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COUPLING CONSTANTS FOR 2^+ AND 1^- MESON NONETS AND
THE 0^- MESON OCTET IN A MODEL WITH INFINITE MULTIPLETS

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ABSTRACT

We investigate the possibility of assigning 2^+ and 1^- meson resonances to states of one infinite unitary representation of the group $SL(2, \mathbb{C})$. We compute vertex functions for the decays of these resonances into two pseudoscalar mesons or one pseudoscalar and one axial vector meson.

1. INTRODUCTION

Non-compact groups and their infinite dimensional unitary representations have now so often been proposed as a tool for classifying the spectrum of strongly interacting particles and resonances that it seems quite interesting to compare the consequences of one such model with experimental data. The idea we are pursuing in this paper is to consider the vector meson nonet and the spin 2^+ nonet as members of one infinite multiplet. Indeed, we generalize this ansatz and consider the octet and singlet components as belonging to independent infinite multiplets. Even in this case our main result will be a negative statement: the 2^+ octet and the 1^- octet cannot belong to one infinite multiplet.

The symmetry group for which we obtain this result is the direct product of Fronsdal's ¹⁾ spin independence group $SL(2,C)$ and the unitary symmetry group $SU(3)$. Both groups commute with the translations. Therefore, a mass formula can be derived with a perturbative treatment of additive terms in the mass operator which break the symmetry. The unitary symmetry can be violated as usual yielding a Gell-Mann - Okubo expression. To break down the $SL(2,C)$ symmetry in a Lorentz invariant fashion we add a term proportional to

$$W^\mu W_\mu$$

where

$$W^\mu = \epsilon^{\mu\nu\lambda\sigma} p_\nu S_{\lambda\sigma}$$

p_ν is the momentum and $S_{\lambda\sigma}$ are the generators of the group $SL(2,C)$. This term yields a contribution to the mass proportional to $S(S+1)$ where S is the dynamical spin of the particle.

2.

The group $SL(2, C) \otimes SU(3)$ is a subgroup of $SL(6, C)$ and the model we are studying is constructed in complete analogy to Fronsda's theory of relativistic $SU(6)$ invariance ¹⁾. The methods we are going to use have been developed in some preceding papers ^{2), 3)}. They simplify considerably in the more elementary case of $SL(2, C)$ symmetry. Since some of the vertex functions can be given explicitly as certain transcend functions, the $SL(2, C)$ model serves as a good example to study the general problems involved in this class of theories. Renormalization problems do, however, not occur, since the infinite representations of $SL(2, C)$ are non-degenerate ³⁾. On the other hand we must be very careful in taking over results from the subgroup model to the $SL(6, C)$ model. As a consequence of the infinite dimension of the representations, characteristic difficulties may arise, which have no counterpart for finite dimensional representations.

The spin 2^+ meson nonet we have in mind is the one defined and investigated by Glashow and Socolow ⁴⁾. The resonances are assigned as

K^{**}	(1410),	$T = \frac{1}{2}$,	width about 100 MeV.
A_2	(1320),	$T = 1$,	" " 85 MeV,
f	(1250),	$T = 0$,	" " 100 MeV,
f'	(1500),	$T = 0$,	" " 100 MeV.

The mixing angle is of the order of 20° .

2. INFINITE MULTIPLETS AND THEIR STATES

In order to fit spin 2^+ and spin 1^- particles into one representation of $SL(2, C)$ we can proceed in two ways. We can let the ladder either start with a 0^+ particle

$$0^+, 1^-, 2^+, \dots \quad (2n)^+, (2n+1)^-, \dots$$

or with a 1^- particle

$$1^-, 2^+, \dots, (2n)^+, (2n+1)^-, \dots$$

The lowest spin of such a multiplet is usually denoted as k_0 . We call the meson multiplet with parity $(-1)^S$ the σ ladder. If $k_0 = 0$ (1) we denote the ladder in particular $\sigma(1)$ ($\sigma(2)$). The parameter k_0 is connected with the invariant "m" by

$$k_0 = \left| \frac{m}{2} \right|.$$

This invariant is therefore fixed up to an unessential sign. The second Casimir invariant " ρ " will be chosen equal to zero in this paper. This choice is supported by a convenient definition of the parity operator⁵⁾. Besides the σ ladders we need a π ladder which starts with 0^- particles,

$$0^-, 1^+, 2^-, \dots, (2n)^-, (2n+1)^+, \dots$$

The invariant " ρ " will again be chosen as zero. We remind the reader that the $SL(6, C)$ multiplets starting with an $SU(6)$ singlet and containing a 35^- plet possess a $\sigma(1)$ singlet ladder, a $\sigma(2)$ octet ladder and a π octet ladder which have members in the lowest $SU(6)$ multiplets.

We are interested in the $\sigma\pi\pi$ coupling and will compute the vertices

$S \rightarrow P + P$	$S \rightarrow A + P$	$\sigma(1)$	
$V \rightarrow P + P$	$V \rightarrow A + P$	$\sigma(1)$	$\sigma(2)$
$T \rightarrow P + P$	$T \rightarrow A + P$	$\sigma(1)$	$\sigma(2)$

Table 1 gives the definition of coupling constants due to Lorentz invariance and unitary symmetry.

4.

In order to construct particle states we apply the technique of generalized tensors explained in Ref. ²⁾. Since the model under discussion is so simple and this technique is correspondingly elementary, we take the opportunity to recall it once more.

We consider homogeneous functions of the spin variables $\xi_1, \xi_2, \bar{\xi}_1, \bar{\xi}_2$, and of the dependent variables

$$\xi^i = \varepsilon^{ij} \xi_j \quad \varepsilon^{ij} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}^{ij}$$

The homogeneity of the functions can be expressed by the Casimir invariants of the representation "m" and "p" as

$$\lambda = -\frac{m}{2} + i\frac{p}{2} - 1 \quad \text{the homogeneity in } \xi$$

$$\mu = +\frac{m}{2} + i\frac{p}{2} - 1 \quad \text{the homogeneity in } \bar{\xi}$$

In addition we make use of the fact that

$$\xi_1 \bar{\xi}_1 + \xi_2 \bar{\xi}_2$$

is invariant under SU(2). At the end we introduce the Gelfand-Neumark variables

$$x = \frac{\xi_1}{\xi_2}, \quad \bar{x} = \frac{\bar{\xi}_1}{\bar{\xi}_2}$$

by extracting the powers $\xi_2^\lambda, \bar{\xi}_2^\mu$.

π ladder and $\sigma(1)$ ladder

$$m = p = 0,$$

$$\lambda = \mu = -1.$$

Spin 0 :

$$F(\xi) = c_0 \cdot \left(\sum_k \xi_k \bar{\xi}_k \right)^{-1},$$

$$f(x) = c_0 \cdot (1 + |x|^2)^{-1}.$$

Spin 1 :

$$F_{ij}(\xi) = c_1 \cdot 2^{-\frac{1}{2}} \cdot (\xi_i \bar{\xi}_j + \xi_j \bar{\xi}_i) \left(\sum_k \xi_k \bar{\xi}_k \right)^{-2},$$

$$f_{11}(x) = c_1 \cdot 2^{-\frac{1}{2}} \cdot 2x (1 + |x|^2)^{-2},$$

$$f_{12}(x) = c_1 \cdot 2^{-\frac{1}{2}} \cdot (1 - |x|^2) (1 + |x|^2)^{-2},$$

$$f_{22}(x) = c_1 \cdot 2^{-\frac{1}{2}} \cdot (-2\bar{x}) (1 + |x|^2)^{-2}.$$

Spin 2 :

$$F_{ijkl}(\xi) = c_2 \cdot 6^{-\frac{1}{2}} \cdot \sum_{\substack{\text{ix independent} \\ \text{permutations}}} \xi_i \bar{\xi}_j \xi_k \bar{\xi}_l \left(\sum_m \xi_m \bar{\xi}_m \right)^{-3},$$

$$f_{1111}(x) = c_2 \cdot 6^{-\frac{1}{2}} \cdot 6x^2 (1 + |x|^2)^{-3},$$

$$f_{1112}(x) = c_2 \cdot 6^{-\frac{1}{2}} \cdot 3x(1 - |x|^2) (1 + |x|^2)^{-3},$$

$$f_{1122}(x) = c_2 \cdot 6^{-\frac{1}{2}} \cdot (|x|^4 - 4|x|^2 + 1) (1 + |x|^2)^{-3},$$

$$f_{1222}(x) = c_2 \cdot 6^{-\frac{1}{2}} \cdot (-3\bar{x})(1 - |x|^2) (1 + |x|^2)^{-3},$$

$$f_{2222}(x) = c_2 \cdot 6^{-\frac{1}{2}} \cdot 6\bar{x}^2 (1 + |x|^2)^{-3}.$$

6.

If we choose

$$C_S = \left(\frac{2S+1}{\pi} \right)^{\frac{1}{2}}$$

we find that f , f_{11} and f_{1111} are normalized to one.

$G(2)$ ladder

$$m = -2, \quad \varphi = 0,$$

$$\lambda = 0, \quad \mu = -2.$$

Spin 1 :

$$F_{ij}(\xi) = C_1 \cdot \xi_i \xi_j \left(\sum_k \xi_k \bar{\xi}_k \right)^{-2},$$

$$f_{11}(x) = C_1 \cdot x^2 (1 + |x|^2)^{-2},$$

$$f_{12}(x) = C_1 \cdot x (1 + |x|^2)^{-2},$$

$$f_{22}(x) = C_1 \cdot (1 + |x|^2)^{-2}$$

Spin 2 :

$$F_{ijkl}(\xi) = C_2 \cdot \frac{1}{2} \cdot \sum_{\text{four indep. perm.}} \xi_i \xi_j \xi_k \bar{\xi}_l \left(\sum_m \xi_m \bar{\xi}_m \right)^{-3}$$

$$f_{1111}(x) = C_2 \cdot \frac{1}{2} \cdot 4x^3 (1 + |x|^2)^{-3}$$

$$f_{1112}(x) = C_2 \cdot \frac{1}{2} \cdot x^2 (3 - |x|^2) (1 + |x|^2)^{-3},$$

$$f_{1122}(x) = C_2 \cdot \frac{1}{2} \cdot 2x (1 - |x|^2) (1 + |x|^2)^{-3},$$

$$f_{1222}(x) = C_2 \cdot \frac{1}{2} \cdot (1 - 3|x|^2) (1 + |x|^2)^{-3},$$

$$f_{2222}(x) = C_2 \cdot \frac{1}{2} \cdot (-4|x|^2) (1 + |x|^2)^{-3}$$

The spinor quantities f_{ij} and f_{ijkl} are connected with the tensors $\hat{f}_\mu$, $\hat{f}_{\mu\nu}$ or the spherical harmonics $\tilde{f}_{S,S_3}$ as follows

$$\tilde{f}_{1,1} = -2^{-\frac{1}{2}} (\hat{f}_1 + i\hat{f}_2) = f_{11},$$

$$\tilde{f}_{1,0} = +\hat{f}_3 = 2^{\frac{1}{2}} f_{12},$$

$$\tilde{f}_{1,-1} = +2^{-\frac{1}{2}} (\hat{f}_1 - i\hat{f}_2) = f_{22},$$

and

$$\tilde{f}_{2,2} = +\frac{1}{2} (\hat{f}_{11} + i\hat{f}_{12} + i\hat{f}_{21} - \hat{f}_{22}) = f_{1111},$$

$$\tilde{f}_{2,1} = -\frac{1}{2} (\hat{f}_{13} + \hat{f}_{31} + i\hat{f}_{32} + i\hat{f}_{23}) = 2f_{1112},$$

$$\tilde{f}_{2,0} = +6^{-\frac{1}{2}} (2\hat{f}_{33} - \hat{f}_{11} - \hat{f}_{22}) = 6^{\frac{1}{2}} f_{1122} \text{ etc.}$$

In general we find

$$\underbrace{f_{111\dots 1}}_{(S+S_3) \text{ times}} \underbrace{222\dots 2}_{(S-S_3) \text{ times}} = \binom{2S}{S+S_3}^{-\frac{1}{2}} \tilde{f}_{S,S_3}.$$

To check our assignments and the phases we can apply the destruction operator

$$A_- = \frac{\partial}{\partial x} + x^2 \frac{\partial}{\partial \bar{x}} - \mu \bar{x}.$$

8.

3. THE VERTEX $\sigma \rightarrow \pi_1 + \pi_2$

We go to the system of reference, where the σ particle is at rest and π_1 moves in direction of the third axis. The vertex function has then the form

$$\left\{ \int K(x, y, z) f_{\sigma}(x) f_{\pi_1}(\alpha^2 y) f_{\pi_2}(\alpha^2 z) dx dy dz \right\} \\ + \text{parity transf.}$$

$$\cdot \left\{ F \cdot \text{Tr} [\sigma(\pi_1 \pi_2)_+] + G \cdot \text{Tr} [\sigma(\pi_1 \pi_2)_-] \right\},$$

where

$$\alpha^2 = \frac{M}{2m} + \left[\left(\frac{M}{2m} \right)^2 - 1 \right]^{\frac{1}{2}}.$$

Later on we shall also use the parameter "v" which represents the velocity of one of the final particles in the particular frame of reference,

$$\alpha^4 = \frac{1+v}{1-v}$$

We can skip the parity transformed integral and take only the correct part of the first integral. Bose symmetrization yields for even (in the velocity v) final states a symmetric $SU(3)$ trace, which brings in the coupling constant F, and for odd final states the antisymmetric trace with the coupling constant G. In this manner parity reflection invariance and Bose symmetrization lead to results which are in accord with the charge conjugation selection rule. The kernel $K(x, y, z)$ is for the $\sigma(1)$ ladder and for the $\sigma(2)$ ladder

$$K(x, y, z) = |x-y|^{-1} |y-z|^{-1} |z-x|^{-1},$$

$$K(x, y, z) = (x-y)^{-1} (x-z)^{-1} (\bar{y}-\bar{z})^{-1},$$

respectively. We note that for three general representations with invariants $m_x, m_y, m_z, p_x, p_y, p_z$, we have the symmetry of the kernel

$$K(x, y, z) = K(x, z, y) \cdot (-1)^{-\frac{1}{2}(m_x + m_y + m_z)}$$

The sum of the invariants m is necessarily even. In our case this formula reduces to

$$K(x, y, z) = K(x, z, y) \cdot (-1)^{k_0(\sigma)}$$

where k_0 is the commonly used notation for the lowest spin of the infinite multiplet.

The integral

$$\mathcal{J}_{\sigma\pi_1\pi_2}(\alpha^2) = \int K \cdot f_\sigma f_{\pi_1} f_{\pi_2} dx dy dz$$

is real

$$\mathcal{J}_{\sigma\pi_1\pi_2}(\alpha^2) = \overline{\mathcal{J}_{\sigma\pi_1\pi_2}(\overline{\alpha^2})}$$

and symmetric due to the symmetry of the kernel

$$\mathcal{J}_{\sigma\pi_1\pi_2}(\alpha^2) = (-1)^{k_0(\sigma)} \mathcal{J}_{\sigma\pi_2\pi_1}(\alpha^2)$$

This latter property is very important. Let us assume that $\pi_1 = \pi_2 = P$. Then it implies that for the $\sigma(1)$ ladder, $k_0 = 0$, only even final states and for the $\sigma(2)$ ladder, $k_0 = 1$, only odd final states are allowed. This consequence of the reflection symmetry of the kernel can be extended to very general representations (for instance those belonging to the supplementary series, etc.).

So we conclude that the 1^- meson octet containing the φ mesons must lie in a $\sigma(2)$ ladder, whereas the 2^+ mesons which decay mainly into two pseudoscalar mesons must lie in a $\sigma(1)$ ladder or a similar ladder starting with spin two.

The experimental situation with 2^+ mesons looks as follows ⁶⁾

$$\begin{aligned} f_0 &\rightarrow 2\pi && \sim 100\% , \\ f_0' &\rightarrow K + \bar{K} && \text{seen} , \\ A_2 &\rightarrow K + \bar{K} && \sim 6\% , \\ K^{**} &\rightarrow K + \pi && \sim 50\% . \end{aligned}$$

If we put the f_0 into a $\sigma(1)$ singlet ladder, evidence against the insertion of the remaining eight particles into the $\sigma(2)$ ladder of the 1^- octet comes from the K^{**} decay. The 1^- singlet ladder does not lead to troubles : $SU(3)$ forbids already its decay into two pseudoscalar mesons.

4. THE COUPLING CONSTANTS

We give now the results for the coupling constants of the vertices quoted in Section 2 in the form of tables (Tables 2 and 3). The functions appearing in these tables are defined as integrals for the case of the $\sigma(1)$ ladder and given explicitly for the $\sigma(2)$ ladder. The function $H(\alpha^4)$ is discussed in the Appendix. It is intimately connected with the logarithmic transcendents (Spence functions) known from quantum electrodynamics.

As an application for the $\sigma(1)$ ladder calculations we evaluate the ratio of the partial widths

$$\frac{\Gamma_{T \rightarrow P+P}}{\Gamma_{S \rightarrow P+P}} = \frac{\frac{2}{3} \left\{ \frac{V}{M} |J_{TPP}|^2 \right\} \text{ at the } T\text{-mass}}{\left\{ \frac{V}{M} |J_{SPP}|^2 \right\} \text{ at the } S\text{-mass}} = \frac{2}{3} R.$$

Values for the function R are contained in Table 4. We do not distinguish between the mass which determines the phase space and the mass on which the coupling constant depends. It could well be argued that only the kinematic mass should exhibit the symmetry breakdown.

The origin of these surprisingly small values for the ratio R can be inspected from another representation of the integrals I_{SPP} and I_{TPP} . We give this form for general even spin.

$$J_{(\text{spin})PP}(\alpha^4) = \frac{1}{2} \int_{-1}^{+1} P_{(\text{spin})}(a) \psi(a) da$$

where $P_J(a)$ is the Legendre polynomial and the positive definite and smooth function $\psi(a)$ is given as the integral

$$\psi(a) = \int_0^{2\pi} d\varphi \int dz dy \cdot \frac{1}{|y-z| \cdot (1+\alpha^4|y|^2)(1+\alpha^{-4}|z|^2)} \\ \cdot \frac{1}{|(1+a)^{\frac{1}{2}} - y e^{-i\varphi} (1-a)^{\frac{1}{2}}| | (1+a)^{\frac{1}{2}} - z e^{-i\varphi} (1-a)^{\frac{1}{2}}|}$$

The integrals in Table 2 are

$$J_{SPP}(\alpha^4) = \int \frac{dx dy dz}{|x-y||y-z||z-x|} \cdot \frac{1}{1+|x|^2} \cdot \frac{1}{1+\alpha^4|y|^2} \cdot \frac{1}{1+\alpha^{-4}|z|^2},$$

$$J_{VPP}(\alpha^4) = 0,$$

$$J_{TPP}(\alpha^4) = \int \frac{dx dy dz}{|x-y||y-z||z-x|} \cdot \frac{|x|^4 - 4|x|^2 + 1}{(1+|x|^2)^3} \cdot \frac{1}{1+\alpha^4|y|^2} \cdot \frac{1}{1+\alpha^{-4}|z|^2},$$

$$J_{SAP}(\alpha^4) = \alpha^4 \frac{d}{d\alpha^4} J_{SPD}(\alpha^4),$$

$$J_{VAP}^{(1)}(\alpha^4) = \int \frac{dx dy dz}{|x-y||y-z||z-x|} \frac{\bar{x}y + \bar{y}x}{(1+|x|^2)^2(1+\alpha^4|y|^2)^2} \frac{1}{1+\alpha^{-4}|z|^2},$$

$$J_{VAP}^{(2)}(\alpha^4) = \int \frac{dx dy dz}{|x-y||y-z||z-x|} \frac{(1-|x|^2)(1-\alpha^4|y|^2)}{(1+|x|^2)^2(1+\alpha^4|y|^2)^2} \frac{1}{1+\alpha^{-4}|z|^2},$$

$$J_{TAP}^{(1)}(\alpha^4) = \int \frac{dx dy dz}{|x-y||y-z||z-x|} \frac{(1-|x|^2)(\bar{x}y + \bar{y}x)}{(1+|x|^2)^3(1+\alpha^4|y|^2)^2} \frac{1}{1+\alpha^{-4}|z|^2},$$

$$J_{TAP}^{(2)}(\alpha^4) = \alpha^4 \frac{d}{d\alpha^4} J_{TDP}(\alpha^4).$$

We have to insert into Table 3

$$J_{VDP}(\alpha^4) = \frac{-2\pi^3}{\alpha^4 - 1} \alpha^4 H(\alpha^4),$$

$$J_{TDP}(\alpha^4) = 0,$$

$$J_{VAP}^{(1)}(\alpha^4) = + \frac{4\pi^3}{(\alpha^4 - 1)^2} \alpha^4 H(\alpha^4),$$

$$J_{VAP}^{(2)}(\alpha^4) = \alpha^4 \frac{d}{d\alpha^4} J_{VDP}(\alpha^4),$$

$$J_{TAP}^{(1)}(\alpha^4) = + \frac{4\pi^3}{(\alpha^4 - 1)^3} \alpha^4 \left[-(\alpha^4 + 1) H(\alpha^4) - 2 + \frac{4\alpha^4}{\alpha^8 - 1} \ln \alpha^4 \right],$$

$$J_{TAP}^{(2)}(\alpha^4) = + \frac{2\pi^3}{(\alpha^4 - 1)^3} \alpha^4 \left[+ 2\alpha^4 H(\alpha^4) + \alpha^4 + 1 - \frac{2\alpha^4}{\alpha^4 - 1} \ln \alpha^4 \right].$$

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A P P E N D I X

We define

$$H(t) = \int_0^{\infty} \frac{d\xi}{(1+\xi)(1+t\xi)} \left[\ln \frac{1+\xi}{t+\xi} + \frac{1}{2} \ln t \right].$$

This function satisfies a differential equation (which we need for the evaluation of some integrals)

$$H'(t) = -\frac{1}{t-1} \cdot H(t) - \frac{\ln t}{t(t+1)}$$

We define always the logarithm as

$$|\operatorname{Im} \ln t| < \pi.$$

Solving the differential equation yields another integral representation for $H(t)$:

$$H(t) = -\frac{1}{t-1} \int_1^t \frac{\xi-1}{\xi(\xi+1)} \ln \xi d\xi.$$

We introduce the Spence function

$$F(x) = \int_0^x \frac{d\xi}{\xi} \ln(1+\xi),$$

which satisfies the functional equations

$$F(x) + F\left(\frac{1}{x}\right) = \frac{1}{2} \ln^2 x + \frac{\pi^2}{6},$$

$$F(x) + F(-x) = \frac{1}{2} F(-x^2).$$

These give

$$F(1) = \frac{\pi^2}{12}, \quad F(-1) = -\frac{\pi^2}{6}.$$

In addition, we obtain a convergent series for $|x| < 1$

$$F(x) = \sum_{v=1}^{\infty} (-1)^{v+1} \frac{x^v}{v^2}.$$

We express H by F as

$$\begin{aligned} H(t) &= \frac{1}{t-1} \left[\frac{1}{2} \ln^2 t - 2 \ln t \ln(1+t) + 2F(t) - 2F(1) \right] \\ &= \frac{1}{t-1} \left[\frac{3}{2} \ln^2 t - 2 \ln t \ln(1+t) + 2F(1) - 2F\left(\frac{1}{t}\right) \right]. \end{aligned}$$

From this relation we find the asymptotic formula for $|\arg(t)| < \pi$,

$$H(t) = \frac{1}{t} \left\{ \frac{\pi^2}{6} - \frac{1}{2} \ln^2 t \right\} + O(t^{-2}), \quad (\text{A.1})$$

the convergent expansion for $|v| < 1$, $t = ((1+v)/(1-v))$

$$H(t) = -2v^2(1-v) \left[\frac{1}{3} + \frac{1}{5} \left(1 + \frac{1}{3}\right) v^2 + \frac{1}{7} \left(1 + \frac{1}{3} + \frac{1}{5}\right) v^4 + \dots \right] \quad (\text{A.2})$$

and the limiting behaviour for $\varepsilon \rightarrow 0+$

$$H(-1+i\varepsilon) = +i\pi \ln \varepsilon + O(\varepsilon). \quad (\text{A.3})$$

	S	V	T
P+P	$+ f_{\text{SPP}}$ SU(3): symmetric in (P,P)	$+ \frac{1}{2m} \cdot f_{\text{VPP}} \epsilon^\mu (q_1 - q_2)_\mu$ SU(3): antisymmetric in (P,P)	$+ \frac{1}{4m^2} \cdot f_{\text{TTP}} \eta^{\mu\nu} (q_1 - q_2)_\mu (q_1 - q_2)_\nu$ SU(3): symmetric in (P,P)
A+P	$+ \frac{1}{M} \cdot f_{\text{SAP}} \epsilon^\mu (q_1 + q_2)_\mu$ SU(3): antisymmetric in (A,P)	$+ f_{\text{VAP}}^{(1)} \cdot \epsilon^\mu (V) \epsilon^\nu (A) \delta_{\mu\nu}$ $+ \frac{1}{2mM} \cdot f_{\text{VAP}}^{(2)} \epsilon^\mu (V) \epsilon^\nu (A)$ $\cdot (q_1 - q_2)_\mu (q_1 + q_2)_\nu$ SU(3): symmetric in (A,P)	$+ \frac{1}{2m} \cdot f_{\text{TAP}}^{(1)} \eta^{\mu\nu} \epsilon^\lambda \delta_{\lambda\mu} (q_1 - q_2)_\nu$ $+ \frac{1}{4mM} \cdot f_{\text{TAP}}^{(2)} \eta^{\mu\nu} \epsilon^\lambda (q_1 - q_2)_\mu \cdot$ $\cdot (q_1 - q_2)_\nu (q_1 + q_2)_\lambda$ SU(3): antisymmetric in (A,P)

Notations: $\delta_{\text{VAP}} = \frac{M}{2m} f_{\text{VAP}}^{(1)} + \left(\frac{M^2}{4m^2} - 1 \right) f_{\text{VAP}}^{(2)}$, $\delta_{\text{TAP}} = \frac{M}{2m} f_{\text{TAP}}^{(1)} + \left(\frac{M^2}{4m^2} - 1 \right) f_{\text{TAP}}^{(2)}$.

$M^2 = (q_1 + q_2)^2$, $m^2 = q_1^2 = q_2^2$.

Table 1

	S	V	T
P + P	$f_{SPP} = + F c_0^3 I_{SPP}(\alpha^4)$	$f_{VPP} = - G c_1 c_0^2 (1 - v^2)^{1/2} \cdot$ $\cdot v^{-1} I_{VPP}(\alpha^4)$	$f_{TPP} = + F c_2 c_0^2 (1 - v^2) \cdot$ $\cdot v^{-2} I_{TPP}(\alpha^4)$
A + P	$f_{SAP} = + G c_1 c_0^2 (1 - v^2)^{1/2} \cdot$ $\cdot v^{-1} I_{SAP}(\alpha^4)$	$f_{VAP}^{(1)} = - F c_1^2 c_0 (1 + v)^{1/2} \cdot$ $\cdot (1 - v)^{-1/2} I_{VAP}^{(1)}(\alpha^4)$ $g_{VAP} = - F c_0 c_1^2 I_{VAP}^{(2)}(\alpha^4)$	$f_{TAP}^{(1)} = + G c_2 c_1 c_0 (1 + v)^{1/2} \cdot$ $\cdot v^{-1} I_{TAP}^{(1)}(\alpha^4)$ $g_{TAP} = + G c_2 c_1 c_0 (1 - v^2)^{1/2} \cdot$ $\cdot v^{-1} I_{TAP}^{(2)}(\alpha^4)$

Table 2

Coupling constants for the σ (1) ladder

	V	T
P + P	$f_{VPP} = - G c_0^2 c_1 2^{1/2} (1 - v^2)^{1/2} v^{-1} \cdot I_{VPP}(\alpha^4)$	$f_{TPP} = + F c_2 c_0^2 6^{1/2} (1 - v^2) v^{-2} \cdot I_{TPP}(\alpha^4)$
A + P	$f_{VAP}^{(1)} = + F c_1^2 c_0 2^{-1/2} (1 - v)^{-1/2} \cdot (1 + v)^{1/2} I_{VAP}^{(1)}(\alpha^4)$ $g_{VAP} = - F c_1^2 c_0 2^{1/2} I_{VAP}^{(2)}(\alpha^4)$	$f_{TAP}^{(1)} = + G c_2 c_1 c_0 2^{-1/2} (1 + v) v^{-1} \cdot I_{TAP}^{(1)}(\alpha^4)$ $g_{TAP} = + G c_2 c_1 c_0 6^{1/2} (1 - v^2)^{1/2} v^{-1} \cdot I_{TAP}^{(2)}(\alpha^4)$

Table 3

Coupling constants for the $\sigma(2)$ ladder

$\left(\frac{M}{2m}\right)_T$ $\left(\frac{M}{2m}\right)_S$	1.5	2	2.5	3	3.5	4
3	0.89	1.74	3.78	7.09	11.4	19.0
4	0.39	0.77	1.68	3.15	5.07	8.46
5	0.25	0.49	1.05	1.98	3.18	5.30
6	0.19	0.38	0.82	1.53	2.47	4.12
7	0.12	0.24	0.51	0.96	1.54	2.57
8	0.08	0.16	0.35	0.65	1.05	1.75

Table 4

Values for $10^3 \cdot R$

R E F E R E N C E S

- 1) P. Budini and C. Fronsdal, Phys.Rev.Letters 14, 968 (1965).
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J. Kirz and D.H. Miller, Phys.Rev.Letters 15, 325 (1965).