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RHO MESON STRUCTURE

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A B S T R A C T

The mass and width of ρ mesons of all charges are studied as a function of momentum transfer t in a large compilation of $\pi^+\pi^-$ mass histograms from bubble-chamber experiments. For small t the ρ masses are found to be t dependent, the ρ^+ and ρ^- masses significantly different, and the ρ^0 mass strongly reaction-dependent. Significant ω decay into $\pi^+\pi^-$ is found. The detailed structure under the ρ meson is discussed.

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The masses in any given place are generally composed of three parts, the relatively active, the intermediate and the relatively backward.

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1. DATA SELECTION

The present work is an analysis of a large compilation of $\pi\pi$ -invariant mass histograms from the reactions

$$\pi^+ + p \rightarrow \pi^+ + \pi^- + \Delta^{++} \xrightarrow{\quad} \pi^+ + p \quad (1)$$

$$\pi^- + p \rightarrow \pi^+ + \pi^- + n \quad (2)$$

$$\pi^+ + d \rightarrow \pi^+ + \pi^- + p + p \quad (3)$$

$$\pi^+ + p \rightarrow \pi^+ + \pi^0 + p \quad (4)$$

$$\pi^- + p \rightarrow \pi^- + \pi^0 + p \quad (5)$$

In order to obtain an enriched ρ sample, we have used only bubble chamber data with relatively low momentum transfer to the baryon. We have divided the data into momentum transfer subsamples, generally with

$$t(= \Delta^2/\mu^2) < 5, 10, 20, 30.$$

Table 1 lists the experiments included in the compilation and their beam momenta. We have not studied the energy dependence of the ρ parameters.

The data from reactions (2) and (3) have been lumped together; all other reactions have been studied separately. For comparison, we have also studied the purest sample of ρ^- mesons from the CERN missing-mass spectrometer¹⁾; reaction

$$\pi^- + p \rightarrow X^- + p. \quad (6)$$

The total number of events in the compilation is about 45,000. With the fitted backgrounds subtracted, this leaves a total number of ρ mesons of about 30,000.

2. ANALYSIS

The ρ (and ω) mesons have been fitted with ordinary Breit-Wigner functions, in which the full width Γ_ρ (and Γ_ω) was assumed to have a p-wave

energy dependence:

$$\Gamma_{\rho}(M) = \frac{\Gamma_{\rho}^0 M_{\rho}}{M} \left(\frac{M^2 - 4\mu^2}{M_{\rho}^2 - 4\mu^2} \right)^{3/2}, \quad (7)$$

where M_{ρ} and μ are the masses of the ρ and π mesons, respectively, M is the invariant mass of the $\pi\pi$ system, and Γ_{ρ}^0 is a parameter.

Since many of the experiments we use have been done with beams of several different momenta, we have not attempted to calculate an appropriate phase space. Quite arbitrarily, we have fitted the background with an empirical function of the form

$$B = C_1 (M - 2\mu) (C_3 - M) e^{MC_2}, \quad (8)$$

where C_1 , C_2 , and C_3 are free parameters. Often it has been possible to find good solutions with C_3 kept fixed at some reasonable value (note that the background is negative for $M > C_3$). This background is always smooth and slowly varying under the ρ peak, and so does not affect the determination of the ρ mass. The correlation between background and ρ width is stronger. This sometimes makes the width determination less accurate, in particular when information on the wings of the ρ is lacking.

It was felt necessary to determine the errors of the parameters with greater accuracy than is done in the customary minimizing programs. (MINFUN and MINROS often return errors which seem ridiculously small). We therefore developed a new general minimizing program, MINUIT (see Ref. 5), which determines the 1 std errors of each parameter, without assuming that the χ^2 function is parabolic (or even symmetric) in any parameter around the minimum point. For an n -parameter fit, this requires at least one n -parameter minimization and $2n$ minimizations with $n-1$ parameters, keeping one parameter fixed at a time. In general, these errors are larger than the MINFUN-type errors, as anticipated, and they are often asymmetric. All errors given here correspond to parameter increments which increase χ^2 by 1, leaving all other parameters free.

Most of the compiled histograms have 10 MeV bin width. We have tried to use as wide ρ wings as possible in the analysis; however, the form of the background (8) sometimes made it possible to obtain good fits at high energies (when, for instance, the f meson is present), or at the lowest one or two bins. In such cases we have restricted the analysis to shorter wing regions.

Sometimes it has been impossible to find acceptable solutions, due to large statistical fluctuations in the wing bins. Since the detailed (10 MeV) structures in the wings have no bearing on the ρ parameters, we have then joined the wing bins into 20 MeV bins.

A Gaussian resolution function has been folded into all theoretical curves. The precise value of the resolution is not known, but the fits are very insensitive to variations in the resolution, as will be shown later. We have therefore used the round values 20 MeV for reaction (1), 30 MeV for reactions (2) to (5) and 28 MeV for reaction (6).

The edge effects near the beginning and the end of the histogram, due to the Gaussian, have been eliminated by discarding the first and last three 10 MeV bins after the Gaussian has been folded in, but before the χ^2 calculation. If this precaution is not taken, the ρ widths may be off by as much as 20 MeV.

The χ^2 fit uses the theoretical errors at each histogram point.

3. $\rho^0 + \omega$ MIXING

The reactions (1) to (3) contain information on the ρ^0 meson, and also on the possible presence of the electromagnetic decay

$$\omega \rightarrow \pi^+ + \pi^- . \quad (9)$$

It is well known that the ω meson is produced relatively more at higher momentum transfers than is the ρ meson. This is apparent from Table 2 which gives the production ratio of ω to ρ^0 in the reactions

$$\begin{array}{l} \pi^+ + p \rightarrow \rho^0 + \Delta^{++} \\ \quad \quad \quad \searrow \\ \quad \quad \quad \pi^+ + \pi^- \end{array} \quad (10)$$

and

$$\begin{array}{l} \pi^+ + p \rightarrow \omega + \Delta^{++} \\ \quad \quad \quad \searrow \\ \quad \quad \quad \pi^+ + \pi^- + \pi^0 \end{array} \quad (11)$$

in selected t bands. One may then expect to see more significant evidence for reaction (9) in the samples of reactions (1) to (3) at higher momentum transfers. In particular, the ω should be enhanced relative to ρ by a lower t cut (in addition to the upper t cut for background suppression). Unfortunately, this reduces the size of the $\pi^+\pi^-$ sample to a level which may again be inconclusive.

The histograms containing the samples of reaction (1) for momentum transfers $t < 10$, $10 < t < 30$, and $t > 30$ are shown in Figs. 1 to 3, respectively. If ω decays are present in these samples, they must be associated with the slight double-peak structure visible in all the three histograms. Note that the " ω peak" is indeed relatively stronger at $10 < t < 30$ (Fig. 2) than in $t < 10$ (Fig. 1). The size of the sample is, however, smaller and the level of the background higher, so that the statistical significance is not improved in Fig. 2.

In the samples of reactions (2) and (3), no such structure is directly visible (Figs. 7 to 9). However, the resolution is expected to be worse in these reactions, and thus 20 MeV structures may well be masked. The statistics is also considerably larger than in the samples of reaction (1).

Note that, even if we may be able to establish the existence of a narrow peak in the ω region, the isospin of this resonance is not known until a comparison with the charged reactions (4) to (6) has been made. We postpone this to Section 5, and use the name " ω " meanwhile.

Since all the information on the ω comes from four or five bins, one cannot expect to obtain information on too many parameters. We therefore restrict ourselves to three types of fits: simple ρ fits, incoherent $\rho + \omega$ fits, and completely coherent $\rho + \omega$ fits. The theoretical distribution of ρ and ω decays then is

$$|A_\rho|^2 + |\sigma A_\omega|^2 + 2x \operatorname{Re}(A_\rho^* \sigma A_\omega) \quad (12)$$

where the A 's are Breit-Wigner amplitudes, σ is a complex number, and $x = 0$ or 1 corresponds to complete incoherence or coherence, respectively.

This procedure leaves six parameters to be determined: M_ρ , M_ω , Γ_ρ^0 , Γ_ω^0 , $\text{Re}\sigma$ and $\text{Im}\sigma$. Since the resolution is 2-3 times larger than the known value of Γ_ω^0 , it is useless to keep Γ_ω^0 free. Γ_ω^0 is therefore fixed at 10 MeV. In test runs with zero resolution and Γ_ω^0 free, we usually obtained ω widths of the order of 25 ± 35 MeV.

In the incoherent fits we set $\text{Im}\sigma = 0$. This leaves us with four free parameters, and five in the coherent fits. In addition, there are the (usually) two free parameters in the empirical background function (8).

In Table 3 we give the results of the fits to reactions (1) to (3), and in Figs. 1 to 9 most of them are drawn on the corresponding experimental histograms.

The simple ρ fits to reaction (1) clearly are so good that no ω contribution is required. The confidence levels, from Table 3, are 54%, 79%, and 85% for the momentum transfer subsamples $t < 10$, $10 < t < 30$, and $t > 30$, respectively. In Figs. 1 to 3 we have drawn bands corresponding to these fits $\pm$ the statistical errors. Clearly, most of the double-peak structure in all the three histograms is well inside these bands. Rather than to compare confidence levels for the existence of ω decays, however, we have chosen the better understood method of establishing whether σ is different from zero, and by how many standard deviations [this was the main reason for developing the program MINUIT⁵].

In Table 3 we next list the $\rho + \omega$ incoherent fits to reaction (1), with M_ω kept fixed at its known value 783.4 MeV. In parentheses, we give the M_ω values found when left free. These free M_ω values generally are higher than 783.4 MeV by 1-3 std. From this, one may conclude either that $\rho + \omega$ should not be added incoherently or that there is a systematic error in the energy scale. We shall return to the latter possibility in Section 4. The former possibility, at least, makes the incoherent fits doubtful.

Let us next turn to the $\rho + \omega$ coherent fits to reaction (1): Table 3 and Figs. 4 to 6. The free M_ω values, again in parentheses in Table 3, now agree with 783.4 MeV within 0.8 std. We therefore fix M_ω at its known value and determine the remaining parameters. The result is that $|\sigma|$ differs from

zero by only 1.1-1.5 std in all t samples. Clearly, the conclusion must be that these samples are too small to furnish definite information on the ω decay (9).

We have also tried to estimate the branching ratio of reaction (9) relative to all charged ω decays, by assuming

$$R = \frac{\Gamma(\omega \rightarrow \pi^+ \pi^-)}{\Gamma(\omega \rightarrow \pi^+ \pi^- \pi^0)} = |\delta|^2 \frac{\sigma_\rho}{\sigma_\omega} \quad (13)$$

where the cross-section ratios $\sigma_\omega/\sigma_\rho$ are as given in Table 2. Our values of R , as given in Table 3, cannot of course, be directly compared with other values quoted in the literature, since we study samples restricted to limited regions of momentum transfer t . Note, further, that the expression (13) assumes that $\sigma_\omega/\sigma_\rho$ can be taken as constants within a given t region, although Table 2 clearly shows that $\sigma_\omega/\sigma_\rho$ varies rapidly as a function of t . Our R values agree roughly, however, with the coherent $\rho + \omega$ fit of Flatte et al.²¹⁾,

$$R = (2.9 \pm 1.1) \% .$$

Turning now to reactions (2) and (3), we cannot expect to determine R , since $\sigma_\omega/\sigma_\rho$ is not directly known*). On the other hand, we may still be able to establish whether $|\sigma|$ is different from zero, and by how many standard deviations.

Simple ρ fits to the histograms in Figs. 8 and 9, corresponding to momentum transfer regions $t < 10$ and $t < 30$, respectively, reveal a considerable excess of events in the ω region. In the $t < 5$ sample (Fig. 7), no significant ω peak is seen. This agrees with the t behaviour of ω relative to ρ . We therefore fit Fig. 7 with a simple ρ , and leave the discussion to the next Section. Figures 8 and 9 we fit with $\rho + \omega$ incoherent. The results are given in Table 3 and in the figures.

When the ω mass is left free, it returns an M_ω value in striking agreement with 783.4 MeV, as given in Table 3 (within parentheses). We therefore fix M_ω at 783.4 MeV. The incoherent fit then requires a $|\sigma|$ -value which is 3.5 std away from zero in Fig. 8 and 2.5 std in Fig. 9.

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*) The $\pi^+ \pi^- \pi^0 n$ final state is difficult to study.

Coherent $\rho + \omega$ fits to the same samples show no improvement and return $\text{Im } \sigma$ values consistent with zero. Note that the resolution is now taken to be 30 MeV, instead of 20 MeV as in reaction (1). This, of course, makes the fit insensitive to the rapidly varying interference term in expression (12). We have tried to use the narrow ω as a calibration signal to determine the true resolution in the sample of Fig. 8. The result is a resolution of

$$(25 \pm 14) \text{ MeV}.$$

Thus we may conclude that 30 MeV is a good estimate, and that the total fit is very insensitive to errors in the resolution.

We conclude that Figs. 8 to 9 require an ω amplitude, different from zero by three standard deviations.

4. ρ MASS AND WIDTH, t -DEPENDENCE

The results of the analysis of reactions (1) to (3) indicate that the ρ^0 mass is not appreciably lowered in the $\rho + \omega$ fits compared with the simple ρ fits. At the most, this shift is (4.2 ± 5.3) MeV. In the following we shall take the $\rho + \omega$ fits [coherent in reaction (1), incoherent in reactions (2) and (3)] as representative of the ρ^0 mesons.

There is an appreciable increase in the ρ^0 mass with t , when going from Fig. 4 to Fig. 5. This increase is, from Table 3, (29 ± 7) MeV. On the other hand, the ρ^0 widths are approximately equal: the difference in $\Gamma_{\rho^0}^0$ in going from Fig. 4 to Fig. 5 is (21 ± 30) MeV. The mean width is then

$$\Gamma_{\rho^0}^0 = (129 \pm 15) \text{ MeV}. \quad (14)$$

If one considers Fig. 6 as being composed of two peaks of the above width, but shifted (29 ± 7) MeV relatively in central mass, one would expect Fig. 6 to have the width (158 ± 17) MeV. The actual width found in the analysis of Fig. 6 is $(150 \pm 14) \text{ MeV}$. Thus the broadening of the ρ^0 peak, observed in Fig. 6, does not require further explanation.

On the other hand, the ρ^0 widths found in the analysis of reactions (2) and (3), Figs. 7 to 9, and Table 4, are considerably larger than Eq. (14). A possible reason for this may be that there are systematic errors in the

energy scale. Figs. 7 and 9 are composed of data from five experiments at eleven energies, and Fig. 8 (which has the largest width) is composed of eight experiments at fourteen energies. If the systematic energy errors are sufficiently randomly distributed among the different experiments, the central masses would not be affected but the widths would be broadened. Indeed, the central masses of the ω peaks in Figs. 8 and 9 agree very well with the known ω mass, as mentioned before. If the ω mass is used for calibration, the mass scale in Fig. 8 is accurate to within (0.4 ± 7) MeV (where the 7 MeV error may be absorbed into the 30 MeV resolution). Correspondingly, the mass scale in Fig. 9 is correct to within (1.4 ± 7) MeV.

Using the same argument, the narrower width of the ρ^0 meson in Figs. 4 and 5 is due to the lower number of experiments -- three experiments at five energies. With so few experiments, the systematic energy errors are not randomly distributed but may well conspire. Thus the ρ peak (and the ω peak) may appear with its natural width but at the wrong mass. We conclude that Figs. 4 and 5 are more reliable in the width determination and less reliable in the mass determination, whereas the opposite case applies to Figs. 7 to 9.

Since the t -shift in the central mass of the ρ seems to produce a broadening of the ρ peak, the best samples for study of the ρ width should be the samples of narrowest t -range, Figs. 10 to 12 for reactions (4) and (5) and Fig. 22 for reaction (6). Figure 12 can be neglected, because it is composed of Figs. 10 and 11, which are $(11 + 4)$ MeV different in central mass. Unfortunately, we have Figs. 10 and 11 only in 30 MeV bins, with very little information in the ρ wings. This makes the background very strongly correlated to the width, in particular in Fig. 10, where the statistics also is very poor. The error in Γ_ρ , of course, contains the background correlation, but since the background function is arbitrary, it may tend to reproduce part of the ρ peak without showing up as a correlation in the Γ_ρ error. This is well demonstrated by the subtracted histogram in the lower part of Fig. 10: although the best fit ρ^+ Breit-Wigner has been subtracted from the experimental points, a broad peak -- ρ or background -- persists.

The ρ^+ samples with higher t , Figs. 13, 16 and 19, definitely require a much larger Γ_{ρ^+} (cf. Table 4). The change in Γ_{ρ^+} from $t < 5$ to $t < 10$ is (60 ± 25) MeV, whereas the change in M_{ρ^+} is only (5.8 ± 6.0) MeV. The simplest explanation is, again, that Fig. 10 is unreliable, due to too low statistics. For still higher t values, Γ_{ρ^+} seems to shrink somewhat, (15 ± 23) MeV from Fig. 13 to Fig. 16, while the mass oscillates. The statistically most significant samples are Figs. 16 and 19, which agree in width. To get a very crude estimate of Γ_{ρ^+} , let us assume that M_{ρ^+} of the sample $t < 30$ (from Table 4, 770 ± 4 MeV) is the weighted mean*) of M_{ρ^+} of the sample $t < 5$ (from Table 4, 756 ± 3 MeV) and the sample $5 < t < 30$ (unknown). This gives us the unknown M_{ρ^+} , and thus the total mass shift. If we now subtract this mass shift from the observed Γ_{ρ^+} of the sample $t < 30$, we obtain

$$\Gamma_{\rho^+} = (125 \pm 24) \text{ MeV} . \quad (15)$$

The error does not contain the uncertainties introduced by the simplifying assumptions. On the other hand, we have added mass and width errors as if they were uncorrelated. This they are not, and thus there is some compensation.

The ρ^- width in Fig. 11 is more reasonable than the corresponding ρ^+ width, but there are still effects of strong background -- Γ_{ρ^-} correlation, which do not enter the Γ_{ρ^-} error. That this 146 MeV width cannot be true is best demonstrated by Fig. 20. Here a sample of much larger statistics, in 10 MeV bins, and containing the t -range of Fig. 11 is analysed, with the result 135 MeV. Thus the true Γ_{ρ^-} must be 135 MeV or smaller.

The t -dependence of Γ_{ρ^-} in Figs. 11, 14, 17, and 20 seems quite logical. From Fig. 11 to Fig. 14, Γ_{ρ^-} increases by (9 ± 14) MeV and M_{ρ^-} by (9 ± 3) MeV. At higher t , M_{ρ^-} remains essentially stationary

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*) The weights are 0.40 for $t < 5$ and 0.60 for $5 < t < 30$, as given by the total number of ρ events in these t -bins in two experiments^{8,11}).

or decreases slightly. The added ρ mesons increase the height of the peak without broadening it, and tend to decrease the width at half-height, as observed. Carrying out the same kind of manipulation as in the ρ^+ case^{*)}, we arrive at a natural ρ^- width of

$$\Gamma_{\rho^-} = (123 \pm 10) \text{ MeV} . \quad (16)$$

It is interesting to compare this width with that of reaction (6), Fig. 22, and Table 4. The sample in Fig. 22 has all the properties one could wish for for a good width determination: large statistics, very narrow t-band (and thus very little mass shift broadening), and sufficient wing regions (to constrain the background function). Its width is

$$\Gamma_{\rho^-} = (131 \pm 7) \text{ MeV} . \quad (17)$$

All the ρ widths (14) to (17) are equal within the errors quoted, and we therefore conclude that they may be combined into a unique width

$$\Gamma_{\rho} = (128 \pm 5) \text{ MeV} . \quad (18)$$

Turning now to the central mass values, plotted versus t in Fig. 23, some points deserve to be restated: the reasons for discrediting the Γ_{ρ} values in the $t < 5$ samples (Figs. 10 and 11) do not discredit the mass determinations. And, the reason for determining Γ_{ρ^0} from Figs. 4 and 5 but not from Figs. 7 to 9, now becomes a reason for trusting the mass values in Figs. 7 to 9, but distrusting the absolute scale of mass values in Figs. 4 to 6. Still, it is very disquieting that the M_{ρ^0} values in the $t < 10$ samples (Figs. 4 and 8) should disagree by as much as 6.5 std!

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^{*)} Using one experiment only¹²⁾.

To make things worse, suppose that it really is the incoherent $\rho + \omega$ fits in Table 3 that we should trust, and suppose further that the M_ω values found can be used for a mass scale renormalization; then the ρ masses would go down, instead of up, to about 742-746 MeV, and the disagreement between Figs. 4 and 8 as well as between Figs. 6 and 9, would exceed 9 std.

Such large discrepancies in the ρ^0 mass, when produced in the two reactions (1) or (2), certainly call for an explanation. Many possibilities may be considered: the ρ meson may actually consist of two nearby resonances which have different production cross-sections in reactions (1) and (2), or the production mechanisms may be different, or there may be effects of background interference which produce the mass shift.

It appears from Fig. 23 that three independent ρ samples, reactions (1), (4) and (5), exhibit strongly t -dependent ρ masses, particularly for low t . Note that the independent ρ^- mass from reaction (6) falls nicely in place where predicted by the data from reaction (5) [see postscript].

The reason for this t -dependence may again be complicated production dynamics, final-state interactions, background interference, or double ρ mesons. (The two ρ mesons only need to have cross-sections $d\sigma/dt$ of different slope in order to reproduce the apparent mass shift with t of the combined peak. Note that this argument for a double ρ meson is completely independent of the previous argument).

There is one more surprising fact in Fig. 23 that needs to be explained: the ρ^+ and ρ^- mesons have different masses at low t . At $t < 5$ this mass split is almost a 3 std effect; at $t < 10$ it is over 3 std.

From OPEM one would expect the ρ mesons to be produced as cleanly as possible at low t , and free from final-state interactions. Thus, if there were charge asymmetric final-state interactions, they might split the ρ^+ and ρ^- masses at higher t , but at low t the OPEM and CPT invariance certainly requires $M_{\rho^+} = M_{\rho^-}$. (In both reactions (4) and (5), a π^0 is exchanged).

The present facts then require more complicated dynamical models than the OPEM. If we have two ρ mesons instead of one, they may require different production mechanisms, and this may be a reason for the charge asymmetry.

For the time being, if one wants to consider the ρ peak as due to one resonance only, and if one requires that $M_{\rho^+} = M_{\rho^-}$, the best mass values of the charged ρ meson are given by the combined $\pm$ histograms, Figs. 12, 15, 18, and 21. $M_{\rho^\pm}$ increases by 9 MeV from Fig. 12 to Fig. 15, but thereafter remains constant (cf. Table 4). Although we see no reason to maintain that this constant value is the "true mass" of the $\rho^\pm$ meson, at least it furnishes a convention until the problems in the ρ region are better understood.

The largest samples are those of Figs. 15 and 21, which yield equal mass values. Rounded off, we suggest

$$M_{\rho^\pm} = (774 \pm 2) \text{ MeV.} \quad (19)$$

The mass difference between ρ^0 from reactions (2) and (3) and $\rho^\pm$ from reactions (4) and (5) is also completely constant from $t < 10$ to $t < 30$ (cf. Table 4):

$$M_{\rho^0} - M_{\rho^\pm} = (5.7 \pm 2.2) \text{ MeV,} \quad (20)$$

and

$$M_{\rho^0} = (780 \pm 2) \text{ MeV.} \quad (21)$$

If the double ρ interpretation is true, the masses of the two ρ mesons should not be far from the limits

$$M_{\rho_1} < 750 \text{ MeV}$$

$$M_{\rho_2} > 780 \text{ MeV.}$$

5. STRUCTURE UNDER THE ρ MESON

All the histograms from reactions (2) to (6) in Figs. 7 to 22 have a lower part, showing an error bar diagram. These represent the true experimental background when the fitted ρ peak has been subtracted. The error bars correspond to the full histograms.

None of the structure in these subtracted diagrams is very significant. In the following we shall discuss only such structure, or lack of structure, as applies to the whole set of data.

In comparing the neutral data (Figs. 7 to 9) with all the charged data, no evidence for the σ meson is seen. Neither do any new $I = 0$ peaks appear. The fluctuations in the 720 MeV region (S^0 meson) are not different from the fluctuations in the charged histograms, notably Figs. 13, 14, 17 and 20. We can therefore not support the S^0 meson.

In the ρ peak region, however, an asymmetry is evident in all histograms with $t < 10$ or higher t cuts. Thus neglecting Fig. 7 (although it has a narrow hole around 745 MeV) and Figs. 10, 11, 12 and 16 (in 30 MeV bins), the subtracted histograms of the remaining figures generally show a hole in the 730-760 MeV region followed by a peak in the 760-800 MeV region. This is clear in the neutral samples, Figs. 8 and 9, the negative samples, Figs. 14, 17 and 20, and to some extent in Fig. 22. The positive samples, Figs. 13 and 19, are smaller and show more complicated fluctuations. Added to the negative samples in Figs. 15, 18 and 21, the asymmetry persists.

The question now arises immediately, whether the ω peak, discovered in Section 3, really is the ω , since there is asymmetry of the same kind in the charged channels. In this case, one should first remember that the simple ρ fits to Figs. 8 and 9 really left a very large enhancement in the ω region unfitted. What is shown in the lower parts of Figs. 8 and 9 is the background, when not only ρ but also ω is subtracted, and the remainder still shows as much asymmetry as in Fig. 15, for example.

Thus, it seems to us that there are two possibilities:

- i) ω and other structure with $I = 1$;
- ii) $I = 1$ structure (second ρ or background interference).

Having discussed the existence of the ω , which should appear only in the neutral histograms with given mass and width, we need not distinguish here between (i) and (ii), which both require some $I = 1$ structure. The mass of a new ρ meson would be roughly 790 MeV, in agreement with the independent conclusions in the previous chapter. Nothing permits us to distinguish, however, between a nonresonant background bump and a resonance, either one coherent with the ρ meson.

In the δ -meson region (960 MeV) we see no indication of a 2π decay. Only the positive sample at high t , Fig. 19, shows a peak in this region, but it is massively contradicted by the corresponding negative sample, Fig. 20. Even in the total charged sample, Fig. 21, the effect has completely disappeared.

We have not tried to interpret the region above 1 GeV where our statistics are very poor.

6. CONCLUSIONS

The neutral $\pi\pi$ histograms from reactions (1) to (3) indicate the presence of a narrow peak in the ω -meson region. The amplitude of an ω peak is about 1 std different from zero in reaction (1) and about 3 std in reactions (2) and (3).

There is, however, some uncertainty as to whether this peak really is an ω , because the charged $\pi\pi$ histograms generally exhibit an asymmetric ρ peak, with an excess in the ω region. We think that one probably has both ω and some other structure in this region, because the ρ asymmetry is about equal in charged and neutral samples, when the best fit ω has been subtracted from the neutral samples. Without this subtraction, the asymmetry in the ρ peak is much larger in the neutral samples than in the charged ones.

The neutral ρ -meson mass is very much smaller in reaction (1) than in reactions (2) and (3). In the samples with $t < 10$, where the signal-to-background ratio is particularly large, the ρ^0 masses are 28 MeV different with 6.5 std. This effect could be due either to a second ρ meson, which has a different ratio of production cross-sections than the ρ meson in the above reactions, or to interference with a complicated background, or to differences in the production process.

The ρ -meson masses show a marked t -dependence at low t values (see Fig. 23). The explanation for this effect may again be due either to a second ρ meson, which has a different $d\sigma/dt$ slope than the ρ -meson, or to background interference or to final-state interactions.

The ρ^+ and ρ^- masses at low t are 12-15 MeV different by 3 std. The simple OPEM is charge symmetric, and is thus untenable in view of this effect. From simple OPEM one could have expected a mass split at higher t , due to final state interactions. Figure 23 shows, however, that the mass split goes to zero in higher t samples.

If we treat the ρ peak as being due to one resonance only, the t -dependence of the masses and the $\rho^+ - \rho^-$ mass split make it difficult to define "the ρ mass". For the time being, we propose to use the mass value at high t , where both the t -dependence and the mass split have vanished. Thus we find the mass values given in (19), (20) and (21).

The width of the ρ meson is less precisely determined. The raw values range from 120 MeV to 170 MeV. We find that if corrections due to the t -dependence of the mass are introduced, all widths are consistent with the value (18), and no width t -dependence.

The structure under the ρ meson shows no significant evidence for the σ meson, the S^0 meson, the δ meson, nor any other object, except for the ρ peak asymmetry.

7. POSTSCRIPT: INDEPENDENT ρ^- DATA

After the completed analysis a large sample of ρ^- have been communicated to us privately by J.P. Baton and G. Laurens, Saclay. The number of events is comparable to our total sample of ρ^- , and the experiment has the virtue of having been done all at one momentum, 2.77 GeV/c, reaction (5). We have fitted the data at $t < 5$, $t < 10$ and $t < 13$ with our program. The resulting ρ^- masses were $(775.5 \pm 1.9)\text{MeV}$, $(779.6 \pm 1.8)\text{MeV}$, and $(779.4 \pm 1.7)\text{MeV}$, respectively. Comparing with Fig. 23, these masses are somewhat heavier than those in Fig. 23. We can think of two reasons for this effect:

- i) There may be a systematic error in the mass scale of this one experiment, relative to the compiled histogram in which the systematic errors of many experiments have cancelled each other.
- ii) If the complications in the ρ region are due to ρ interference with either a second ρ or with background, the relative phase may be energy dependent. Then one large experiment at one energy may show much more peak asymmetry than does a compilation of experiments at several energies. Indeed, the reason why a simple ρ fit to the Saclay data wants such a high mass, is a marked peak asymmetry of the kind pointed out in Section 5.

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REFERENCES

- 1) H.R. Blieden, D. Freytag, J. Geibel, A.R.F. Hassan, W. Kienzle, F. Lefebvres, B. Levrat, B.C. Maglič, J. Seguinot and A.J. Smith, Nuovo Cimento 43, 71 (1965) (Fig. 3d).
- 2) M.A. Abolins, D.D. Carmony, R.L. Lander, Ng. H. Xuong, and P.M. Yager, Proc. Second Topical Conf. on Resonant Particles, Athens, Ohio, 1965 (Ed. B.A. Munir, Ohio University, 1965), p. 198.
- 3) Aachen-Berlin-Birmingham-Bonn-Hamburg-London(I.C.)-München Collaboration, Nuovo Cimento 35, 659 (1965).
- 4) C. Alff-Steinberger, D. Berley, D. Colley, N. Gelfand, D. Miller, U. Nauenberg, J. Schultz, T.H. Tan, H. Brugger, P. Kramer and R. Plano, Phys.Rev. 145, 1072 (1966), and private communications from N. Gelfand.
- 5) F. James and M. Roos, MINUIT, A program to minimize a general function of n variables and calculate the true error, CERN 6600 Computer Program Library, D 506 (1967, unpublished).
- 6) M.A. Abolins, R.L. Lander, W.A. Mehlhop, Ng. H. Xuong and P.M. Yager, Phys.Rev. Letters 11, 381 (1963).
- 7) G. Goldhaber, J.L. Brown, S. Goldhaber, J.A. Kadyk, B.C. Shen and G.H. Trilling, Phys.Rev. Letters 12, 336 (1964), and private communications from G. Goldhaber.
- 8) Aachen-Berlin-Birmingham-Bonn-Hamburg-London(I.C.)-München Collaboration, Physics Letters 10, 226 and 229 (1964), Phys.Rev. 138, B 897 (1965), and private communications from M. Deutschmann.
- 9) F.E. James and H.L. Kraybill, Phys.Rev. 142, 896 (1966), and private communications from F.E. James.
- 10) D.D. Carmony, D.N. Hoa, R.L. Lander, Ng. H. Xuong, P. Yager, Proc. XII Int. Conf. on High-Energy Physics, Dubna 1964 (Atomizdat, Moscow, 1966), 1, 486.
- 11) P. Daronian, A. Daudin, M.A. Jabiol, C. Lewin, C. Kochowski, B. Ghilini, S. Mongelli and V. Picciarelli, Nuovo Cimento 41, 503 (1966), and private communications from M.A. Jabiol.
- 12) L.D. Jacobs, UCRL-16877 (unpublished thesis, 1966), and private communications.

- 13) E. West, J.H. Boyd, A.R. Erwin and W.D. Walker, Phys.Rev. 149, 1089 (1966).
- 14) V. Hagopian, W. Selove, J. Alitti, J.P. Baton and M. Neveu-René, Phys.Rev. 145, 1128 (1966).
V. Hagopian and Yu-Li Pan, Phys.Rev. 152, 1183 (1966), and private communications from V. Hagopian.
- 15) D.D. Allen, G.P. Fisher, G. Godden, J.B. Kopelmann, L. Marshall and R. Sears, Phys.Rev. Letters 17, 53 (1966), and private communications from L. Marshall.
- 16) Bari-Bologna-Firenze-Orsay-Collaboration, private communications from J. Loskiewicz, Nguyen Huu Khanh, and J. Quinquard.
- 17) D. Huwe, E. Marquit, F. Oppenheimer, W. Schultz and J.R. Wilson, Physics Letters 24B, 252 (1967), and private communications from W. Schultz.
- 18) Saclay-Orsay-Bari-Bologna Collaboration, Nuovo Cimento 29, 515 (1963).
- 19) L.J. Gutay, P.B. Johnson, F.J. Loeffler, R.L. McIlwain, D.H. Miller, R.B. Willmann and P.L. Csonka, Phys.Rev. Letters 18, 142 (1967), and private communications from D.H. Miller.
- 20) T.C. Bacon, W.J. Fickinger, D.G. Hill, H.W.K. Hopkins, D.K. Robinson and E.D. Salant, Proc. Second Topical Conference on Resonant Particles, Ohio University (Ed. B.A. Munir, Athens, Ohio, 1965), p.129, and private communications from D.K. Robinson.
- 21) S.M. Flatté, D.O. Huwe, J.J. Murray, J. Button-Shafer, F.T. Solmitz, M.L. Stevenson and C. Wohl, Phys.Rev. 145, 1050 (1966). The errors in R may have been somewhat larger if Flatté et al. would have kept M_ρ and Γ_ρ free in their coherent fit.

Table 1

Experiments compiled

Reference	Reaction	Beam momenta (GeV/c)	Laboratories
1	6	4.5	CERN
2, 6, 10	1, 4	3.43, 3.54	UC San Diego - Michigan
3, 8	1, 4	4.0	Aachen - Berlin - Birmingham - Bonn - Hamburg - London (I.C.) - München
4	1, 4	2.35, 2.62, 2.90	Columbia - Rutgers
7	1	3.65	LRL
9	4	2.08	Yale - BNL
11	4	1.6	Saclay - Bari
12	2, 5	2.05, 2.17, 2.36, 2.60, 2.86, 3.22	LRL
13	2, 5	2.1	Wisconsin
14	2, 5	2.14, 2.75, 3.00	Penn. - Saclay - LRL
15	5	1.7	Colorado
16	3	5.0	Bari - Bologna - Firenze - Orsay
17*)	2	2.36	Colorado
18	2, 5	1.59	Saclay - Orsay - Bari - Bologna
19	2, 5	2.7	Purdue
20	2, 3	1.7	BNL

*) Includes the LRL 2.36 GeV/c experiment, Ref. 12).

Table 2

Number of ρ and ω events in selected t bands

t	$\sigma_{\rho}(\pi^+p \rightarrow \rho\Delta^{++})$				$\sigma_{\omega}(\pi^+p \rightarrow \omega\Delta^{++})$				$\sigma_{\omega}/\sigma_{\rho}$			
	82	461	303	846	39	114	65	218	0.48	0.25	0.21	0.26 ± 0.02
< 10												
$10 < t < 30$	143	242	112	497	199	308	122	629	1.39	1.27	1.09	1.26 ± 0.08
< 30	225	703	415	1343	238	422	187	847	1.06	0.60	0.45	0.63 ± 0.03
Reference	^a 4	2	3	2+3+4	4	2	3	2+3+4	4	2	3	2+3+4

Table 3

Parameters for $\rho + \omega$ fits to reactions (1)-(3)

t	< 10				10 < t < 30				< 30			
Reaction	1				1				1			
Figure Fit	1 no ω	ω incoherent	ω coherent	2, 3	2 no ω	ω incoherent	ω coherent	3 no ω	ω incoherent	ω coherent	2, 3	
M_ρ (MeV)	755.0 ± 3.3	753.0 ± 3.8	750.8 ± 4.1 $- 5.3$	779.0 ± 1.4	782.4 ± 5.5 $- 5.1$	782.5 ± 5.0 $- 5.9$	779.2 ± 6.0 $- 5.6$	771.7 ± 3.0	771.1 ± 3.1 $- 2.7$	768.3 ± 3.8 $- 3.2$	779.4 ± 1.7	
	148 ± 12 $- 11$	150 ± 13	141 ± 14 $- 23$	172 ± 7	149 ± 19 $- 16$	148 ± 16 $- 20$	120 ± 20 $- 18$	167 ± 12 $- 11$	163 ± 12 $- 10$	150 ± 14 $- 12$	161 ± 7	
		783.4 (795 ± 10) $- 9$	783.4 (785 ± 15) $- 11$	783.4 (783 ± 7) $- 6$		783.4 (805 ± 19) $- 17$	783.4 (774 ± 15) $- 13$		783.4 (809 ± 12) $- 8$	783.4 (792 ± 19) $- 11$	783.4 (782 ± 7)	
M_ω (MeV)												
Γ_ω^0 (MeV)												
Re σ	0	0.136 ± 0.055 $- 0.093$	0.047 ± 0.048 $- 0.040$	0.121 ± 0.027 $- 0.035$	0	0.004 ± 0.083 $- 0.091$	-0.39 ± 0.31 $- 0.06$	0	0 ± 0.073	-0.031 ± 0.031 $- 0.036$	-0.116 ± 0.047 $- 0.032$	
Im σ	0	0	0.046 ± 0.060 $- 0.049$	0	0	0	0.05 ± 0.08	0	0	0.048 ± 0.035 $- 0.032$	0	
Resolution (MeV)	0	20	20	30	0	20	20	0	20	20	30	
Background %	21	21	22	14	41	41	45	23	24	26	24	
$ \sigma $	0	0.14 ± 0.06 $- 0.09$	0.07 ± 0.07 $- 0.05$	0.12 ± 0.03 $- 0.04$	0	0.0 ± 0.1	0.39 ± 0.06 $- 0.35$	0	0 ± 0.07	0.06 ± 0.05 $- 0.04$	0.12 ± 0.03 $- 0.05$	
R %		7 ± 6 $- 11$	1.7 ± 3.5 $- 2.6$	73		0 ± 1	12 ± 4 $- 22$		0 ± 1.2	0.5 ± 0.8 $- 0.7$		
χ^2	66.2	63.5	55.0	76.1	75.0	74.5	67.3	75.9	76.2	69.4	58.4	
$\langle \chi^2 \rangle$	68	67	60	73	90	89	82	90	89	82	69	
C.L. %	54	60	66	38	79	86	87	85	83	83	81	
Events	1250				1127				3236			
References	4, 7, 8				4, 7, 8				4, 6, 7, 8			
					12, 13, 14, 16, 17, 18, 19, 20,				12, 14, 16, 17, 19			

Table 4

Parameters for rho meson fits to reactions (2) - (6)

t	< 5			< 10			< 20			< 30		
Q Reaction Figure	+	-	$\pm$	+	-	$\pm$	+	-	$\pm$	+	-	$\pm$
	4 10	5 11	4, 5 12 7	4 13	5 14	4, 5 15 8	4 16	5 17	4, 5 18 9	4 19	5 20	4, 5 21 9
M_ρ (MeV)	755.5 ± 3.4			761.3 ± 4.1			776.0 ± 3.5			769.8 ± 4.0		
ΔM_ρ (MeV)	767.2 ± 2.1			776.1 ± 2.0			777.3 ± 2.4			773.5 ± 1.8		
Γ_ρ (MeV)	764.3 ± 1.7			773.3 ± 1.6			777.3 ± 2.2			773.7 ± 1.9		
Resolution (MeV)	776.2 ± 1.9			779.0 ± 1.4			776.8 ± 2.2			779.4 ± 1.7		
Background %	776.2 ± 2.2			776.1 ± 2.0			777.3 ± 2.2			773.5 ± 1.8		
	767.2 ± 4.0			776.1 ± 2.0			777.3 ± 2.2			773.5 ± 1.8		
	11.7 ± 4.0			14.8 ± 4.6			1.3 ± 4.1			3.7 ± 4.4		
	106 ± 13			166 ± 19			151 ± 16			149 ± 22		
	146 ± 8			152 ± 7			143 ± 9			135 ± 8		
	143 ± 6			152 ± 8			143 ± 7			146 ± 7		
	146 ± 13			149 ± 7			30			30		
	30			30			30			30		
	35.5			23.8			31.0			22.8		
	15.1			25.8			27.3			34.1		
	8.7			13.6			24.9			26.2		
Events	1275			1947			3209			2522		
χ^2	4255			6830			5275			7879		
	4934			15160			64.7			58.4		
	6.4			63.8			52.7			54.9		
	67.0			47.2			59			60.2		
	69			69			67			60		
	54			65			56			47		
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	54			65			56			47		
	54			6								

Figure captions

- 1 : ρ^0 and arbitrary background fitted to reaction (1) , $t < 10$.
- 2 : ρ^0 and arbitrary background fitted to reaction (1) , $10 < t < 30$.
- 3 : ρ^0 and arbitrary background fitted to reaction (1) , $t < 30$.
- 4 : Coherent $\rho^0 + \omega$ and arbitrary background fitted to reaction (1) , $t < 10$.
- 5 : Coherent $\rho^0 + \omega$ and arbitrary background fitted to reaction (1) , $10 < t < 30$.
- 6 : Coherent $\rho^0 + \omega$ and arbitrary background fitted to reaction (1) , $t < 30$.
- 7 : ρ^0 and arbitrary background fitted to reaction (2)-(3), $t < 5$. Below, experimental histogram less fitted ρ^0 .
- 8 : Incoherent $\rho^0 + \omega$ and arbitrary background fitted to reaction (2)-(3), $t < 10$. Below, experimental histogram less fitted $\rho^0 + \omega$.
- 9 : Incoherent $\rho^0 + \omega$ and arbitrary background fitted to reaction (2)-(3), $t < 30$. Below, experimental histogram less fitted $\rho^0 + \omega$.
- 10 : ρ^+ and arbitrary background fitted to reaction (4) , $t < 5$. Below, experimental histogram less fitted ρ^+ .
- 11 : ρ^- and arbitrary background fitted to reaction (5) , $t < 5$. Below, experimental histogram less fitted ρ^- .
- 12 : ρ^+ and arbitrary background fitted to reaction (4)-(5), $t < 5$. Below, experimental histogram less fitted ρ^+ .
- 13 : ρ^+ and arbitrary background fitted to reaction (4) , $t < 10$. Below, experimental histogram less fitted ρ^+ .
- 14 : ρ^- and arbitrary background fitted to reaction (5) , $t < 10$. Below, experimental histogram less fitted ρ^- .
- 15 : ρ^+ and arbitrary background fitted to reaction (4)-(5), $t < 10$. Below, experimental histogram less fitted ρ^+ .
- 16 : ρ^+ and arbitrary background fitted to reaction (4) , $t < 20$. Below, experimental histogram less fitted ρ^+ .
- 17 : ρ^- and arbitrary background fitted to reaction (5) , $t < 20$. Below, experimental histogram less fitted ρ^- .
- 18 : ρ^+ and arbitrary background fitted to reaction (4)-(5), $t < 20$. Below, experimental histogram less fitted ρ^+ .
- 19 : ρ^+ and arbitrary background fitted to reaction (4) , $t < 30$. Below, experimental histogram less fitted ρ^+ .
- 20 : ρ^- and arbitrary background fitted to reaction (5) , $t < 30$. Below, experimental histogram less fitted ρ^- .
- 21 : ρ^+ and arbitrary background fitted to reaction (4)-(5), $t < 30$. Below, experimental histogram less fitted ρ^+ .
- 22 : Experimental histogram from reaction (6), $5 < t < 8.5$, less fitted ρ^- .
- 23 : ρ masses versus maximum momentum transfer t .

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6.

FIG. 7

FIG.9

FIG. 10

FIG.11

FIG.12

FIG. 13

FIG. 14

FIG. 15

FIG.16

FIG. 17

FIG.19

FIG.20

FIG. 21

FIG.22

FIG.23