.4) with S replaced by $r(= P, D, \dots)$.

Table II gives some results. $R(\omega_1)$ is calculated, as R_0 in Section 3, from the total cross-sections. The error is mainly determined by the uncertainty in the K^-N total cross-sections just above $p_L = 275$ MeV/c. This error decreases rapidly as ω_1 goes further from this region to the $\sum \pi$ threshold. We estimate it ≈ 5 MeV mb at $\omega_1 = 494$ MeV and ≈ 1 MeV mb at $\omega_1 = 358$ MeV. $I_S(\omega_1)$ is computed with

$$\alpha = -1.51 f, \beta^2 = .42 f^2, \gamma = \tau = 0 \quad (4.5)$$

but the result does not depend much on these precise values; the variation is small compared with the uncertainty in $R(\omega_1)$.

The fourth column of Table II gives $I_r(\omega_1)$ calculated from (4.2) assuming $g_{\Lambda KN}^2 = 7.0$. It is clear that there is an important P wave contribution: near the K^-p threshold it is of the same order of magnitude as the S wave contribution. It is very easy to fit these residue terms $I_r(\omega_1)$ if we assume an effective range approximation for the P wave. We have then at least eight parameters at our disposal: the analogues of α , β and γ for $P_{\frac{1}{2}}$ and $P_{\frac{3}{2}}$ waves and at least two effective range parameters. Because of this large number of parameters, the serious doubt on the validity of an effective range approximation for P wave over such a large energy range and the rather large uncertainty in $I_r(\omega_1)$ [because of $R(\omega_1)$ and $g_{\Lambda KN}^2$] we have not performed such a fit. Instead we have taken all the parameters for the P wave equal to zero, except the α for the $P_{\frac{3}{2}}$:

$$\alpha_p = \alpha_p^0 + r_p q^2 \quad (4.6)$$

So we have two parameters only. The last column of Table II shows that we obtain a reasonable fit with

$$\alpha_p^0 = .029 f^3, r_p = .049 f^5 \quad (4.7)$$

From the smallness of these values it is evident that experimental detection will be impossible : at $p_L \approx 150$ MeV/c the contribution of the P wave to the total cross-section ≈ 0.05 mb (with $\beta_p^2 = 0.03$, we find ~ 2.0 mb), to be compared with about 180 mb for the S wave. We calculated also the contribution to I_0 (3.3) and thus $g_{\Lambda \bar{K}N}^2$ for the parameters (4.7) and found it negligible; this contribution increases rapidly for $\beta_p^2 \neq 0$; e.g., for $\beta_p^2 = 0.03$, it becomes -40 MeV mb, but there is some evidence that b_p (and thus β_p^2) is small compared with a_p (and thus α_p) from the analysis around $p_L = 400$ MeV/c⁹).

Thus we may conclude that the value of $g_{\Lambda \bar{K}N}^2$ will not be changed appreciably by the P wave.

5. DISCUSSION

We summarize our conclusions as follows :

- i) use of the correct formulae for the $T=0$, S wave scattering, which have automatically the right threshold behaviour, increases $g_{\Lambda \bar{K}N}^2$ considerably, compared with the estimate $g_{\Lambda \bar{K}N}^2 \approx 4.8$ by LRSV. We found

$$g_{\Lambda \bar{K}N}^2 = 7.4 \pm 1.2 \quad (5.1)$$

The error is mainly due to the uncertainty in the total cross-sections for the $\bar{K}N$, just above $p_L = 275$ MeV/c, and the K^+n , near threshold, scattering.

- ii) a more complete use of the forward dispersion relations shows the presence of P wave ($T=0$) around the $\bar{K}N$ threshold. Its contribution relatively to that of the S wave is so small that it seems impossible to detect this P wave experimentally. Perhaps a future use of partial wave dispersion relations can give more detailed information on the P wave in the low energy region. Before one needs to have an as accurate as possible phase shift analysis at higher energies.

We restricted ourselves to $T=0$ scattering by subtracting in a suitable way the Kn from the Kp dispersion relations. The treatment of the $T=1$ case near threshold is much more complicated. Experimentally the $T=1$, S wave scattering length is badly determined, which is perhaps also due to a more important P wave contribution, with the $Y_1^*(1385)$ below $\bar{K}N$ threshold. Further the scattering matrix is now a 3×3 matrix, so that we have at least three more parameters. The unphysical region is much larger, going down to $\omega_{\Lambda\pi} = 235$ MeV, which makes an effective range approximation even more doubtful certainly for the P wave. We tried without success to describe the $Y_1^*(1385)$ within a zero range approximation.

Several authors ^{13),14)} used the experimentally known real parts at higher energies to obtain some conclusions about the coupling constants $g_{\Lambda KN}^2$ and $g_{\Sigma KN}^2$. It should be remarked that in that way one does not avoid the uncertainty at the lower and unphysical energies. Table I of Ref. ¹³⁾ shows clearly the big influence of the low energy region on the real parts also compared with the pole contribution.

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α (fm)	β^2 (fm ²)	γ (fm)	r (fm ³)	E_Σ (MeV)	Γ_Σ (MeV)	E_N (MeV)	Γ_N (MeV)	I_0 (MeV mb)
-1.41	0.3	0.	0.	1401	22	1403	19	972
	0.4			1400	32	1404	25	966
	0.5			1399	49	1405	31	963
-1.51	0.4	0.	0.	1404	26	1407	21	973
	0.5			1404	37	1408	26	970
	0.6			1403	56	1409	30	968
-1.61	0.4	0.	0.	1408	21	1410	18	983
	0.5			1408	30	1411	21	976
	0.6			1407	41	1412	25	973
-1.51	0.3	0.1	0.	1405	19	1407	16	991
	0.4			1404	29	1408	20	986
	0.5			1404	45	1409	25	984
	0.3	0.2	0.	1405	21	1408	15	1001
	0.4			1404	35	1409	19	997
	0.5			1404	58	1410	23	995
	0.3	0.3	0.	1405	24	1408	14	1008
	0.4			1404	43	1409	19	1006
	0.5			1404	43	1409	19	1006
-1.55	0.3	0.	-0.05	1406	18	1407	16	1020
	0.4			1405	27	1408	20	1008
	0.5			1405	38	1409	25	1000
-1.57	0.3	0.	-0.10	1406	19	1407	16	1066
	0.4			1405	28	1408	21	1047
	0.5			1404	43	1409	25	1034
-1.62	0.3	0.	-0.20	1406	21	1407	17	1177
	0.4			1405	33	1408	22	1143
	0.5			1404	x	1409	27	1095

TABLE I : Dependence of the resonance energy and width on α , β , γ and r for the $\Sigma \pi \rightarrow \Sigma \pi$ and $Kp \rightarrow \Sigma \pi$ channels. E_Σ and Γ_Σ are the resonance energy and width for $\Sigma \pi$ (2.5), E_N and Γ_N the analogues for $\text{Im } F$ (2.1). I_0 is the contribution of the $T=0$, S wave up to $p_L = 275 \text{ MeV}/c$ to the value of $g_{\Lambda KN}^2$ [cf., Section 3].

ω_1 (MeV)	$R(\omega_1)$	$I_s(\omega_1)$	$I_r(\omega_1)$	$I_p(\omega_1)$
358	31.9	-14.0	2.6	1.4
370	33.6	-14.9	0.2	- 0.6
382	35.4	-15.9	- 2.2	- 2.6
395	37.6	-17.1	- 4.4	- 4.8
407	40.0	-18.4	- 6.6	- 7.2
420	42.8	-20.0	- 9.0	- 9.6
432	46.1	-21.8	-11.4	-12.0
445	49.8	-23.9	-13.8	-14.6
457	54.3	-26.0	-17.0	-17.4
469	60.0	-29.2	-20.0	-20.2
482	66.8	-33.9	-22.8	-23.0
494	75.8	-39.4	-26.8	-26.0
506	77.6	-46.5	-32.0	-29.2
519	104.1	-56.6	-39.0	-32.4
531	128.8	-72.2	-48.6	-36.2
543	170.0	-97.9	-64.4	-40.4

TABLE II : The presence of $T=0$, P wave. $R(\omega_1)$ and $I_s(\omega_1)$ are the contributions of the higher energies and the low energy S wave to the combination of dispersion relation given in (4.2). $I_r(\omega_1)$ are the rest terms ($g_{\Lambda \bar{K} N}^2 = 7$), which are fitted by introducing some P wave, $I_p(\omega_1)$, with parameters (4.7).

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